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Abstract

This paper studies how a tariff reduction influences the productive efficiency of a public firm competing against a private firm, by focusing on the public firm's incentive to make cost-reducing R&D investment. It is shown that if a tariff reduction induces a foreign firm to enter into the domestic market wherein public monopoly prevails, the public firm's productive efficiency deteriorates dramatically. However, subsequent trade liberalization steadily improves the efficiency. Furthermore, it is also shown that efficiency under trade liberalization can be higher than that under public monopoly, depending on the curvature of the public firm's reaction curve and demand curve.

JEL Classification: L13; L32; F13

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1 Introduction

Economists typically presume that public firms produce less efficiently than private firms or other types of firms, and also presume that their inefficiency is persistent. Indeed, most of empirical studies substantiate the presumption of their inefficiency.[1] The presumption of persistency, however, does not seem to be evident. Perelman and Pestieau (1994), taking the difference between the 1985-1989 and 1975-1979 efficiency scores of postal service industries in 16 developed countries, conclude that the technical efficiency is stable in most of these countries, whereas the efficiency gains can beeb seen in Japan, Luxembourg, and Switzerland. The empirical results of Zhang, Zhang and Zhao (2001) reveal that state-owned firms in China have a greater efficiency growth rate than collectively owned firms. Despite these findings, many economists continue to hold the presumption that the inefficiency of public firms is persistent, in part, arguably because of X-inefficiency.

Then, what are the factors that keep public firms' productive efficiency from remaining stable? One of them is a change in the economic environment. Empirical works point out that a removal of regulation might lead to an improvement in public firms' productive efficiency. Ahuja and Majumdar (1998) show that economic liberalization and reforms aimed at improving the performance of state-owned firms in India induced efficiency gains over time. On the other hand, however, theoretical works explain a deterioration in public firms' productive efficiency with policy reforms. Nishimori and Ogawa (2002), comparing public firms' incentives to make cost-reducing R&D investment in a public monopoly and in an oligopoly with public and private firms, show that the removal of an entry regulation discourages public firms to reduce their costs, and as a result, the public firm's productive efficiency decreases.

The purpose of this paper is to reconcile these mixed results from the empirical and theoretical

[1]For example, Fries and Taci (2005), using the data of 289 banks in 15 East European countries, show that private banks are significantly more cost efficient compared to state-owned banks. Based on a sample of about 600 public firms in China from 1980 to 1994, Zheng, Liu and Bigsten (2003) investigate their productivity performance. Their empirical results show that the average technical efficiency was low for these firms. For other empirical results of public firms' inefficiency, see Megginson and Netter (2001) who present an excellent survey of empirical studies on public firms' inefficiency and privatization.

works. To investigate a process in which public firms change their costs, we need some sorts of dynamic viewpoints. Nevertheless, it might be desirable to establish a simple, yet general, model. Consequently, we choose trade liberalization as a key explanatory factor, and analyze a relationship between a tariff rate and a public firm's incentive to make cost-reducing R&D investment under a duopoly comprising a welfare-maximizing public firm and a profit-maximizing private firm.[2] In this simple model, public monopoly prevails for prohibitively high tariffs, whereas a reduction in tariff rates would accommodate foreign firms' entry. This is consistent with Nishimori and Ogawa's removal of entry regulation. In addition, taking into account the current wave of global trade liberalization, we shall be able to use the tariff rate as a good substitutable variable of time. This conception goes along with Ahuja and Majumdar (1998).

To our knowledge, there is one paper that explores the relationship between public firms' productive efficiency and trade liberalization. Ghosh and Whalley (2008) show that trade liberalization facilitates the production of public firms through a fall in workers' shirking in public firms. Consequently, their marginal costs become large, since the public firms are assumed to have increasing marginal costs. This result is crucially dependant on the assumptions that goods produced by public firms differ from those produced by private firms and that all the goods markets are competitive. As observed frequently, however, the industries with public firms, such as oil, gas, airline, hospitals, and education, are characterized by competition with a few private firms against a backdrop of deregulation in public monopolistic markets. As such, different from Ghosh and Whalley (2008), this paper focuses on an imperfect competitive market, in particular, a duopolistic market involving one domestic public firm and one foreign private firm.

Our model has some interesting implications. Developed countries that have pursued trade liberalization (or countries whose extant tariff rates are sufficiently low) will see that a further tariff reduction makes their public firms more efficient. On the other hand, developing countries whose

[2] Such a type of market is called a *mixed oligopoly*. The analysis of mixed oligopoly dates back to Merrill and Schneider (1966). Recently, the studies on mixed oligopoly have become popular. In particular, for recent works on mixed oligopoly with R&D investment and cost differentials between public and private firms, see Matsumura and Matsushima (2004), Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), and Poyago-Theotoky, Gil-Molto, and Zikos (2011).

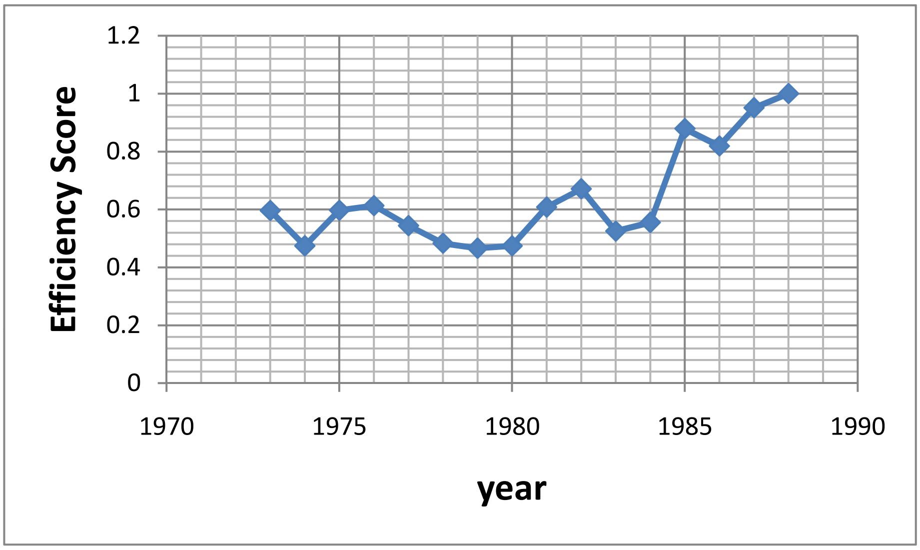

Figure 1: Efficiency of national firms in India

tariff rates are prohibitively high suffer from a drastic deterioration in public firms' productive efficiency, provided that they open their monopolistic markets to foreign firms through decreases in tariff rates. Another implication from our model is that even if developing countries experience deterioration in public firms' productivity, a large progress in trade liberalization may recuperate their productivity, and in some cases, can allow it to be higher than before the opening of the markets.

There is evidence upholding the results. Majumdar (1998), using aggregate, industry-level survey data in India for the period 1973–1974 to 1988–1989, derives national public firms' average efficiency scores on a scale of 0 to 1. Figure 1 illustrates the efficiency score changes over time. Public firms' efficiency is relatively stable from 1973 to 1976. However, it stagnates in the late 1970s and shoots up after 1980. During 1950–1975, the Indian government was an extreme protectionist. India was virtually an autarky economy in this period. After this period, it has moved toward trade liberalization partially and intermittently. It was by the 1980s that the movement toward trade liberalization accelerated. In sum, the history of tariff reductions affect public firms' efficiency in such a way that is consistent with our result.

The remainder of this paper is organized as follows. Section 2 presents our basic model and shows our main results described above. In section 3, we discuss two extensions. One is R&D competition between public and private firms. In order to elicit a pure effect of a tariff reduction on public firm's productive efficiency, we focus on the situation where it is the sole investor in section 2. However, this would not be vindicated from the viewpoint of reality. Then, we consider both public and private firms that engage in R&D competition and show that such a modification of our model leaves our results intact. The other extension is privatization of the public firm. By this extension, it is shown that the privatized firm's investment level is increasing in the tariff rate once the market is opened to the foreign firm, but it is decreasing when the tariff rate is low. This is in sharp contrast to the results from pre-privatization analysis. Finally, in section 4, we conclude this paper.

2 Model

Consider a country wherein there is one public firm that competes against one foreign private firm. The demand in this country is given by $P = P(Q)$, where P is the price, $Q = q_0 + q_1$ is the total output, q_0 is the output of the public firm, and q_1 is the output of the foreign private firm. As usual in the existing works on mixed oligopoly such as De Fraja and Delbono (1989), we assume that the public firm maximizes domestic welfare, whereas the foreign private firm maximizes its profits.

Each firm is assumed to have a constant marginal cost technology. To focus on the public firm's effort toward cost reduction, we assume that the public firm is the sole cost-reducing R&D investor. Its marginal cost is represented as $C(x_0)$, where x_0 is the level of R&D investment. For reducing the marginal cost, the public firm must incur investment cost $f(x_0)$.

