

**Chukyo University Institute of Economics**

**Discussion Paper Series**

**February 2026**

**No. 2504**

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R&D-based Growth Model with Automation**

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# Effects of Industrial Regulation Policies in an R&D-based Growth Model with Automation

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February 27, 2026

## Abstract

To examine how regulatory policies affect automation and inequality, we develop an endogenous growth model with two household types (workers and capitalists) and two R&D activities (innovation and automation). The two R&D activities generate automated and nonautomated industries. The model features the unique balanced growth path (BGP). We consider two types of regulation for both industries: one that operates through prices and another that operates through production costs. The results show that tighter regulation in automated industries and more relaxed regulation in non-automated industries reduce inequality between workers and capitalists and improve the social welfare. The price-based regulation is more effective than the cost-based regulation in terms of the magnitude of the welfare improvement.

*Keywords:* Automation; Inequality; Price regulation; Schumpeterian growth

*JEL Classification:* E20; O31; O38

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# 1 Introduction

With advances in artificial intelligence (AI), it is now used across a wide range of activities. As the use of AI raises various concerns, governments are considering regulations. In practice, the European Union (EU) has introduced legislation on AI: the EU AI Act, to ensure a high level of protection of fundamental rights. The use of AI in production can be viewed as the latest form of automation that replaces labor with capital. As shown by Acemoglu and Restrepo (2020), automation has proceeded rapidly, and this trend may widen inequality between workers and capitalists. Although practical AI regulations such as the EU AI Act are not designed to address economic outcomes such as inequality, they can affect the economy by restricting firm behavior in automated industries that use AI in production. Therefore, AI restriction can be interpreted as a regulation for automated industries. To address inequality, the government can also target regulation toward specific industries. This paper examines the effects of regulation targeted at specified industries.

To investigate the effects of regulation policies, we formulate the Schumpeterian growth model with automation as in Chu et al. (2023) and Maeda et al. (2025). In the model, there are two types of households: workers and capitalists, and two types of industries: nonautomated and automated industries. Firms in nonautomated industries use only labor for production, whereas firms in automated industries use capital only. As regulatory policies for specified industries, we consider price regulation and production subsidies in both nonautomated and automated industries, as the former restricts firm behavior and the latter affects production costs. The former corresponds to price-based regulation and the latter corresponds to cost-based regulation. In the model, price regulation is characterized as an exogenous upper bound on the markup ratio determined by the government. A lower (higher) upper bound on the markup ratio, and a reduction (increase) in subsidies for a specific type of industry, represent

strengthening (relaxing) the regulation for these industries.

We show the existence and uniqueness of the balanced growth path (BGP) analytically, and examine the effects of the policies on the BGP analytically and numerically. Furthermore, we numerically demonstrate that both relaxing regulation for nonautomated industries and strengthening regulation for automated industries reduce inequality with welfare improvement. The price-based regulation is more effective than the cost-based regulation because the former has the greater impact on the welfare improvement than the latter. In addition, we examine the effects of uniform changes in policies.

This paper contributes to the literature on price regulation and economic growth. Evans et al. (2003) and Zeng et al. (2014) examine the effects of price regulation on growth and welfare by extending Barro and Sala-i-Martin's (1995) model. Sorek (2021) considers industrial policies, patent breadth, price controls, and production subsidies within Young's (1998) model. However, none of those papers consider automation and examine inequality.

Patent breadth has a similar effect to price regulation in our model, as both policies restrict the upper bound of the markup ratio.<sup>1</sup> Therefore, our paper also contributes to the literature on patent breadth and inequality in economic growth models. Because our model is a Schumpeterian growth model, the following three papers are closely related to ours. Chu (2010) and Chu and Cozzi (2018) show that a higher level of patent breadth positively affects income inequality. Chu et al. (2021) suggests that a higher level of patent breadth may reduce income inequality in the long run. However, these papers do not consider automation.

Many papers analyze the relationship between automation and inequality. Specifically, Acemoglu and Restrepo (2018), Prettnner and Strulik (2020), and Hémous and Olsen (2022), and Maeda et al. (2025) examine this relationship in an R&D-based growth model. Acemoglu and Restrepo (2018), Prettnner and Strulik (2020) and Hémous and Olsen (2022) con-

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<sup>1</sup>A higher level of patent breadth allows a higher markup ratio, which corresponds to a lower level of price regulation.

struct variety expansion models with automation, focusing on inequality among workers, specifically between skilled and unskilled workers. In contrast, we develop a Schumpeterian growth model with automation and analyze the inequality between workers and capitalists. Maeda et al. (2025) also analyze inequality between workers and capitalists in a Schumpeterian growth model with automation. The difference between our work and Maeda et al. (2025) lies in the research questions and the structure of economic growth. Maeda et al. (2025) consider a capital-use tax and R&D subsidies for automation and innovation in a semi-endogenous growth setting with human capital accumulation. On the other hand, we focus on price regulation and production subsidies in an endogenous growth setting.

The rest of this paper is organized as follows. Section 2 describes the model. Section 3 characterizes the BGP and analyzes the effects of price regulation. Section 4 presents a quantitative analysis. Concluding remarks are provided in the final section.

## **2 The model**

To examine the effects of price regulation in the nonautomated and automated sectors, we formulate a Schumpeterian growth model with automation. In the model, there are two types of households: workers and capitalists. Workers supply labor and hold financial assets, whereas capitalists hold financial assets and physical capital. The final good is competitively produced by using a unit continuum of intermediate goods. The intermediate goods industries are categorized into automated industries, in which the production factor is labor only, and nonautomated industries, in which the production factor is capital only. All intermediate goods industries are monopolies, and firms in both automated and nonautomated industries face price regulation and are subsidized by the government. The productivity of intermediate goods improves as a result of R&D activities for innovation, which are conducted in both

automated and nonautomated industries. Given the productivity level, nonautomated industries can become automated through R&D activities for automation. Automated industries revert to being nonautomated when innovation occurs. All R&D activities are conducted using labor.