The profits of the public and private firms are given by

$$\Pi_0(q_0, q_1, x_0) := [P(Q) - C(x_0)]\, q_0 - f(x_0),$$

$$\Pi_1(q_0, q_1, t) := [P(Q) - c - t]\, q_1,$$

respectively, where t is the tariff rate levied by the domestic government. The domestic welfare is given by

$$W(q_0, q_1, x_0, t) := \int_0^Q P(z)dz - P(Q)q_1 - C(x_0)q_0 - f(x_0) + tq_1.$$

For our subsequent analysis, we assume the following:

Assumption 1. *$P(Q)$ is twice continuously differentiable with $P'(Q) < 0$ for all $Q \geq 0$ such that $P(Q) > 0$. Furthermore, it satisfies*

$$\varepsilon(Q) \in [0, 1) \quad \textit{and} \quad \varepsilon'(Q) = 0, \quad \textit{where} \quad \varepsilon(Q) := \frac{QP''(Q)}{P'(Q)}.$$

Assumption 2. *$C(x_0)$ is twice continuously differentiable and satisfies $C(x_0) > 0$, $C'(x_0) < 0$, and $C''(x_0) \geq 0$ for all $x_0 \geq 0$.*

Assumption 3. *$f(x_0)$ is twice continuously differentiable and satisfies $f(x_0) \geq 0$ for all $x_0 \geq 0$ with $f(x_0) = 0$ if and only if $x_0 = 0$. Furthermore, it satisfies $f'(x_0) > 0$ and $f''(x_0) > 0$ for all $x_0 > 0$.*

For tractability, we assume that $c = 0$.[3] This and Assumption 2 ensure that the public firm cannot produce at a lower cost than the private firm, and thus, the private firm becomes active when no tariff is imposed. Assumption 3 indicates that R&D investment is subject to a decreasing return to scale. As for Assumption 1, Okuno-Fujiwara and Suzumura (1993), which show that excess entry emerges in private oligopoly with R&D investment decisions, justify this assumption by pointing out that it accommodates a wide class of normal inverse demand functions.

We consider the following two-stage game. In the first stage, the domestic public firm determines its cost-reducing R&D investment level x_0. Observing this investment level, both the domestic public and foreign private firms simultaneously select their outputs q_i ($i = 0, 1$) in the second stage. As usual, we apply backward induction to solve this game.

[3] If $\lim_{x_0 \to \infty} C_0(x_0) > c$ is added to Assumption 2 and the assumption $c = 0$ is not made, there is no change in the results obtained.

2.1 Analysis of the second stage

The first-order conditions for the public firm and the private firm are given by

$$\frac{\partial W}{\partial q_0} = P(Q) - P'(Q)q_1 - C(x_0) \leq 0, \quad q_0 \geq 0, \quad q_0 \cdot \frac{\partial W}{\partial q_0} = 0, \tag{1}$$

$$\frac{\partial \Pi_1}{\partial q_1} = P(Q) + P'(Q)q_1 - t \leq 0, \quad q_0 \geq 0, \quad q_1 \cdot \frac{\partial \Pi_1}{\partial q_1} = 0. \tag{2}$$

As can be easily seen, the second-order conditions are satisfied under Assumption 1. The first-order condition Eq.(1) states that the public firm's marginal cost exceeds the domestic consumer price. This can be attributed to an improvement in the terms-of-trade owing to its more aggressive behavior than the private firm.[4]

From the first-order conditions, we derive the reaction function of each firm: $R_0(q_1, x_0)$ and $R_1(q_0, t)$. Under Assumption 1, the private firm's reaction curve is downward-sloping and its slope is less than unity in terms of its absolute value. Thus, its strategy is strategic substitute. On the other hand, the public firm's reaction curve is upward-sloping, but its slope is less than unity. Indeed, for q_1 that satisfies $\partial W(R_0(q_1, x_0), q_1, x_0, t)/\partial q_0 = 0$,

$$\frac{\partial R_0}{\partial q_1} = \frac{P''(Q)q_1}{P'(Q) - P''(Q)q_1} = \frac{\varepsilon\theta}{1 - \varepsilon\theta} \geq 0, \quad \text{where} \quad \theta := \frac{q_1}{Q}.$$

Hence, the second-stage equilibrium is unique and stable.

Let $q_i^*(x_0, t)$ denote the second-stage Cournot Nash equilibrium ($i = 0, 1$). Note that even though the public firm conducts no investments, public monopoly would prevail when a high tariff rate is set. For brevity of our analysis, we preclude such high tariff rates as follows.

Assumption 4. *Let $\bar{t}$ denote the tariff level satisfing $q_1^*(0, \bar{t}) = 0$. Then, $t < \bar{t}$ holds.*

Unfortunately, this assumption does not fully preclude the possibility of corner solutions. A

[4]For a detailed discussion on the behaviors of public firms that confront foreign private firms, see Fjell and Pal (1996). In addition, the mixed oligopoly literature includes a wide variety of studies on foreign competition. See also Pal and White (1998), Matsushima and Matsumura (2006), Long and Stähler (2009), and Mukherjee and Suetrong (2009).

high level of investment can excel the private firm out of the market, because of $\partial R_0/\partial x_0 > 0$. On the basis of this observation, q_i^* is defined as follows:

$$q_0^*(x_0,t) = \begin{cases} q_0^e(x_0,t) & \text{if } x_0 \le \bar{x}_0(t), \\ q_0^m(x_0) & \text{otherwise,} \end{cases} \qquad q_1^*(x_0,t) = \begin{cases} q_1^e(x_0,t) & \text{if } x_0 \le \bar{x}_0(t), \\ 0 & \text{otherwise,} \end{cases}$$

where $q_i^e(x_0,t)$ is the output satisfying both $\partial W(q_0^e, q_1^e, x_0, t)/\partial q_0 = 0$ and $\partial \Pi_1(q_0^e, q_1^e, t)/\partial q_1 = 0$, $q_0^m(x_0)$ is the public firm's output under public monopoly, and $\bar{x}_0(t)$ is the least investment level satisfying $q_0 = R_0(0, x_0)$ and $0 = R_1(q_0, t)$ for any given $t \in [0, \bar{t})$. Note that $\bar{x}_0(t)$ is a decreasing function.

The Cournot equilibrium outputs have the following properties.

Lemma 1. *Define $Q^e(x_0,t) := q_0^e(x_0,t) + q_1^e(x_0,t)$ and $\theta^e(x_0,t) := q_1^e(x_0,t)/Q^e(x_0,t)$. For each x_0 and t,*

$$\frac{\partial q_0^e}{\partial x_0} = \frac{[2+\varepsilon\theta^e(x_0,t)]\,C'(x_0)}{2P'(Q^e(x_0,t))} > 0, \quad \frac{\partial q_1^e}{\partial x_0} = -\frac{[1+\varepsilon\theta^e(x_0,t)]\,C'(x_0)}{2P'(Q^e(x_0,t))} < 0,$$
$$\frac{\partial q_0^e}{\partial t} = \frac{\varepsilon\theta^e(x_0,t)}{2P'(Q^*(x_0,t))} \le 0, \quad \frac{\partial q_1^e}{\partial t} = \frac{1-\varepsilon\theta^e(x_0,t)}{2P'(Q^e(x_0,t))} < 0, \quad q_0^{m\prime}(x_0) = \frac{C'(x_0)}{P'(Q)} > 0.$$

As expected, a decrease in one firm's effective marginal cost expands its production. The public firm's investment reduces the output of the private firm through strategic substitutability. On the other hand, a tariff reduction increases the output of the public firm through strategic complementarity.

2.2 Analysis of the first stage

We now consider the first stage. The payoff function that the public firm confronts in this stage is given by $W^*(x_0,t) := W(q_0^*(x_0,t), q_1^*(x_0,t), x_0, t)$. Let us denote the level of R&D investment maximizing W^* as $x_0^*(t)$. For convenience, we define two welfare functions:

$$W^e(x_0,t) := W(q_0^e(x_0,t), q_1^e(x_0,t), x_0, t), \quad W^m(x_0) := W(q_0^m(x_0), 0, x_0, 0),$$

and two investment levels:

$$x_0^e(t) = \underset{x_0}{argmax}\ W^e(x_0,t), \quad x_0^m = \underset{x_0}{argmax}\ W^m(q_0^m(x_0), 0, x_0, 0).$$

To analyze the properties of $x_0^*(t)$, we present the following lemma.

Lemma 2. *The following four properties hold:*

(a) $x_0^{e\prime}(t) < 0$*,*

(b) there exists a unique $\tilde{t} \in [0, \bar{t})$ *such that* $\bar{x}_0(\tilde{t}) = x_0^m$*,*

(c) if $x_0^e(t) > 0$ *for any* $t \in [0, \bar{t})$*, then there exist some* $\hat{t} \in (0, \bar{t})$ *such that* $x^e(\hat{t}) = \bar{x}(\hat{t})$*,*

(d) $\tilde{t} < \hat{t}$ *if* $\hat{W}^{e\prime\prime}(t) < 0$ *where* $\hat{W}^e(t) := W(q_0^e(x_0^e(t),t), q_1^e(x_0^e(t),t), x_0^e(t), t)$*.*

Proof: See Appendix A.

Figure 2 briefly illustrates the results from this lemma. The demarcation curve $\bar{x}_0(t)\bar{t}$ is downward-sloping, whose upper side entails public monopoly and whose lower side entails mixed duopoly. The locus of x_0 that maximizes welfare W^e (i.e., curve $Ax_0^e(t)$) is also downward-sloping. The intuition behind this negative correlation is explained as follows. A tariff reduction decreases the tariff revenue and worsens the terms-of-trade. In order to compensate these welfare losses, the public firm, raising its R&D investment level, gets more profits.

As also observed from Figure 2, the curve $Ax_0^e(t)$ intersects with the curve $\bar{x}_0(t)\bar{t}$ at $\hat{t}$ and the former curve lies above the latter for $t < \hat{t}$. Therefore, an inner solution is warranted for such ts. This is because the public firm has to exert great efforts to exclude from the market the private firm having low effective marginal cost.

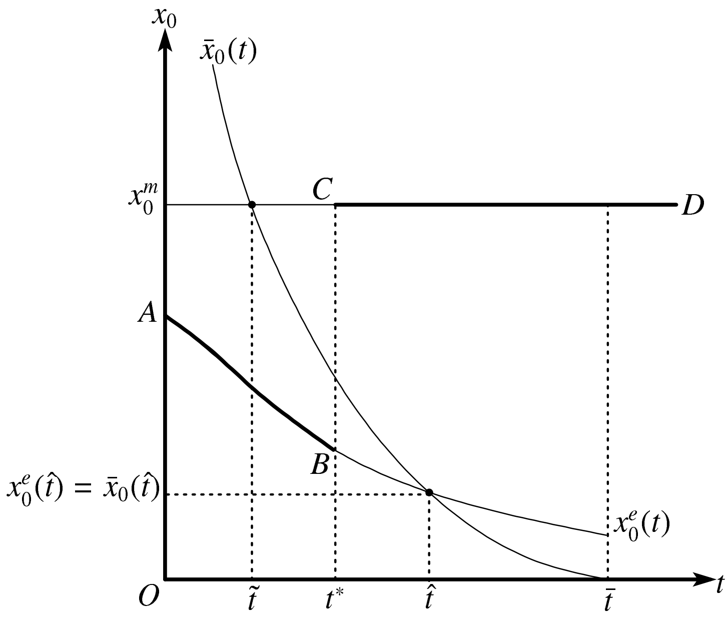

Figure 2: The locus of $x_0^*(t)$

One question might arise here: why does x_0^m differ from $x_0^e(t)$ at $\hat{t}$ at which the private firm becomes inactive in the second stage? This is closely related to the contestable market theory. For $x_0 = \bar{x}_0(t)$, the private firm is inactive, but is a potential entity in the market, and hence, its output decision necessarily affects the public firm's. As a result, the public firm select its output such that $P(q_0^e(\bar{x}_0(t), t)) = C(\bar{x}_0(t)) = t$. This implies that t is a floor of cost reduction. Thus, the public firm underinvests at $t = \hat{t}$, as compared to under public monopoly.

From Lemma 2, we have the following.

Lemma 3. *Suppose that $\hat{W}^{e\prime\prime}(t) < 0$ for any $t \in [0, \bar{t})$ and that $x_0^e(t)$ and $\bar{x}_0(t)$ have only one intersection, i.e., $x_0^{e\prime\prime}(t) < \bar{x}_0''(t)$. Then,*

(a) there exists a unique $t^* \in (\tilde{t}, \hat{t})$ such that $W^e(x_0^e(t^*), t^*) = W^m(x_0^m)$,

(b) $x_0^m > x_0^e(t^*)$.

Proof: See Appendix A.

Using the results from Lemma 3, we now have the following.

Proposition 1. *Suppose that* $\hat{W}^{e\prime\prime}(t) < 0$ *for any* $t \in [0, \bar{t})$ *and that* $x_0^{e\prime\prime}(t) < \bar{x}_0^{\prime\prime}(t)$*. Then,*

(a) $x_0^* = x_0^m$ *for* $t \geq t^*$ *and* $x_0^*(t) = x_0^e(t)$ *for* $t < t^*$*,*

(b) $x_0^{*\prime}(t) = 0$ *for* $t \geq t^*$ *and* $x_0^{*\prime}(t) < 0$ *for* $t < t^*$*.*

Proof: See Appendix A.

When a tariff rate is sufficiently high, public monopoly prevails as expected and a small tariff reduction leaves the public firm's productive efficiency intact. On the other hand, once a tariff rate becomes low, mixed duopoly emerges and a small tariff reduction increases the public firm's productive efficiency. It may be interesting that the public firm's productive efficiency drastically worsens immediately after the markets are opened to foreign firms.

In addition, we can elicit another important implication from Proposition 1. The assumption $\hat{W}''(t) < 0$ ensures a unique optimal tariff. Defining the optimal tariff as t^e, we know that this tariff rate satisfies the following equation:

$$\begin{aligned} 0 = \hat{W}^{e\prime}(t^e) &= \frac{\partial W^e}{\partial x_0} \cdot x_0^{e\prime}(t^e) + \frac{\partial W^e}{\partial t}, \\ &= \frac{\partial W}{\partial q_1} \cdot \frac{\partial q_1^e}{\partial t} + \frac{\partial W}{\partial t}, \\ &= \left[P(Q^e(x_0^e(t^e), t^e))q_1^e(x_0^e(t^e), t^e) + t^e\right] \frac{\partial q_1^e}{\partial t} + q_1^e(x_0^e(t^e), t^e). \end{aligned}$$

Solving this equation with respect to t^e, we have

$$\begin{aligned} t^e &= P'(Q^e(x_0^e(t^e), t^e))q_1^e(x_0^e(t^e), t^e) - \frac{q_1^e(x_0^e(t^e), t^e)}{\partial q_1^e/\partial t}, \\ &= -q_1^e(x_0^e(t^e), t^e)P'(Q^e(x_0^e(t^e), t^e)) \left[\frac{1 + \varepsilon\theta^e(x_0^e(t^e), t^e)}{1 - \varepsilon\theta^e(x_0^e(t^e), t^e)}\right], \\ &> 0, \end{aligned}$$

and

$$\begin{aligned}\hat{W}^{e\prime}(\hat{t}) &= [P(Q^e(x_0^e(\hat{t}),\hat{t}))q_1^e(x_0^e(\hat{t}),\hat{t}) + \hat{t}]\,\frac{\partial q_1^e}{\partial t} + q_1^e(x_0^e(\hat{t}),\hat{t}),\\ &= \hat{t}\frac{\partial q_1^e}{\partial t},\\ &< 0.\end{aligned}$$

Thus, the optimal tariff lies in the range $(0, \hat{t})$. This implies that the domestic countries can enjoy gains from trade that are generated by a small tariff reduction from $t = t^*$. However, this does not imply that the tariff levels which is lower than $\hat{t}$ warrant welfare levels higher than that in autarky. Suppose that there exists $t^a < t^*$ satisfying $W^e(x_0^e(t^a), t^a) = W^m(x_0^m)$. In this case, $W^e(x_0^e(t^a), t^a) < W^m(x_0^m)$ for $t < t^a$, and thus, the public firm selects its R&D to again entail autarky, if possible, when the tariff rate becomes sufficiently low. However, Proposition 1 ensures that there does not exist such a tariff rate. Thus, the domestic country recieves gains from trade for any $t \in [0, t^*)$, and the ensuing trade liberalization does not impede trade through the public firm's high level of investment.

2.3 Tariff reduction and cost efficiency

We have observed in Proposition 1 that a tariff reduction definitely enhances the public firm's productive efficiency when the tariff rate is sufficiently low. In this subsection, from two viewpoints, we examine how trade liberalization improves the public firm's efficiency. One viewpoint is a comparison between $x_0^*(t)$ and x_0^m. The other is a comparison between $x_0^*(t)$ and the investment level $x_0^{**}(t)$ that minimizes $F(q_0, x_0) := C(x_0)q_0 + f(x_0)$.

As for the first viewpoint, we find the following.

Proposition 2. *There exists a unique t^{**} such that $x_0^e(t^{**}) = x_0^m$, and $x_0^e(t) \geq x_0^m$ for $t \leq t^{**}$,*

$x_0^e(t) < x_0^m$ for $t > t^{**}$, provided that the following condition holds:

$$\frac{q_0^e(x_0^m, 0) - q_0^m(x_0^m)}{q_1^e(x_0^m, 0)} \geq \frac{1 + \varepsilon\theta^e(x_0^m, 0)}{2}. \tag{3}$$

Proof: See Appendix A.

Proposition 3. *Suppose that the inverse demand function is linear. Then, $x_0^m > x_0^e(t)$ for $t \in [0, \bar{t})$.*

Proof: See Appendix A.

Propositions 2 and 3 indicate that the slope of the public firm's reaction function R_0 plays an important role in determining whether trade liberalization can raise the public firm's productive efficiency to a level higher than that under public monopoly. To show this, we rearrange Eq.(3) as

$$\left[\frac{q_0^e(x_0^m, 0) - q_0^m(x_0^m)}{q_1^e(x_0^m, 0)}\right] - \frac{\varepsilon}{2}\left[\frac{1}{1 + \frac{q_0^e(x_0^m, 0)}{q_1^e(x_0^m, 0)}}\right] \geq \frac{1}{2}.$$

The first term in the left-hand side is the mean rate of change between the public monopoly equilibrium and the mixed duopoly equilibrium. The second term is a function of the ratio of q_0^e to q_1^e. For a given x_0, the first term rises and the second term lowers as the reaction curve of the public firm gets steeper. Thus, the steeper reaction curve would allow the above condition to hold. Conversely, the condition would not hold if the slope is not so steep. Indeed, the horizontal reaction curve, which is given by linear inverse demand, inverts the inequality.