## 2.1 Households

In the model, there are identical workers and identical capitalists. The superscript  $\ell$  represents workers and  $k$  represents capitalists. The workers' population size is constant  $\bar{L}$  and the size of capitalists is normalized to one. Both types of households can hold financial assets. Only capitalists can accumulate physical capital, and only workers supply labor. In the model, the final good is numéraire and thus all values are real terms. The lifetime utility function of the representative workers is specified as:

$$U^\ell = \int_0^\infty e^{-\rho t} \ln c_t^\ell dt, \quad (1)$$

where  $\rho$  is the subjective discount rate and  $c_t^\ell$  is the consumption of a worker. The representative worker maximizes the lifetime utility function subject to the following budget constraint:

$$\dot{a}_t^\ell = r_t a_t^\ell + w_t - c_t^\ell - \tau_t, \quad (2)$$

where  $a_t^\ell$  is the value of assets held by a worker and  $r_t$  is the rate of return on assets at time  $t$ . A worker inelastically supplies one unit of labor to earn the wage  $w_t$ , and  $\tau_t$  is the lump-sum tax. From dynamic optimization, we obtain the following familiar Euler equation:

$$\frac{\dot{c}_t^\ell}{c_t^\ell} = r_t - \rho. \quad (3)$$

The representative capitalist maximizes the following lifetime utility function:

$$U^k = \int_0^{\infty} e^{-\rho t} \log c_t^k dt \quad (4)$$

subject to:

$$\dot{a}_t^k + \dot{k}_t = r_t a_t^k + (R_t - \delta^k) k_t - c_t^k - \tau_t. \quad (5)$$

$a_t^k$  is the value of assets held by a capitalist.  $k_t$  is the volume of physical capital, and the return rate on  $k_t$  is  $R_t - \delta$ , where  $R_t$  denotes the rental price and  $\delta$  indicates the capital depreciation rate.  $c_t^k$  is the consumption of a capitalist. Similarly, from dynamic optimization, we obtain the following familiar Euler equation and no arbitrage condition between financial assets and physical capital:

$$\frac{\dot{c}_t^k}{c_t^k} = r_t - \rho, \quad (6)$$

$$r_t = R_t - \delta. \quad (7)$$

## 2.2 Production

The final good is competitively produced using a unit continuum of intermediate goods. The production function is specified as:

$$Y_t = \exp\left(\int_0^1 \ln x_t(i) di\right), \quad (8)$$

where  $x_t(i)$  represents intermediate goods produced in industry  $i \in [0, 1]$ . Solving the profit-maximizing problem yields the following conditional demand function:

$$x_t(i) = \frac{Y_t}{p_t(i)}, \quad (9)$$

where  $p_t(i)$  is the price of  $x_t(i)$ .

There are two categories of intermediate goods industries: automated and nonautomated industries. Let  $\theta_t \in [0, 1]$ ,  $\Theta_t$ , and  $\mathcal{N}_t$  denote the share of automated industries, the set of automated, and nonautomated industries, respectively. An intermediate good  $i \in \Theta_t$  is produced using physical capital  $k_t(i)$  in an automated industry, as in Zeira (1998), and an intermediate good  $i \in \mathcal{N}_t$  is produced by using labor  $\ell_t(i)$  in a nonautomated industry. The states of industries, automated and nonautomated, shift as a result of R&D for innovation and automation. R&D activities for innovation are conducted in both automated and nonautomated industries, whereas R&D activities for automation are conducted only in nonautomated industries. If R&D activity for innovation succeeds in an industry, the productivity in that industry improves, given the state of the industry. If R&D activity for automation succeeds in a nonautomated industry, the state of this industry shifts from nonautomated to automated, keeping the number of productivity improvements constant. Following Chu et al. (2023) and Maeda et al. (2025), we assume that in each industry, the firm that has succeeded in the latest R&D monopolizes the industry, as incumbents exit due to cost disadvantages. Depending on the state, the production function for an intermediate good  $i$  is as follows:

$$x_t(i) = \begin{cases} z^{n_t(i)} \ell_t(i) & \text{if nonautomated,} \\ \frac{A}{Z_t} z^{n_t(i)} k_t(i) & \text{if automated,} \end{cases} \quad (10)$$

where  $z > 1$  is the common and constant step size of productivity, and  $n_t(i)$  is the number

of innovations that have occurred in industry  $i$  as of time  $t$ .  $A > 0$  is an exogenous parameter representing the productivity difference between automated and nonautomated.  $Z_t$  represents aggregate productivity improvement, defined as  $Z_t \equiv \exp\left(\int_0^1 n_t(i) di \ln z\right)$ . This term captures the erosion effect of physical capital, as in Chu et al. (2023). The government subsidizes intermediate goods industries in the form of cost reduction, and the subsidy rates differ depending on the state of the industries, as they distinguish between nonautomated and automated industries. The subsidy rates for nonautomated and automated industries are  $s^\ell$  and  $s^k$ , respectively. Then, the profit of firms is:

$$\pi_t(i) = \begin{cases} p_t(i)x_t(i) - (1 - s^\ell)w_t\ell_t(i) & \text{if nonautomated,} \\ p_t(i)x_t(i) - (1 - s^k)R_tk_t(i) & \text{if automated.} \end{cases} \quad (11)$$

Here, we consider the price regulation by the government because, without price regulation, the profit-maximizing price would be infinite. The strength of price regulation differs between automated and nonautomated industries. As in Evans et al. (2003), we assume that the exogenous markup ratio is a policy variable for price regulation by the government. Let  $\mu^\ell > 1$  and  $\mu^k > 1$  denote the exogenous markup ratios in nonautomated and automated industries, respectively. Under this settings, the profit-maximizing price of the firm in industry  $i$  becomes:

$$p_t(i) = \begin{cases} \mu^\ell \frac{(1 - s^\ell)w_t}{z_t^{n_t(i)}} & \text{if nonautomated,} \\ \mu^k \frac{Z_t(1 - s^k)R_t}{Az_t^{n_t(i)}} & \text{if automated.} \end{cases} \quad (12)$$