The intuition behind this result is simple. In the case wherein the reaction curve of the public firm is steep, a shift from public monopoly to mixed duopoly expands its market share. In order to save on production cost that increases due to such a shift, it attempts to raise its R&D investment level.

We now conduct on the analysis from the second viewpoint. Simple computation yields the following result.

Proposition 4. *Suppose that $x_0^{**}(t)$ gives rise to $q_i^*(x_0^{**}(t), t) > 0$ for any $t \in [0, \hat{t}]$. The public firm makes an insufficient investment for $t < t^*$ and an efficient investment for $t \geq t^*$.*

Proof: See Appendix A.

Let us explain Proposition 4, which suggests that competition ensures the public firm make an insufficient investment. To this end, we decompose the welfare effects into the following four components:

$$\begin{aligned}\frac{\partial W^e}{\partial x_0} &= [t - P'(Q^e)q_1^e]\,\frac{\partial q_1}{\partial x_0} + [P(Q^e) - P'(Q^e)q_1^e - C(x_0)]\,\frac{\partial q_0}{\partial x_0} - [C'(x_0)q_1^e + f'(x_0)]\,, \\ &= t\frac{\partial q_1^e}{\partial x_0} + [P(Q^e) - C(x_0)]\frac{\partial q_0^e}{\partial x_0} - P'(Q^e)q_1^e\frac{\partial Q^e}{\partial x_0} - [C'(x_0)q_1^e + f'(x_0)]\,.\end{aligned}$$

In the right-hand side of this equation, the first term is *the tariff revenue effect*, the second is *the allocation effect*, the third is *the terms-of-trade effect*, and the fourth is *the cost efficiency effect*. The tariff revenue effect and the allocation effect are negative welfare effects. On the other hand, the terms-of-trade effect is a positive effect. However, since the welfare effect of an infinitesimal change in q_0 is negligible, the sum of the allocation effect and the terms-of-trade effect is negative. Therefore, all the three effects (the tariff revenue effect, the allocation effect, and the terms-of-trade effect), the sum of which is definitely negative, reduces the public firm's incentive to make its R&D investments.

3 Discussion

3.1 R&D competition

In the previous section, we have observed that a tariff reduction enhances the productive efficiency of the public firm when the tariff rate is sufficiently low, whereas it drastically lowers the efficiency when the tariff rate is high. However, since we obtained this result by concentrating on only the public firm's incentive to reduce costs, it does not seem to be clear as to whether or not the result

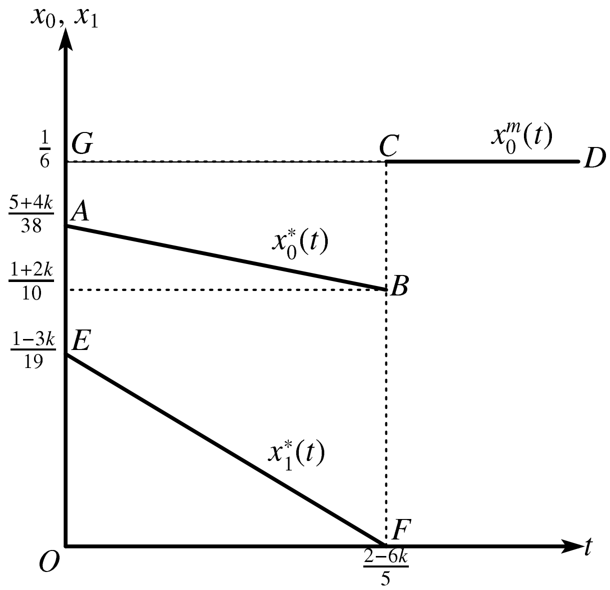

Figure 3: Locii of $x_0^*(t)$ and $x_1^*(t)$

also holds if both public and private firms engage in R&D. As pointed out in Poyago-Thetoky et al. (2011), there are many examples where both types of firms are involved in R&D investment in real mixed oligopolistic industries. An ignorance of these facts would lead to an incomplete analysis. As such, in this subsection, we attempt to examine how the introduction of the private firm's investment influences our previous results.

For this sake, we use the following specific model. The inverse demand is linear and is given by $P = 1 - Q$. The public firm's production cost is given by $C(x_0) = (\frac{1}{2} - x_0)q_0$. For ensuring Assumption 2, we assume that $x_0 < \frac{1}{2}$. Similarly, we assume that the private firm's production cost is $c(x_1) = (k - x_1)q_1$ and $k \in (\frac{1}{22}, \frac{1}{3})$. The upper bound of k precludes the possibility that the private firm makes R&D investment at a level higher than k, and the lower bound excludes the possibility that public monopoly emerges for any t.

In this setting, we find that for $t \leq \frac{2-6k}{5}$, which gives rise to mixed duopoly,[5]

$$x_0^*(t) = \frac{5 + 4k - 3}{38}, \quad x_1^*(t) = \frac{2 - 6k - 5t}{38},$$

[5]For derivation of all the results in this subsection, see Appendix B.

and for $t > \frac{2-6k}{5}$, which yields public monopoly,

$$x_0^*(t) = \frac{1}{6}, \quad x_1^*(t) = 0.$$

These results are illustrated in Figure 3. The public firm's investment level $x_0^*(t)$ is represented as the discontinuous curve $ABCD$. As observed from Figure 3, $t = t^* = \frac{2-6k}{5}$ is a threshold of mixed duopoly and public monopoly, and a jump of investment level exists at the tariff rate. At tariff rates higher than $t = t^*$, public monopoly prevails and a change in t does not affect the investment level selected by the public firm. On the other hand, mixed duopoly prevails and a decrease in the tariff rate encourages the public firm to conduct more investment at tariff rates lower than $t = t^*$. These results indicate that Propositions 1 and 3 hold even when we take into account R&D competition.

Additionally, Figure 3 also demonstrates that Proposition 4 is robust. Indeed, simple calculation reveals that $x_0^{**}(t) = \frac{1}{6}$ regardless of t, which is drawn as a horizontal line GCD. As seen in the figure, the investment level selected by the public firm under public monopoly is commensurate to $x_0^{**}(t)$, whereas that under mixed duopoly is always lower than $x_0^{**}(t)$.

The introduction of the private firm's R&D investment not only proves the robustness of our propositions in the previous section, but also adds a new insight. Figure 3 shows that for $t \leq t^*$ for which mixed duopoly emerges, a decrease in t induces the private firm to raise its investment level. Furthermore, an increment in the private firm's investment is always larger than the public firm's. In this sense, trade liberalization, of course, enhances the public firm's productive efficiency, but also has a major role in drastically improving the private firm's productive efficiency.

3.2 Privatization

In this subsection, we use a specific model to clarify the differences in the results obtained from pre-privatization and post-privatization. Following many works on privatization such as De Fraja and Delbono (1989) and Poyago-Thetoky et al. (2011), we assume that the privatized firm is

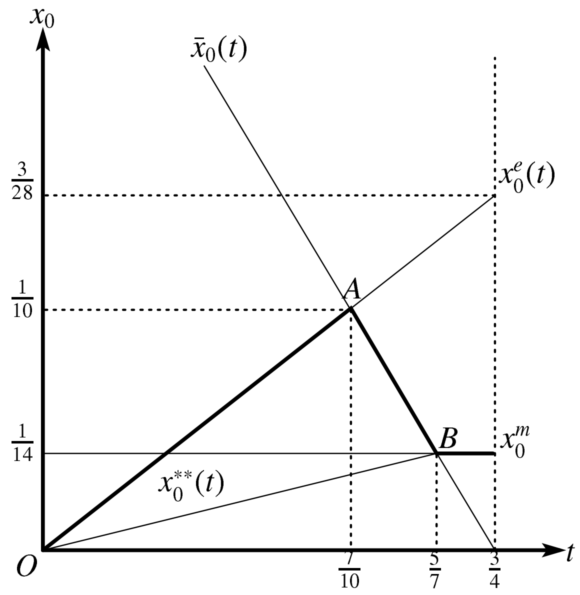

Figure 4: Locus of $x_0^*(t)$ after privatization

not a welfare-maximizer but a profit-maximizer. As in the analysis of R&D competition, we apply linear demand ($P = 1 - Q$) and assume that the privatized firm's cost is assumed to be $C(x_0)q_0 + f(x_0) = (\frac{1}{2} - x_0)q_0 + 2x_0^2$. On the other hand, the existing private firm's cost is assumed to be zero as in Section 2. By using this setting, we obtain[6]

$$\bar{x}_0(t) = \frac{3-4t}{2}, \quad x_0^m(t) = \frac{1}{14}, \quad x_0^e(t) = \frac{t}{7}, \quad x_0^{**}(t) = \begin{cases} \frac{t}{10}, & \text{if } t \leq \frac{5}{7}, \\ \frac{1}{14}, & \text{otherwise.} \end{cases}$$

Figure 4 illustrates some crucial differences between $x_0^*(t)$ under pre- and post-privatization. First, $x_0^*(t)$ after privatization is not strictly decreasing under mixed duopoly. For a higher tariff rate (i.e., $\frac{5}{7} < t \leq \frac{7}{10}$), a tariff reduction definitely increases the investment level of the public firm, but for a lower tariff rate (i.e., $t < \frac{7}{10}$), it rather decreases the investment level. This can be explained as follows. In the higher tariff case, the effective marginal cost of the private firm is high. Even though the existing private firm expands its market share due to a tariff reduction,

[6]The derivation of all the results here is relegated to Appendix B.

a small increase in investment regains the privatized firm's market share. However, in the lower tariff case wherein the effective marginal cost of the private firm is sufficiently low, the public firm faces two large negative losses from a rise in x_0. One is a loss from the large investment cost $f(x_0)$ because of the convexity of f. The other is a loss from the drastic reduction in the market price. These two additional losses dominate the additional gain from the raising market share.