Combining (9), (10), (12) and (11) yields:

$$\pi_t(i) = \begin{cases} \frac{\mu^\ell - 1}{\mu^\ell} Y_t \equiv \pi_t^\ell & \text{if nonautomated,} \\ \frac{\mu^k - 1}{\mu^k} Y_t \equiv \pi_t^k & \text{if automated.} \end{cases} \quad (13)$$

Combining (9), (10), and (12), the aggregate demands for labor and physical capital,  $L_t$  and  $K_t$ , respectively, are:

$$L_t \equiv \int_{i \in N_t} \ell_t(i) di = (1 - \theta_t) \frac{Y_t}{\mu^\ell (1 - s^\ell) w_t}, \quad (14)$$

$$K_t \equiv \int_{i \in \Theta_t} k_t(i) di = \theta_t \frac{Y_t}{\mu^k (1 - s^k) R_t}. \quad (15)$$

### 2.3 R&D activities

R&D activities for innovation and automation are competitively conducted using labor. To ensure incentives for automation R&D, we must assume the following condition:

$$\frac{(1 - s^\ell) w_t}{z_t^{n_t(i)}} > \frac{Z_t (1 - s^k) R_t}{A z_t^{n_t(i)}}. \quad (16)$$

This condition implies that the marginal cost of producing intermediate goods decreases as a result of automation. Furthermore, to ensure incentives for innovation R&D, we must assume the following condition:

$$\frac{Z_t (1 - s^k) R_t}{A z_t^{n_t(i)}} > \frac{(1 - s^\ell) w_t}{z_t^{n_t(i)+1}}, \quad (17)$$

which implies that the marginal cost of production decreases when an automated industry becomes a non-automated industry via innovation. To satisfy (16) and (17) simultaneously,

we impose the following condition:

$$1 < \frac{A(1 - s^\ell)w_t}{Z_t(1 - s^k)R_t} < z. \quad (18)$$

The profits are symmetric for each category of industry (see (13)), and following the literature, we focus on the symmetric equilibrium. Hence, the values of innovation are the same (i.e.  $v^\ell(i) = v^\ell$ ) and the values of automation are also the same (i.e.  $v^k(i) = v^k$ ). The no-arbitrage condition for  $v^\ell$  is:

$$r_t = \frac{\pi_t^\ell + \dot{v}_t^\ell - (\lambda_t + \alpha_t)v_t^\ell}{v_t^\ell}, \quad (19)$$

where  $\lambda_t$  is a symmetric arrival rate of innovation and  $\alpha_t$  is the symmetric arrival rate of automation. The return on  $v^\ell$  is the sum of (i) the dividend  $\pi_t^\ell$ , (ii) capital gain  $\dot{v}_t^\ell$ , and (iii) capital loss  $(\lambda_t + \alpha_t)v_t^\ell$ . Similarly, the no-arbitrage condition for  $v^k$  is:

$$r_t = \frac{\pi_t^k + \dot{v}_t^k - \lambda_t v_t^k}{v_t^k}, \quad (20)$$

which means that the return on  $v^k$  is the sum of (i) the dividend  $\pi_t^k$ , (ii) capital gain  $\dot{v}_t^k$ , and (iii) capital loss  $\lambda_t v_t^k$ . The arrival rate of innovation in industry  $i \in [0, 1]$  is:

$$\lambda_t(i) = \varphi \ell_t^\ell(i), \quad (21)$$

where  $\ell_t^\ell(i)$  is the labor input for innovation in industry  $i$ , and  $\varphi$  is the R&D productivity of innovation. Then, the symmetric arrival rate of innovation is  $\lambda_t = \varphi \ell_t^\ell$ , where  $\ell_t^\ell$  is the

symmetric R&D labor for innovation. The free entry condition for innovation is given by:

$$v_t^\ell = \frac{w_t}{\varphi}. \quad (22)$$

The arrival rate of automation in industry  $i \in \mathcal{N}$  is:

$$\alpha_t(i) = \phi(1 - \theta_t)\ell_t^k(i), \quad (23)$$

where  $\ell_t^k(i)$  is the labor input for automation in industry  $i$ , and  $\phi$  is the R&D productivity of automation. The term  $1 - \theta$  captures the negative externality for automation, as in Chu et al. (2023). Then, the symmetric arrival rate of automation is  $\alpha_t = \phi(1 - \theta_t)\ell_t^k$ , where  $\ell_t^k$  is the symmetric R&D labor for automation. The free entry condition for automation is given by:

$$v_t^k = \frac{w_t}{\phi(1 - \theta_t)}. \quad (24)$$

The share of automated industries,  $\theta_t$ , changes according to the following law of motion:

$$\dot{\theta}_t = \alpha_t(1 - \theta_t) - \lambda_t\theta_t, \quad (25)$$

where the first term on the right-hand side represents the inflow due to automation, and the second term represents the outflow due to innovation.

## 2.4 Government

The government subsidizes capital and labor use in the production sector and levies the lump-sum tax on both types of households to satisfy the following balanced budget constraint:

$$\tau_t(1 + \bar{L}) = s^\ell w_t L_t + s^k R_t K_t. \quad (26)$$

## 2.5 Markets equilibrium

An effective labor supply  $\bar{L}$  is used for the production of intermediate goods by nonautomated industries  $L_t = (1 - \theta_t)Y_t/[\mu^\ell(1 - s)w_t]$ , for innovation  $\int_0^1 \ell_t^\ell di = \ell_t^\ell$ , and for automation  $\int_{i \in \mathcal{N}} \ell_t^k di = (1 - \theta_t)\ell_t^k$ . Then, the labor market-clearing condition becomes:

$$\bar{L} = (1 - \theta_t) \frac{Y_t}{\mu^\ell(1 - s^\ell)w_t} + \ell_t^\ell + (1 - \theta_t)\ell_t^k. \quad (27)$$

The physical capital market-clearing condition is given by:

$$k_t = K_t. \quad (28)$$

Final goods are used for consumption and the formation of physical capital. The final good market-clearing condition is:

$$Y_t = C_t + \dot{K}_t + \delta K_t, \quad (29)$$

where  $C_t \equiv c_t^k + \bar{L}c_t^\ell$  represents aggregate consumption. The financial asset market-clearing condition is given by:

$$a_t \equiv a_t^k + \bar{L}a_t^\ell = (1 - \theta_t)v_t^\ell + \theta_tv_t^k. \quad (30)$$

### 3 Balanced growth path

In this section, we present the definition of balanced growth path (BGP) and prove its existence and uniqueness. Hereafter, we focus on the case in which  $\alpha_t$  and  $\lambda_t$  are positive.