Second, the investment level under mixed duopoly is excessive compared to the efficient investment level. Indeed, for any $t < \frac{5}{7}$, the schedule of $x_0^*(t)$, OAB, lies above that of $x_0^{**}(t)$, OB. The minimal cost requires less investment because the privatized firm underproduces.

4 Concluding remarks

Although the literature on mixed oligopoly has studied a broad range of issues relating to public firms' inefficiency, very little attention has been paid to the effects of trade liberalization on the public firms' productive efficiency. Usually, it is said that trade liberalization facilitates fierce competition among firms, and as a result, firms endeavor to produce its good more efficiently. Is this statement always true when we include a public firm among such firms? In this paper, we have tackled this issue by exploring a model wherein one public firm competes against one foreign private firm and makes cost-reducing R&D investment.

Our findings suggest that the investment level is constant for a sufficiently high tariff rate at which public monopoly prevails, whereas it is decreasing in the tariff rate for a low tariff rate at which mixed duopoly prevails. More interesting is that at a threshold tariff rate of mixed duopoly and public monopoly, the investment level under mixed duopoly is lower than that under public monopoly. In other words, once this tariff is reduced and the foreign firm enters the domestic market, the public firm's productive efficiency deteriorates dramatically.

Moreover, we have examined, from two viewpoints, whether such investment is effective. First, we compared the investment levels before and after the opening of the market. In a linear demand model, for any tariff rate, the investment level in a closed economy is higher than that in an open economy. However, in the case where the public firm's reaction curve in the output setting stage is very steep, the latter could outweigh the former for lower tariff rates. Second, we compared the investment level selected by the public firm and the efficient level that minimizes the sum of production cost and investment cost. We have shown that the public firm sets an investment level efficiently for a high tariff rate, but underinvestment for a low tariff rate.

Appendix A

Proof of Lemma 2

We first prove Lemma 2-(a). This proof consists of three steps.

Step 1. We start by deriving the second-order derivatives of R_0 and R_1. Differentiating the first-order condition for the private firm with respect to q_0 and arranging the equation yields

$$P'(q_0 + R_1(q_0, t)) \left[1 + 2\frac{\partial R_1}{\partial q_0} + \left(1 + \frac{\partial R_1}{\partial q_0} \right) \varepsilon\hat{\theta} \right] = 0,$$

where $\hat{\theta} = R_1(q_0, t)/(q_0 + R_1(q_0, t))$. Differentiating both sides of this equation with respect to q_0, we get

$$(2 + \varepsilon\hat{\theta})\frac{\partial^2 R_1}{\partial q_0^2} + \frac{\varepsilon}{q_0 + R_1(q_0, t)} \left(1 + \frac{\partial R_1}{\partial q_0} \right) \left[\frac{\partial R_1}{\partial q_0} - \left(1 + \frac{\partial R_1}{\partial q_0} \right) \hat{\theta} \right] = 0,$$

where we use the envelope theorem. Thus, it follows that

$$\frac{\partial^2 R_0}{\partial q_0^2} = -\frac{\dfrac{\varepsilon}{Q}\left(1+\dfrac{\partial R_1}{\partial q_0}\right)\left[\dfrac{\partial R_1}{\partial q_0} - \left(1+\dfrac{\partial R_1}{\partial q_0}\right)\hat{\theta}\right]}{2+\varepsilon\hat{\theta}},$$

$$= -\frac{\dfrac{\varepsilon}{2+\varepsilon\hat{\theta}}\left[-\dfrac{1+\varepsilon\hat{\theta}}{2+\varepsilon\hat{\theta}} - \dfrac{\hat{\theta}}{2+\varepsilon\hat{\theta}}\right]}{Q(2+\varepsilon\hat{\theta})} \qquad \text{(from the proof of Lemma 1),}$$

$$= \frac{\varepsilon\left[1+\hat{\theta}(1+\varepsilon)\right]}{Q(2+\varepsilon\hat{\theta})^3}.$$

Analogously, the cross second-order derivative of R_1 can be derived as follows:

$$\frac{\partial^2 R_1}{\partial t \partial q_0} = -\frac{\varepsilon(1-\hat{\theta})}{QP'(Q)(2+\varepsilon\hat{\theta})^3}.$$

Next, we proceed with the derivation of the second-order derivatives of R_0. Differentiating the first-order condition with respect to q_1 and rearranging it yields

$$P'(R_0(q_1, x_0) + q_1)\left[\frac{\partial R_0}{\partial q_1} - \varepsilon\tilde{\theta}\left(1+\frac{\partial R_0}{\partial q_1}\right)\right] = 0,$$

where $\tilde{\theta} := q_1/(R_0(q_1, x_0) + q_1)$. By differentiating this equation with respect to q_1 and x_0, we have

$$\frac{\partial^2 R_0}{\partial q_1^2} = \frac{\varepsilon\left[1-\tilde{\theta}(1+\varepsilon)\right]}{Q(1-\varepsilon\tilde{\theta})^3}, \qquad \frac{\partial^2 R_0}{\partial x_0 \partial q_1} = -\frac{\varepsilon\tilde{\theta}C'(x_0)}{QP'(Q)(1-\varepsilon\tilde{\theta})^3}.$$

Step 2. We calculate $\partial^2 q_i^e/\partial t \partial x_0$ $(i = 0, 1)$. By differentiating the second-stage equilibrium conditions with respect to x_0 and then with respect to t, we have

$$\begin{pmatrix} 1 & -\dfrac{\partial R_0}{\partial q_1} \\ -\dfrac{\partial R_1}{\partial q_0} & 1 \end{pmatrix}\begin{pmatrix} \dfrac{\partial^2 q_0^e}{\partial t \partial x_0} \\ \dfrac{\partial^2 q_1^e}{\partial t \partial x_0} \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix},$$

where

$$A_1 = \frac{\partial q_1^e}{\partial t}\left(\frac{\partial q_1^e}{\partial x_0}\cdot\frac{\partial^2 R_0}{\partial q_1^2}+\frac{\partial^2 R_0}{\partial q_1 \partial x_0}\right) = -\frac{\varepsilon(1-\varepsilon\theta^e)\left[-1+3\theta^e+\varepsilon(1+\varepsilon)\theta^{e2}\right]C'(x_0)}{4P'(Q^e)^2 Q^e(1-\varepsilon\theta^e)^3},$$
$$A_2 = \frac{\partial q_0^e}{\partial x_0}\left(\frac{\partial q_0^e}{\partial t}\cdot\frac{\partial^2 R_1}{\partial q_0^2}+\frac{\partial^2 R_1}{\partial t \partial q_0}\right) = \frac{\varepsilon(-1+\theta^e+\varepsilon\theta^e)C'(x_0)}{4P'(Q^e)(2+\varepsilon\theta^e)}.$$

Therefore, we have $\partial^2 q_i^e/\partial t\partial x_0 = \Delta_i^{tx}/\Delta$ $(i = 0, 1)$, where

$$\Delta := 1 - \frac{\partial R_0}{\partial q_1}\cdot\frac{\partial R_1}{\partial q_0} = \frac{2}{[2+\varepsilon\theta^*(x_0,t)]\,[1-\varepsilon\theta^*(x_0,t)]} > 0,$$
$$\Delta_0^{tx} := A_1 + \frac{\partial R_0}{\partial q_1}A_2 = -\frac{\varepsilon\left[-1+3\theta^e+2\varepsilon\theta^{e2}+\varepsilon^2(1+\varepsilon)\theta^{e3}\right]C'(x_0)}{2Q^e P'(Q^e)(2+\varepsilon\theta^e)(1-\varepsilon\theta^e)^2},$$
$$\Delta_1^{tx} := A_2 + \frac{\partial R_1}{\partial q_0}A_1 = \frac{\varepsilon\left[-1+(2+\varepsilon)\theta^e+(1-\varepsilon)\varepsilon\theta^{e2}+\varepsilon^2(1+\varepsilon)\theta^{e3}\right]C'(x_0)}{2Q^e P'(Q^e)(2+\varepsilon\theta^e)(1-\varepsilon\theta^e)^2}.$$

Step 3. Recall that $x_0^e(t)$ satisfies

$$\begin{aligned} 0 = \frac{\partial W^e(x_0^e(t),t)}{\partial x_0} &= \frac{\partial W}{\partial q_1}\cdot\frac{\partial q_1^e}{\partial x_0}+\frac{\partial W}{\partial x_0}, \\ &= \left[t - P'(Q^e(x_0^e(t),t))q_1^e(x_0^e(t),t)\right]\cdot\frac{\partial q_1^e}{\partial x_0} - \left[C'(x_0^e(t))q_0^e(x_0^e(t),t)+f'(x_0^e(t))\right]. \end{aligned} \quad (4)$$

Applying the implicit function theorem, we obtain

$$x_0^{e\prime}(t) = -\frac{\partial^2 W^e(x_0^e(t),t)/\partial t\partial x_0}{\partial^2 W^e(x_0^e(t),t)/\partial x_0^2}.$$

Along with the second-order condition, this implies that $\mathrm{sgn}(x^{*\prime}(t)) = \mathrm{sgn}(\partial^2 W^*(x_0^*(t),t)/\partial t\partial x_0)$. Simple calculation yields that $\partial^2 W^*/\partial t\partial x_0$ is decomposed into three parts as follows:

$$\frac{\partial^2 W^*}{\partial t\partial x_0} = \left[1 - P'(Q^*)\frac{\partial q_1^*}{\partial t} - q_1^* P''(Q^*)\frac{\partial Q^*}{\partial t}\right]\frac{\partial q_1^*}{\partial x_0} + \left[t - P'(Q^*)q_1^*\right]\frac{\partial^2 q_1^*}{\partial t\partial x_0} - C'(x_0)\frac{\partial q_0^*}{\partial t}. \quad (5)$$

From Lemma 1, the first term in the right-hand side can be reduced to

$$\left[1 - P'(Q^e)\frac{\partial q_1^e}{\partial t} - q_1^e P''(Q^e)\frac{\partial Q^e}{\partial t}\right]\frac{\partial q_1^e}{\partial x_0} = \frac{1}{2}\cdot\frac{\partial q_1^e}{\partial x_0}.$$

Suppose that $\partial^2 q_1^e/\partial t\partial x_0 < 0$. Then, Eq.(5) can be rewritten as

$$\begin{aligned}\frac{\partial^2 W^e}{\partial t\partial x_0} &= \frac{1}{2}\cdot\frac{\partial q_1^e}{\partial x_0} + [t - P'(Q^e)q_1^e]\frac{\partial^2 q_1^e}{\partial t\partial x_0} - C'(x_0)\frac{\partial q_0^e}{\partial t},\\ &= [t - P'(Q^e)q_1^e]\frac{\partial^2 q_1^e}{\partial t\partial x_0} - \frac{1}{2}\left[\frac{(1+\varepsilon\theta^e)C'(x_0)}{2P'(Q^e)}\right] - \frac{\varepsilon\theta^e C'(x_0)}{2P'(Q^e)},\\ &= [t - P'(Q^e)q_1^e]\frac{\partial^2 q_1^e}{\partial t\partial x_0} - \frac{(1+3\varepsilon\theta^e)C'(x_0)}{4P'(Q^e)}.\end{aligned}$$

Therefore, we have $x^{e\prime}(t) < 0$. Next, on the contrary, suppose that $\partial^2 q_1^e/\partial t\partial x_0 \geq 0$. From Eq.(4) and Lemma 1, Eq.(5) as evaluated at $x_0 = x_0^e(t)$ can be rearranged as follows:

$$\begin{aligned}\frac{\partial^2 W^e(x_0^e(t), t)}{\partial t\partial x_0} &= \frac{1}{2}\cdot\frac{\partial q_1^e}{\partial x_0} - C'(x_0^e)\frac{\partial q_0^e}{\partial t} + \frac{C'(x_0^e)q_0^e + f'(x_0^e)}{\partial q_1^e/\partial x_0}\cdot\frac{\partial^2 q_1^e}{\partial t\partial x_0},\\ &= -\frac{(1+\varepsilon\theta^e)C'(x_0^e)}{4P'(Q^e)} - \frac{(1-\varepsilon\theta^e)C'(x_0^e)}{2P'(Q^e)} - \frac{2P'(Q^e)}{(1+\varepsilon\theta^e)C'(x_0^e)}\cdot\frac{\partial^2 q_1^e}{\partial t\partial x_0}\left[C'(x_0^e)q_0^e + f'(x_0^e)\right].\end{aligned}$$

Furthermore, from the result of Step 2 ($\partial^2 q_1^e/\partial t\partial x_0 = \Delta_1^{tx}/\Delta$), this can be rewritten as

$$\frac{\partial^2 W^e(x_0^e(t), t)}{\partial t\partial x_0} = -\frac{C'(x_0^e)B}{4P'(Q^e)(1+\varepsilon\theta^e)(1-\varepsilon\theta^e)} - \frac{2P'(Q^e)f'(x_0^e)}{(1+\varepsilon\theta^e)C'(x_0^e)}\cdot\frac{\partial^2 q_1^e}{\partial t\partial x_0}, \tag{6}$$

where

$$\begin{aligned}B &= (1+\varepsilon\theta^e)(1-\varepsilon\theta^e)(3-\varepsilon\theta^e) + 2(1-\theta^e)\varepsilon\left[-1 + (2+\varepsilon)\theta^e + (1-\varepsilon)\varepsilon\theta^{e2} + \varepsilon^2(1+\varepsilon)\theta^{e3}\right],\\ &> (1-\theta^e)(3-\varepsilon\theta^e-2\varepsilon) + 2(1-\theta^e)\varepsilon\left[(2+\varepsilon)\theta^e + (1-\varepsilon)\varepsilon\theta^{e2} + \varepsilon^2(1+\varepsilon)\theta^{e3}\right],\\ &= (1-\theta^e)\left[3 - 2\varepsilon(1-\theta^e) + 2\varepsilon^4\theta^e + 2\varepsilon^2\theta^e(1+\theta^e) - 2\varepsilon^3\theta^{e2}(1-\theta^{e2})\right],\\ &> 0.\end{aligned}$$

Since both the first and second terms in the right-hand side of Eq.(6) are negative, it follows that $x^{e\prime}(t) < 0$.

We now prove Lemma 2-(b). From the definitions of $\bar{t}$ and $\bar{x}_0(t)$, we obtain $\bar{x}_0(\bar{t}) = 0$ and $\lim_{t\to 0} \bar{x}_0(t) = \infty$. Since $x_0^m > 0$ holds and $\bar{x}_0$ is continuous and monotone, there exists a unique $t \in [0, \bar{t})$ such that $x_0^m = \bar{x}_0(t)$.

We now consider the proof of Lemma 2-(c). It follows from the continuity of x_0^e and $\bar{x}_0$, $x_0^e(t) > 0$, $\bar{x}_0(\bar{t}) = 0$, and $\lim_{t\to 0} \bar{x}_0(t) = \infty$ that there exists some $t \in [0, \bar{t})$ satisfying $x_0^e(t) = \bar{x}_0(t)$.

Finally, we consier the proof of Lemma 2-(d). Suppose that $\hat{t} \leq \tilde{t}$. From the definitions of $\hat{t}$ and $\tilde{t}$ and from Lemma 2-(a), $x_0^e(\hat{t}) = \bar{x}_0(\hat{t}) \geq \bar{x}_0(\tilde{t}) = x_0^m$. For the proof, we define $\Delta W(t) := W(q_0^e(x_0^e(t), t), q_1^e(x_0^e(t), t), x_0^e(t), t) - W(q_0^m(x_0^m), 0, x_0^m, 0)$ and we first consider $\hat{t} < \tilde{t}$. Evidently, $\Delta W'(t) = \hat{W}^{e\prime}(t)$. For $\hat{t}$ and $\tilde{t}$, $\Delta W(t)$ satisfies

$$\begin{aligned}
\Delta W(\hat{t}) &= W(q_0^e(x_0^e(\hat{t}), \hat{t}), q_1^e(x_0^e(\hat{t}), \hat{t}), x_0^e(\hat{t}), \hat{t}) - W(q_0^m(x_0^m), 0, x_0^m, 0), \\
&= W(q_0^e(\bar{x}_0(\hat{t}), \hat{t}), 0, \bar{x}_0(\hat{t}), \hat{t}) - W(q_0^m(x_0^m), 0, x_0^m, 0) \quad \text{(by the definition of } \hat{t} \text{ and } \bar{x}_0(t)), \\
&= W(q_0^e(\bar{x}_0(\hat{t}), \hat{t}), 0, \bar{x}_0(\hat{t}), 0) - W(q_0^m(x_0^m), 0, x_0^m, 0), \\
&= W(q_0^m(\bar{x}_0(\hat{t})), 0, \bar{x}_0(\hat{t}), 0) - W(q_0^m(x_0^m), 0, x_0^m, 0), \\
&< 0 \quad \text{(by the definition of } x_0^m),
\end{aligned}$$

and

$$\begin{aligned}
\Delta W(\tilde{t}) &= W(q_0^e(x_0^e(\tilde{t}), \tilde{t}), q_1^e(x_0^e(\tilde{t}), \tilde{t}), x_0^e(\tilde{t}), \tilde{t}) - W(q_0^m(x_0^m), 0, x_0^m, 0), \\
&> W(q_0^e(\bar{x}_0(\tilde{t}), \tilde{t}), q_1^e(\bar{x}_0(\tilde{t}), \tilde{t}), \bar{x}_0(\tilde{t}), \tilde{t}) - W(q_0^m(x_0^m), 0, x_0^m, 0) \quad \text{(by the definition of } x_0^e(t)), \\
&= W(q_0^e(\bar{x}_0(\tilde{t}), \tilde{t}), 0, \bar{x}_0(\tilde{t}), 0) - W(q_0^m(x_0^m), 0, x_0^m, 0), \\
&= W(q_0^m(\bar{x}_0(\tilde{t})), 0, \bar{x}_0(\tilde{t}), 0) - W(q_0^m(x_0^m), 0, x_0^m, 0), \\
&= 0 \quad \text{(by the definition of } \tilde{t}).
\end{aligned}$$