**Definition 1.** *The BGP is a path that satisfies the following conditions (i) all of  $Y_t$ ,  $C_t$ ,  $a_t$ ,  $K_t$ ,  $v_t^\ell$ ,  $v_t^k$ ,  $w_t$ , and  $Z_t$  grow at the same constant rate  $g$ , and (ii)  $\theta_t$ ,  $R_t$ ,  $r_t$ ,  $\lambda_t$  and  $\alpha_t$  are constant over time.*

In what follows, we focus on the BGP and let the variables without the subscript  $t$  represent constant BGP values.

**Proposition 1.** *Suppose that*

$$\varphi \frac{\mu^\ell - 1}{\mu^\ell} < \phi \frac{\mu^k - 1}{\mu^k}. \quad (31)$$

*There exists a unique BGP with  $\theta \in (0, 1)$  and the following two equations determine the BGP values of  $w_t/Y_t$  and  $\theta$ :*

$$\theta = \frac{\phi \frac{\mu^k - 1}{\mu^k} - \varphi \frac{\mu^\ell - 1}{\mu^\ell}}{\rho \frac{w_t}{Y_t} + \phi \frac{\mu^k - 1}{\mu^k} - \varphi \frac{\mu^\ell - 1}{\mu^\ell}} \Leftrightarrow \frac{w_t}{Y_t} = \omega(\theta; \mu^\ell, \mu^k) \equiv \frac{\phi \frac{\mu^k - 1}{\mu^k} - \varphi \frac{\mu^\ell - 1}{\mu^\ell}}{\rho} \frac{1 - \theta}{\theta}, \quad (32)$$

$$\frac{w_t}{Y_t} = \Omega(\theta; \mu^\ell, \mu^k, s^\ell) \equiv \frac{1}{\bar{L} + \frac{\rho}{\varphi}} \left\{ \left[ \frac{1}{\mu^\ell(1 - s^\ell)} + \left( \frac{\phi}{\varphi} - 1 \right) \frac{\mu^k - 1}{\mu^k} \right] (1 - \theta) + \frac{\varphi \mu^\ell - 1}{\phi \mu^\ell} \right\}. \quad (33)$$

The constant growth rate is given by:

$$g = \left[ \phi(1 - \theta) \frac{\mu^k - 1}{\mu^k} \frac{Y_t}{w_t} - \rho \right] \ln z = \left[ \frac{\theta}{1 - \frac{\phi(\mu^\ell - 1)/\mu^\ell}{\phi(\mu^k - 1)/\mu^k}} - 1 \right] \rho \ln z. \quad (34)$$

**Proof.** See Appendix A.

□

Eq. (32) represents the negative relationship between  $\theta$  and  $w_t/Y_t$  for constant  $\theta$ . An increase in the wage rate stifles both R&D activities. From the no-arbitrage conditions, the value of innovation suffers from a higher rate of creative distraction because nonautomated industries are targeted by both R&D activities, whereas automated industries are targeted only by R&D activities for innovation. Hence, lower R&D activities make investment in innovation more attractive than in automation, as the rate of creative distraction in nonautomated industries falls more than in automated industries. Therefore, an increase in  $w_t/Y_t$  stimulates innovation relative to automation and reduces the share of automated industries,  $\theta$ .

Eq. (33) represents the relationship between  $\theta$  and  $w_t/Y_t$  that clear the labor market. A change in  $\theta$  affects  $w_t/Y_t$  through three types of labor demand: for production, innovation, and automation. An increase in  $\theta$  means shrinking the share of nonautomated industries, and thus the labor demand for production declines. An increase in  $\theta$  makes R&D for automation more difficult due to the negative externality, and thus both the labor use for R&D for automation per targeted industry and the value of automation rise. However, the former effect is offset by a decrease in the number of industries targeted by R&D for automation. Therefore, a change in  $\theta$  affects the labor use for both R&D activities only through a change in the value of automation. A higher value of automation attracts more labor inputs to R&D for automation, and thus decreases labor input to R&D for innovation. In summary, an increase

in  $\theta$  decreases labor demand for production and innovation-R&D, but increases labor demand for automation-R&D. Each effect influences  $w_t/Y_t$  linearly through the labor market-clearing condition. Therefore, if the negative (positive) effects dominate, an increase in  $\theta$  raises (declines)  $w_t/Y_t$  linearly.

Next, we examine the effects of the policies regarding industrial regulations by using (32) – (34). We consider the following policies: (i) a cut of subsidies for nonautomated industries (i.e., a decrease in  $s^\ell$ ), (ii) a cut in subsidies for automated industries (i.e., a decrease in  $s^k$ ), (iii) a strengthening of price regulation in nonautomated industries (i.e., a decrease in  $\mu^\ell$ ), and (iv) a strengthening of price regulation in automated industries (i.e., a decrease in  $\mu^k$ ). We can show the analytical results of comparative statics regarding cuts in subsidies and summarize these results in the following proposition. However, the effects of price regulation are complex, and thus, we decompose and discuss the effects later.

**Proposition 2.** *A decrease in  $s^\ell$  increases  $\theta$  and  $g$ , and decreases  $w_t/Y_t$ . A decrease in  $s^k$  affects none of  $\theta$ ,  $w_t/Y_t$ , and  $g$ .*

**Proof.** *Eqs. (32) and (33) determine the BGP values of  $\theta$  and  $w_t/Y_t$ . The subsidy rate  $s^\ell$  appears only in (33). A decrease in  $s^\ell$  increases the value of the coefficient of  $\theta$  in (33), but does not affect the equation when  $\theta = 1$ . Hence, a decrease in  $s^\ell$  causes the line in (33) to rotate counterclockwise about the point  $\theta = 1$ , as shown in Figure 1. Because of the downward sloping of (32), the counterclockwise rotation moves the intersect to the lower right. From this and (34), a decrease in  $s^\ell$  increases  $\theta$  and  $g$ , and decreases  $w_t/Y_t$ . In contrast,  $s^k$  does not affect (32) – (34), and thus,  $\theta$ ,  $w_t/Y_t$ , and  $g$  are independent of  $s^k$ .*

□

We can interpret Proposition 2 as follows. Firstly, a cut in subsidies for nonautomated industries (a decrease in  $s^\ell$ ) increases the share of automated industries,  $\theta$ , decreases the wage

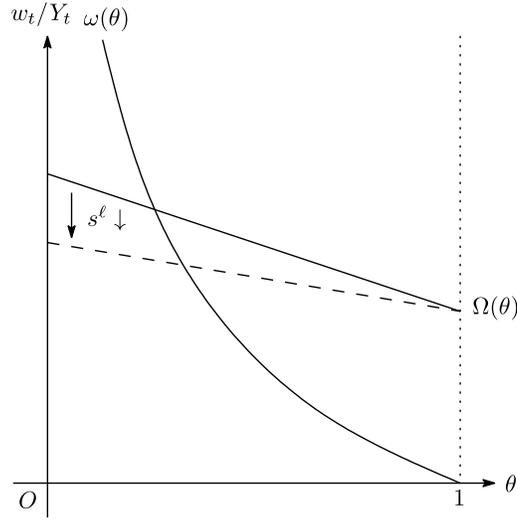


Figure 1: The effects of a decrease in  $s^\ell$

rate,  $w_t/Y_t$ , and increases the growth rate,  $g$ . The production function (8) gives the uniform expenditure among industries, as shown in (9), and thus, the profits are independent of the marginal costs. This means that the subsidies do not alter the incentives for R&D activities and, consequently, do neither affect the labor demands for R&D activities nor (32). Therefore, the subsidy rate,  $s^\ell$ , affects the BGP only through the labor demand for production in (33). The cut in subsidies raises the labor-use costs and reduces the labor demand for production. This leads to a lower wage rate,  $w_t/Y_t$ , through the labor market-clearing condition, (33). A lower  $w_t/Y_t$  encourages automation relative to innovation and raises the share of automated industries  $\theta$ , in (32) for the reverse of the reasons described above. Further, encouraged R&D activities lead to a higher growth rate,  $g$ .

Secondly, the subsidies for automated industries,  $s^k$ , affect none of the share of automated industries,  $\theta$ , the wage rate,  $w_t/Y_t$ , or the growth rate,  $g$ . The subsidies  $s^k$  do not affect (32) for the same reason as  $s^\ell$ . In addition, the subsidies do not affect the relative prices of capital because capital is used only for production in automated industries, and the subsidy rates are uniform among automated industries. Therefore, the subsidies do not affect the capital

$g_b$	$w\bar{L}/GDP$	$R\&D/GDP$	$a_0^k/a_0$
0.0230	0.600	0.0460	0.300

Table 1: Target values in the numerical analysis

allocation nor the demand for other production factors, namely, labor demand. This implies that the subsidies do not affect the labor market-clearing condition, (33).

Next, we discuss the effects of  $\mu^\ell$  and  $\mu^k$  on (32) and (33). Eq. (32) can be rewritten as  $\theta = \alpha/(\alpha + \lambda)$ , in which  $\theta$  is characterized as the ratio of creative destruction rates. For the constant  $\theta$  in (32), the wage rate must rise (fall) when  $\mu^k$  increases ( $\mu^\ell$  increases). Eq. (33) represents the labor market-clearing condition. A higher  $\mu^k$  (a higher  $\mu^\ell$ ) increases the rate of return on  $v^k$  (the rate of return on  $v^\ell$ ). To equalize both rates of return, R&D activities must be discouraged (encouraged) for the same reason as described above. Then,  $\mu^k$  and  $\mu^\ell$  change the labor demands for R&D activities. Moreover,  $\mu^\ell$  also changes the labor demand for production. Both  $\mu^k$  and  $\mu^\ell$  have positive and negative effects on labor demand, affecting (33) ambiguously. Therefore, we cannot obtain definite results of the comparative statics with respect to  $\mu^k$  and  $\mu^\ell$ , and thus, we conduct the numerical analysis.

## 4 Numerical analysis

To investigate the effects of the policies on the share of automated industries, the wage rate, and the economic growth rate under realistic parameter values, we conduct numerical analysis. Further, we also investigate the inequality between workers and capitalists and the social welfare. In this section, we focus on the BGP.

Hereafter, the subscript  $b$  represents the benchmark values. To calibrate some parameter

$\delta$	$\rho$	$z$	$\mu_b^\ell$	$\mu_b^k$	$s_b^\ell$	$s_b^k$	$A$	$\varphi$	$\phi$
0.0430	0.0500	1.20	1.20	1.20	0.10	0.10	0.133	0.0107	0.0119

Table 2: Parameter values in the numerical analysis

values, we use US data as shown in Table 1,<sup>2</sup> following Maeda et al. (2025).<sup>3</sup> We also follow Maeda et al. (2025) to set and calibrate the depreciation rate of physical capital,  $\delta$ , the subjective discount rate,  $\rho$ , and the step size,  $z$ , as shown in Table 2. Jones and Williams (2000) report that the range of empirical estimates of the markup ratio is 1.05 to 1.4. Therefore, we set the benchmark markup ratios,  $\mu_b^\ell$  and  $\mu_b^k$ , to 1.2. We consider the subsidy cut as industrial regulation and thus set positive symmetric subsidies as the benchmark case,  $s_b^\ell = s_b^k = 0.1$ . Although the government does not distinguish between the types of industries in practice, we will change the individual values of  $\mu^\ell$ ,  $\mu^k$ ,  $s^\ell$ , and  $s^k$  separately to investigate which industries the government should regulate. In addition, we investigate the case where the government uniformly regulates all industries (i.e., changes in  $\mu^\ell = \mu^k$  and in  $s^\ell = s^k$ ). The values of  $A$ ,  $\varphi$ , and  $\phi$  are set to satisfy the target values, the economic growth rate,  $g_b$ , the labor income share,  $w\bar{L}/GDP$ , and the ratio of R&D investment to GDP,  $R\&D/GDP$  in Table 1.<sup>4</sup> The target value  $a_0^k/a_0$  is used to calculate the inequality, which is represented by  $c_t^k/c_t^\ell$ .<sup>5</sup>

We numerically show the effects of changes in  $s^\ell$ ,  $s^k$ , and both  $s^\ell$  and  $s^k$  simultaneously (i.e.,  $s^\ell = s^k$ ). Also, we show the effects of changes in  $\mu^\ell$ ,  $\mu^k$ , and both  $\mu^\ell$  and  $\mu^k$  simultane-

<sup>2</sup>The data is available upon request.