Thus, we have $\Delta W(\hat{t}) < \Delta W(\tilde{t})$. Differentiating ΔW, we also have

$$\begin{aligned}
\Delta W'(\hat{t}) &= \hat{W}^{e\prime}(\hat{t}), \\
&= [\hat{t} - P'(Q^e(x_0^e(\hat{t}), \hat{t}))q_1^e(x_0^e(\hat{t}), \hat{t})] \left.\frac{\partial q_1^e}{\partial t}\right|_{t=\hat{t}} + q_1^e(x_0^e(\hat{t}), \hat{t}), \\
&= [\hat{t} - P'(Q^e(\bar{x}_0(\hat{t}), \hat{t}))q_1^e(\bar{x}_0(\hat{t}), \hat{t})] \left.\frac{\partial q_1^e}{\partial t}\right|_{t=\hat{t}} + q_1^e(\bar{x}_0(\hat{t}), \hat{t}), \\
&= \hat{t} \cdot \left.\frac{\partial q_1^e}{\partial t}\right|_{t=\hat{t}}, \\
&< 0.
\end{aligned}$$

Since $\hat{W}^{e\prime\prime}(t) < 0$, we get $\hat{W}^{e\prime}(t) < 0$ for any $t \in [\hat{t}, \tilde{t}]$. This implies that $\Delta W(\tilde{t}) < \Delta W(\hat{t})$, which is a contradiction. Next, we consider $\tilde{t} = \hat{t}$. In this case, since $x_0^e(\hat{t}) = \bar{x}_0(\tilde{t}) = x_0^m$, we have $q_0^e(x_0^e(\hat{t}), \hat{t}) = q_0^e(x_0^m, \hat{t})$ and $q_1^e(x_0^e(\hat{t}), \hat{t}) = q_1^e(x_0^m, \hat{t}) = 0$. For $t = \hat{t}$,

$$\begin{aligned}
\left.\frac{\partial W^e}{\partial x_0}\right|_{x_0 = x_0^m} &= [\hat{t} - P(Q^e(x_0^m, \hat{t}))q_1^e(x_0^m, \hat{t})] \left.\frac{\partial q_1^e}{\partial x_0}\right|_{x_0 = x_0^m} - [C'(x_0^m)q_0^e(x_0^m, \hat{t}) + f'(x_0^m)], \\
&= \hat{t} \cdot \left.\frac{\partial q_1^e}{\partial x_0}\right|_{x_0 = x_0^m} \quad \text{(by the definition of } x_0^m\text{)}, \\
&< 0.
\end{aligned}$$

Thus, we obtain $x_0^m > x_0^e(\hat{t})$, which is a contradiction. ■

Proof of Lemma 3

First, we prove Lemma 3-(a). From the proof of Lemma 1, we have $\Delta W(\tilde{t}) > 0$ and $\Delta W(\hat{t}) < 0$. Since $\Delta W(t)$ is continuous, there exists $t^* \in (\tilde{t}, \hat{t})$. Suppose that $\Delta W'(\tilde{t}) \leq 0$. In this case, $\Delta W(t)$ is monotonically decreasing since $\Delta W''(t) < 0$, and thus, t^* is unique. On the contrary, suppose that $\Delta W'(\tilde{t}) > 0$. Then, there exists $t = t_{max} \in (\tilde{t}, \hat{t})$ such that $\Delta W'(t) = 0$. Evidently, $\Delta W(t) > 0$ for $t \in [\tilde{t}, t_{max}]$ and $\Delta W'(t) < 0$ for $t \in (t_{max}, \hat{t}]$. Thus, there exists a unique $t^* \in (t_{max}, \hat{t})$.

We now prove Lemma 3-(b). From $x_0^e(\hat{t}) = \bar{x}_0(\hat{t})$, the single crossing property, and $x_0^{e\prime}(t) < 0$, we have $x_0^e(t^*) < x_0^e(\tilde{t}) < \bar{x}_0(\tilde{t}) = x_0^m$. ■

Proof of Proposition 1

We first prove Proposition 1-(a). Since $x_0^e(t)$ and $\bar{x}_0(t)$ have a single crossing property and $x_0^e(0) < \lim_{t\to 0} \bar{x}_0(t)$ holds, $x_0^e(t) < \bar{x}_0(t)$ for $t \leq t^*$. Thus, for this range of t, $x_0^e(t)$ yields an inner solution in the second stage, i.e., $q_i^e(x_0^e(t), t) > 0$ $(i = 0, 1)$. Then, we define the following function:

$$\Omega(\lambda) = W(\lambda q_0^e(x_0^m, 0) + (1-\lambda) q_0^m(x_0^m), \lambda q_1^e(x_0^m, 0), x_0^m, 0).$$

Differentiating this yields

$$\begin{aligned}
\Omega'(\lambda) &= \frac{\partial W}{\partial q_0}(q_0^e(x_0^m, 0) - q_0^m(x_0)) + \frac{\partial W}{\partial q_1} \cdot q_1^e(x_0^m, 0), \\
&= \frac{\partial W}{\partial q_1} \cdot q_1^e(x_0^m, 0) \quad \text{(by the envelope theorem)}, \\
&> 0 \quad (\because\ \partial W/\partial q_1 > 0).
\end{aligned}$$

From this monotonically increasing property, we get

$$\begin{aligned}
\Delta W(0) &= W(q_0^e(x_0^m, 0), q_1^e(x_0^m, 0), x_0^m, 0) - W(q_0^m(x_0^m), 0, x_0^m, 0), \\
&= \Omega(1) - \Omega(0), \\
&> 0.
\end{aligned}$$

Therefore, we have $\Delta W(t) \geq 0$ for $t \in [0, t^*]$ and $\Delta W(t) < 0$ for $t > t^*$, from $\Delta W(0) > 0$, $\Delta W(t^*) = 0$, and the concavity of $\Delta W(t)$. Proposition 1-(b) can be shown from Lemma 2 and Proposition 1-(a). ■

Proof of Proposition 2

For $t = 0$,

$$\begin{aligned}\left.\frac{\partial W^e}{\partial x_0}\right|_{x_0=x_0^m} &= -P'(Q^e(x_0^m,0))q_1^e(x_0^m,0)\left.\frac{\partial q_1^e}{\partial x_0}\right|_{x_0=x_0^m} - \left[C'(x_0^m)q_0^e(x_0^m,0)+f'(x_0^m)\right],\\ &= -P'(Q^e(x_0^m,0))q_1^e(x_0^m,0)\left.\frac{\partial q_1^e}{\partial x_0}\right|_{x_0=x_0^m} - C'(x_0^m)\left[q_0^e(x_0^m,0)-q_0^m(x_0^m)\right],\\ &= -C'(x_0^m)\left[-\frac{1+\varepsilon\theta^e(x_0^m,0)}{2}q_1^e(x_0^m,0)+q_0^e(x_0^m,0)-q_0^m(x_0^m)\right].\end{aligned}$$

Thus, if Eq.(3) holds, we have $\partial W^e(x_0^m,0)/\partial x_0 \geq 0$, and thus, $x_0^m \leq x_0^e(0)$. Since $x_0^e(t)$ is a decreasing function, there exists a unique t^{**} such that $x_0^e(t^{**}) = x_0^m$, and $x_0^e(t) \geq x_0^m$ for $t \leq t^{**}$ and $x_0^e(t) < x_0^m$ for $t > t^{**}$. ■

Proof of Proposition 3

From the proof of Proposition 2 and the linearity of inverse demand, we get

$$\left.\frac{\partial W^e}{\partial x_0}\right|_{x_0=x_0^m} = -C'(x_0^m)\left[-\frac{1}{2}q_1^e(x_0^m,0)+q_0^e(x_0^m,0)-q_0^m(x_0^m)\right].$$

Recall that the reaction curve of the public firm is horizontal if demand is linear. Thus, $q_0^e(x_0^m,0) = q_0^m(x_0^m)$ is obtained, which leads to $\partial W^e(x_0^m,0)/\partial x_0 < 0$. Accordingly, we have $x_0^m > x_0^e(0)$. Since $x_0^e(t)$ is a decreasing function, we also have $x_0^m > x_0^e(t)$ for $t \in [0,\bar{t})$. ■

Proof of Proposition 4

Note that $x_0^{**}(t)$ is derived from $\partial F(q_0^*(x_0^{**}(t), t), x_0^{**}(t))/\partial x_0 = C'(x_0^{**}(t))q_0^*(x_0^{**}(t), t) + f'(x_0^{**}(t)) = 0$. Then, for $t < t^*$,

$$\begin{aligned}\left.\frac{\partial W^*}{\partial x_0}\right|_{x_0 = x_0^{**}(t)} &= [t - P'(Q^e(x_0^{**}(t), t))q_1^e(x_0^{**}(t), t)] \left.\frac{\partial q_1^e}{\partial x_0}\right|_{x_0 = x_0^{**}(t)} + C'(x_0^{**}(t))q_0^e(x_0^{**}(t), t) + f'(x_0^{**}(t)),\\ &= [t - P'(Q^e(x_0^{**}(t), t))q_1^e(x_0^{**}(t), t)] \left.\frac{\partial q_1^e}{\partial x_0}\right|_{x_0 = x_0^{**}(t)},\\ &< 0.\end{aligned}$$