<sup>3</sup>There is a difference in the notation of population size and the share of assets. Maeda et al. (2025) normalize the population size of workers to 1 and set that of capitalists to 0.01. We normalize the population size of capitalists to 1 and set that of workers to 100 (i.e.,  $\bar{L} = 100$ ) to match the ratios of population sizes. Further, we use the labor income share as the target value, but Maeda et al. (2025) use the capital income share.

<sup>4</sup>These values also satisfy (18) and (31) under the calibrated parameters. We show the derivation in Appendix B.

<sup>5</sup>The capitalists and the workers face the same growth rate, and their utility functions are of the same logarithm form. Hence, the gap in lifetime utility on the BGP between the capitalists and the workers is determined by the ratio of per capita consumption:  $U^k - U^\ell = (1/\rho) \ln(c^k/c^\ell)$ .

ously (i.e.,  $\mu^\ell = \mu^k$ ).

## 4.1 Numerical results

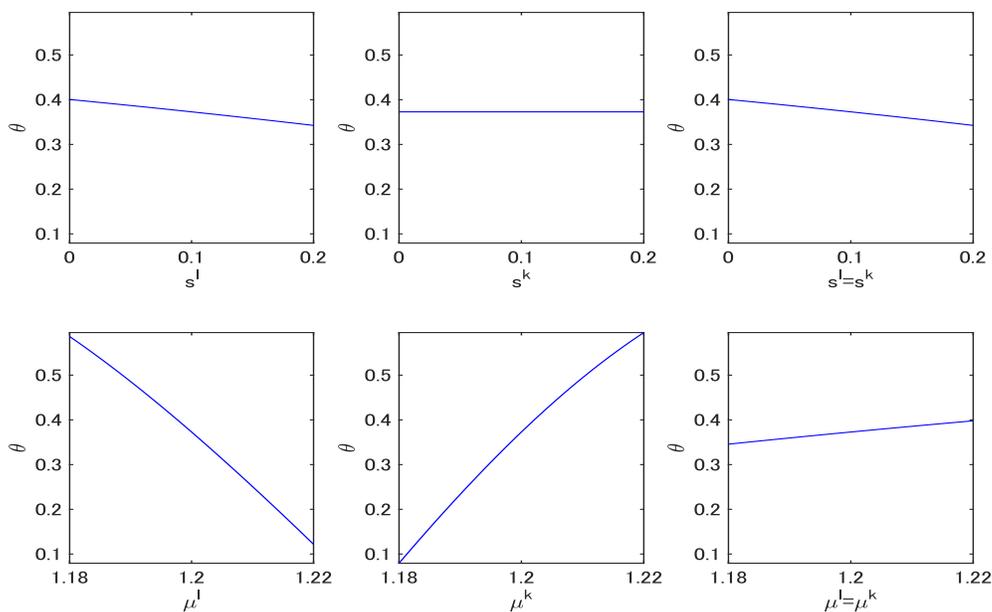


Figure 2: The effects on  $\theta$

Figure 2 shows the effects on the share of automated industries,  $\theta$ . As in Proposition 2, a cut in  $s^\ell$  has a positive effect, and a cut in  $s^k$  has no effect. Therefore, a uniform cut of  $s^\ell$  and  $s^k$  has a positive effect. A decrease in  $\mu^\ell$  has a positive effect, and a decrease in  $\mu^k$  has a negative effect. This implies that strengthening price regulation in nonautomated and automated industries decreases the share of regulated industries. A uniform decrease in  $\mu^\ell$  and  $\mu^k$  decreases the share of automated industries. This implies that the effect in automated industries is stronger than that in nonautomated industries.

Figure 3 shows the effects on the wage rate,  $w_t/Y_t$ . As in Proposition 2, a cut in  $s^\ell$  has a negative effect, and a cut in  $s^k$  has no effect. Therefore, a uniform cut in  $s^\ell$  and  $s^k$  negatively

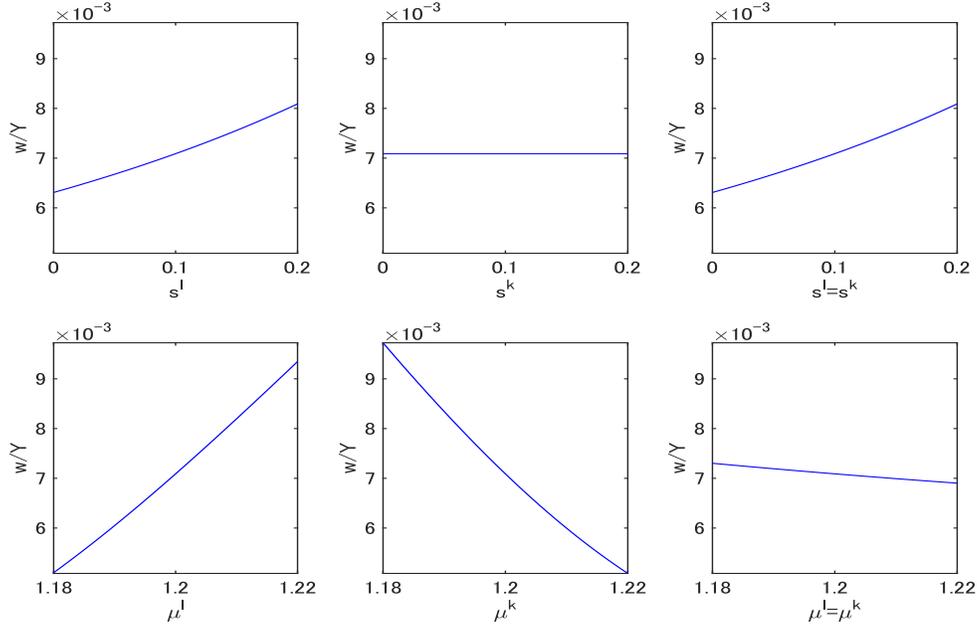


Figure 3: The effects on  $w_t/Y_t$

affects the wage rate. A decrease in  $\mu^\ell$  has a negative effect, and a decrease in  $\mu^k$  has a positive effect. This implies that strengthening price regulation in nonautomated industries decreases the prices of goods in the regulated industries, and thus, the wage rate decreases. In contrast, strengthening price regulation in automated industries increases the prices of goods in the non-regulated industries relatively, and thus, the wage rate increases. A uniform decrease in  $\mu^\ell$  and  $\mu^k$  increases the wage rate. This implies that the effect in automated industries is stronger than that in nonautomated industries, as in the result for  $\theta$ .

Figure 4 shows the effects on the economic growth rate,  $g$ . As in Proposition 2, a cut in  $s^\ell$  has a positive effect, and a cut in  $s^k$  has no effect. Therefore, a uniform cut in  $s^\ell$  and  $s^k$  positively affects the growth rate. A decrease in  $\mu^\ell$  has a negative effect. The relationship between  $\mu^k$  and  $g$  is an inverted-U shape; there exists a growth-maximizing level of price regulation in automated industries. A uniform decrease in  $\mu^\ell$  and  $\mu^k$  decreases the growth

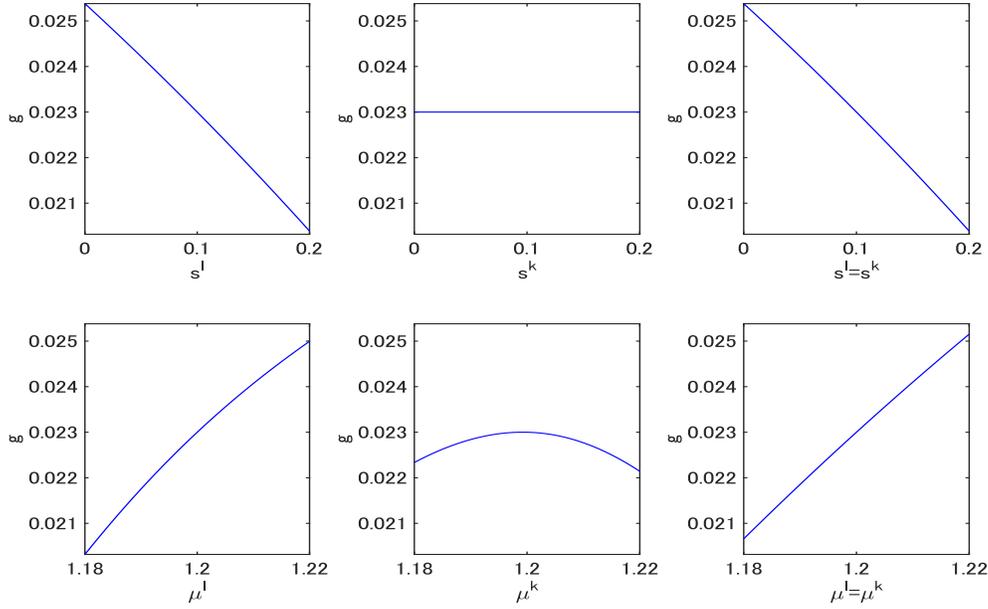


Figure 4: The effects on  $g$

rate. This implies that the effect in automated industries is weaker than that in nonautomated industries.

Both a cut of  $s^\ell$  and a decrease in  $\mu^\ell$  represent regulation in nonautomated industries and have positive effects on  $\theta$  and negative effects on  $w_t/Y_t$ . However, it is interesting that the policies have the opposite effects on  $g$ . A cut in  $s^\ell$  decreases labor demand for production, which encourages R&D for innovation due to labor reallocation. A decrease in  $\mu^\ell$  reduces profits in nonautomated industries, which discourages R&D for innovation. The opposite effects on  $g$  arise from these opposite effects on R&D for innovation.

Figure 5 shows the effects on the relative consumption between capitalists and workers,  $c_t^k/c_t^\ell$ , which represents inequality. A cut in  $s^\ell$  has a positive effect on  $c_t^k/c_t^\ell$ , implying that strengthening regulation in labor-use industries makes workers poorer and widens the inequality. Although  $s^k$  has no effect on  $\theta$ ,  $w_t/Y_t$ , and  $g$ , a cut in  $s^k$  has a negative effect