Thus, from the second-order condition, we obtain $x_0^{**}(t) > x_0^*(t)$ for $t \in [0, t^*]$. For $t \geq t^*$, public monopoly emerges and the public firm selects $x_0 = x_0^m$. From the definition of x_0^m, the investment level $x_0^*(t) = x_0^m$ is efficient. ■

Appendix B

Derivation of $x_0^*(t)$, $x_1^*(t)$, and the efficient investment level schedule

We first derive $x_0^*(t)$ and $x_1^*(t)$. From the first-order conditions in the second stage, we have

$$q_0^*(x_0) = \frac{1}{2} + x_0, \qquad q_1^*(x_1, t) = \begin{cases} \frac{1-2k-2t-2x_0+2x_1}{4}, & \text{if } x_1 \geq x_0 + t + k - \frac{1}{2},\\ 0, & \text{otherwise.}\end{cases}$$

Using these equilibrium outputs, both firms' payoffs in the first stage are given by

$$W^*(x_0, x_1, t) = \begin{cases} W^e(x_0, x_1, t) = \frac{5+4k^2-12t^2+12x_0-44x_0^2+4x_1-8x_0x_1+4x_1^2-4k(1+2t-2x_0+2x_1)+t(4-8x_0+8x_1)}{32}, \\ \quad \text{if } x_1 \geq x_0 + t + k - \frac{1}{2}, \\ W^m(x_0) = -\frac{1}{8}(-1 + 2x_0)(1 + 6x_0), \quad \text{otherwise,} \end{cases}$$

$$\Pi_1^*(x_0, x_1, t) = \begin{cases} \Pi_1^e(x_0, x_1, t) = \frac{1+4k^2+4t^2-4x_0+4x_0^2+t(-4+8x_0-8x_1)+k(-4+8t+8x_0-8x_1)+4x_1-8x_0x_1-28x_1^2}{16}, \\ \quad \text{if } x_1 \geq x_0 + t + k - \frac{1}{2}, \\ \Pi_1^m(x_1) = -f(x_1), \quad \text{otherwise.} \end{cases}$$

These functions satisfy the second-order conditions.

On the basis of the above results, we consider the first stage. First, let us derive the reaction curve of the public firm. To this end, we compare the welfare levels of public monopoly and mixed duopoly:

$$W^e(x_0, x_1, t) - W^m(x_0) = \frac{(-1 + 2k - 6t + 2x_0 - 2x_1)(-1 + 2k + 2t + 2x_0 - 2x_1)}{32}.$$

Thus, we can divide the (x_0, x_1) plane into three regions;

(i) the region wherein $W^e \geq W^m$ holds and mixed duopoly prevails, i.e., $A = \{\, (x_0, x_1) \mid x_1 \geq x_0 + t + k - \frac{1}{2}\}$,

(ii) the region wherein $W^e < W^m$ holds and public monopoly prevails, i.e., $B = \{\, (x_0, x_1) \mid x_1 < x_0 + t + k - \frac{1}{2}$ and $x_1 > x_0 - 3t - 1 + 2k\}$, and

(iii) the region wherein $W^e \geq W^m$ holds and public monopoly prevails, i.e., $C = \{\, (x_0, x_1) \mid x_1 \leq x_0 - 3t - 1 + 2k\}$.

Depending on how $\tilde{R}_0^x(x_1, t) = \underset{x_0}{\text{argmax}}\ W^e(x_0, x_1, t)$ and $\hat{R}_0^x = \underset{x_0}{\text{argmax}}\ W^m(x_0)$ go through in regions A, B, and C, the curvature of the public firm's reaction curve changes. Noting that

$\partial W^e/\partial x_0 = 0 \Leftrightarrow x_0 = \frac{3+2k-2t-2x_1}{22}$ and $\partial W^m/\partial x_0 = 0 \Leftrightarrow x_0 = \frac{1}{6}$, we get the public firm's reaction function $R_0^x(x_1, t)$ as follows:

(a) for $t < \frac{2-6k}{5}$,

$$x_0 = R_0^x(x_1, t) = \begin{cases} \frac{3+2k-2t-2x_1}{22}, & \text{if } x_1 \leq \frac{3+2k-2t}{2}, \\ 0, & \text{otherwise,} \end{cases}$$

(b) for $\frac{2-6k}{5} \leq t < 1$,

$$x_0 = R_0^x(x_1, t) = \begin{cases} \frac{1}{6}, & \text{if } x_1 \leq \frac{-2+6k+5t}{6}, \\ \frac{3+2k-2t-2x_1}{22}, & \text{if } \frac{-2+6k+5t}{6} < x_1 \leq \frac{3+2k-2t}{2}, \\ 0, & \text{otherwise,} \end{cases}$$

(c) for $t \geq 1$,

$$x_0 = R_0^x(x_1, t) = \begin{cases} \frac{1}{6}, & \text{if } x_1 \leq t + k - \frac{1}{2}, \\ 0, & \text{otherwise,} \end{cases}$$

By the same procedure, noting that $\partial \Pi_1^e/\partial x_1 = 0 \Leftrightarrow x_1 = \frac{1-2k-2t-2x_0}{14}$ and $\partial \Pi_1^m/\partial x_1 < 0$, we also have the private firm's reaction function as follows:

(a) for $t \leq \frac{1}{2} - k$,

$$x_1 = R_1^x(x_0, t) = \begin{cases} \frac{1-2k-2t-2x_0}{14}, & \text{if } x_0 \leq \frac{1}{2} - k - t, \\ 0, & \text{otherwise,} \end{cases}$$

(b) for $t > \frac{1}{2} - k$, $x_0 = R_1^x(x_0, t) = 0$ for any $x_0 \geq 0$.

We thus obtain the equilibrium investment levels $x_i^*(t)$ given in the main text.

We now derive the efficient investment level $x_0^{**}(t)$. Evaluating $C'(x_0)q_0 + f'(x_0)$ at $q_0 = q_0^*(x_0, t)$, we have

$$C'(x_0)q_0^*(x_0,t) + f'(x_0) = \frac{-1+6x_0}{3} = 0 \iff x_0^{**}(t) = \frac{1}{6}.$$

Derivation of $x_0^*(t)$ and the efficient investment level schedule after privatization

We first derive $x_0^*(t)$. From the first-order conditions in the second stage, we have

$$q_0^*(x_0,t) = \begin{cases} \frac{t+2x_0}{3}, & \text{if } x_0 \le \frac{3-4t}{2}, \\ \frac{1+2x_0}{4}, & \text{otherwise,} \end{cases} \qquad q_1^*(x_0,t) = \begin{cases} \frac{3-4t-2x_0}{6}, & \text{if } x_0 \le \frac{3-4t}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

The payoff function that the privatized firm confronts in the first stage is given by

$$\Pi_0^*(x_0,t) = \begin{cases} \Pi_0^e(x_0,t) = \frac{t^2+4tx_0-14x_0^2}{9}, & \text{if } x_0 \le \frac{3-4t}{2}, \\ \Pi_0^m(x_0) = \frac{1+4x_0-28x_0^2}{16}, & \text{otherwise.} \end{cases}$$

For convenience, we derive $x_0^e(t) = argmax_{x_0}\Pi_0^e(x_0,t)$ and $x_0^m = argmax_{x_0}\Pi_0^m(x_0)$ as follows:

$$\frac{\partial \Pi_0^e}{\partial x_0} = 0 \iff x_0^e(t) = \frac{t}{7},$$
$$\frac{d\Pi_0^m}{dx_0} = 0 \iff x_0^m = \frac{1}{14},$$

where the second-order conditions are satisfied. Note that $\Pi_0^e(x_0,t) \gtreqless \Pi_0^m(x_0) \Leftrightarrow x_0 \gtreqless \frac{3-4t}{2}$, $x_0^e(t) \gtreqless x_0^m \Leftrightarrow t \gtreqless \frac{1}{2}$, $x_0^e(t) \gtreqless \frac{3-4t}{2} \Leftrightarrow t \gtreqless \frac{7}{10}$, and $x_0^m \gtreqless \frac{3-4t}{2} \Leftrightarrow t \gtreqless \frac{5}{7}$. Therefore, we obtain the privatized firm's optimal investment level $x_0^*(t)$ as follows:

$$x_0^*(t) = \begin{cases} \frac{1}{14}, & \text{if } t \geq \frac{5}{7}, \\ \frac{3-4t}{2}, & \text{if } \frac{7}{10} \leq t < \frac{5}{7}, \\ \frac{t}{7}, & \text{otherwise.} \end{cases}$$

We focus on the derivation of $x_0^{**}(t)$. Evaluating $C'(x_0)q_0 + f'(x_0)$ at $q_0 = q_0^*(x_0, t)$, we have

$$C'(x_0)q_0^e(x_0, t) + f'(x_0) = \frac{-t + 10x_0}{3} = 0 \iff x_0^{**}(t) = \frac{t}{10}, \quad \text{for } t \leq \frac{5}{7},$$
$$C'(x_0)q_0^m(x_0) + f'(x_0) = \frac{-1 + 14x_0}{4} = 0 \iff x_0^{**}(t) = \frac{1}{14}, \quad \text{for } t > \frac{5}{7}.$$

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