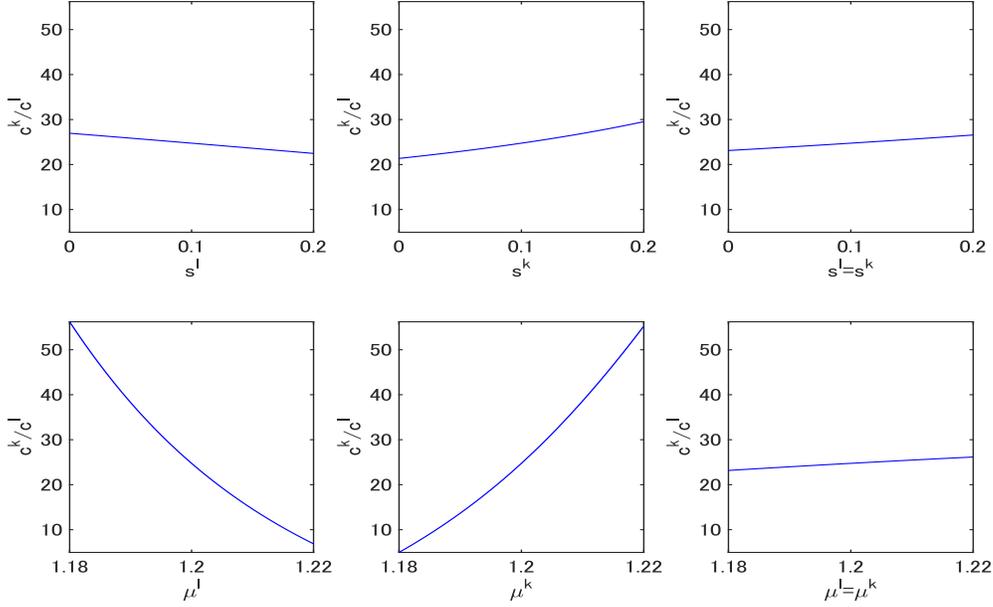


Figure 5: The effects on the inequality

on  $c_i^k/c_i^\ell$  through income redistribution due to the lump-sum tax. A uniform cut in  $s^\ell$  and  $s^k$  shrinks the inequality.<sup>6</sup> A decrease in  $\mu^\ell$  ( $\mu^k$ ) has a positive (negative) effect on  $c_i^k/c_i^\ell$ , implying that strengthening regulation in labor-use (capital-use) industries makes workers (capitalists) poorer and widens (shrinks) the inequality. A uniform decrease in  $\mu^\ell$  and  $\mu^k$  shrinks the inequality. This implies that the effect in automated industries is stronger than that in nonautomated industries.

Figure 6 shows the changes in the social welfare resulting from policy changes.<sup>7</sup> The social welfare is given by  $U^\ell \bar{L} + U^k$  and  $\Delta \text{welfare}$  in Figure 6 represents the percent change in the social welfare from the benchmark value. A cut in  $s^\ell$  ( $s^k$ ) has a negative (positive) effect on the welfare. A uniform cut in  $s^\ell$  and  $s^k$  negatively affects the welfare. A decrease in  $\mu^\ell$

<sup>6</sup>The effect in automated industries is stronger than that in nonautomated industries under the current parameter set. Because the size of capitalists is smaller than that of workers, the effects of income redistribution on per capita capitalists are sufficiently strong. If we change the population size, the opposite result will arise.

<sup>7</sup>To calculate the social welfare, we normalize the initial values capital and aggregate productivity improvement to one (i.e.,  $K_0 = 1$  and  $Z_0 = 1$ ).

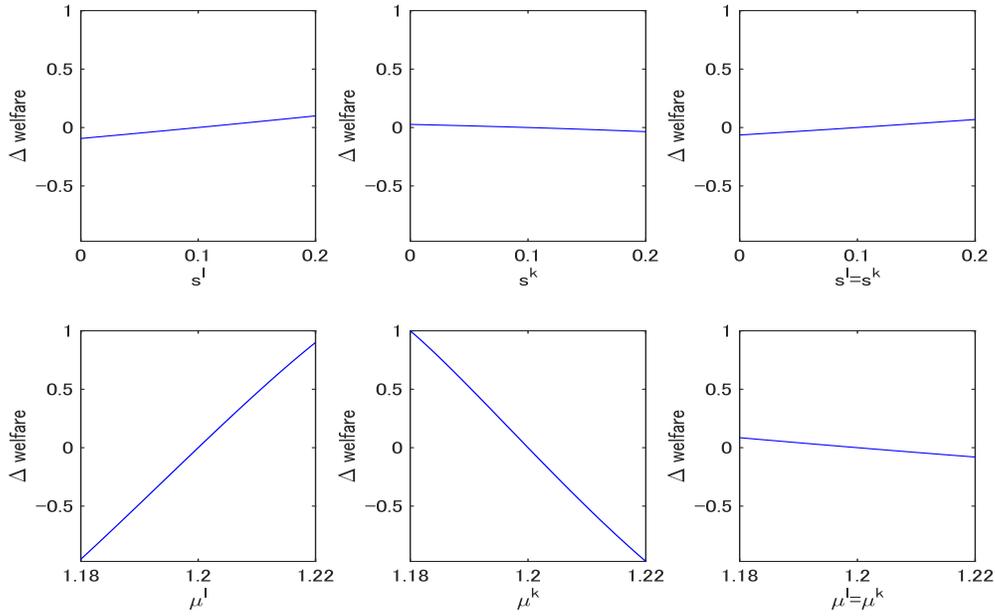


Figure 6: The effects on the welfare

$(\mu^k)$  has a negative (positive) effect on the welfare. A uniform decrease in  $\mu^\ell$  and  $\mu^k$  positively affects the welfare.

	$s^\ell$	$s^k$	$s^\ell = s^k$	$\mu^\ell$	$\mu^k$	$\mu^\ell = \mu^k$
1% up	0.0968	-0.0305	0.0663	55.9	-60.6	-4.87
1% down	-0.0967	0.0304	-0.0663	-57.7	62.1	5.07

Table 3: The percent changes in the social welfare resulting from one percent changes in the policies from the benchmark values

In relation to the welfare analysis, we calculate the percent changes in the social welfare resulting from one percent changes in the policies. Table 3 shows that the magnitude of the welfare changes due to the changes in the price regulations are greater than those due to the changes in the subsidies.

## 4.2 Policy implication

From Figures 5 and 6, increases in  $s^\ell$  and  $\mu^\ell$  shrink the inequality and improve the welfare. Further, decreases in  $s^k$  and  $\mu^k$  have the same effects. The increases in  $s^\ell$  and  $\mu^\ell$  can be considered as the relaxing the regulation in nonautomated industries or the strengthening indirect regulation for automation. The decreases in  $s^k$  and  $\mu^k$  are the strengthening direct regulation for automation. Therefore, regardless of direct or indirect, the regulation for automation shrinks inequality and improves the welfare. The regulation via the changes in the subsidies affects the production costs and can be considered as cost-based regulation. In contrast, that via the changes in markup ratios affects the prices and can be considered as price-based regulation. Regardless of cost-based or price-based, the regulation for automation shrinks inequality and improves the welfare but the magnitudes of slopes significantly differ between cost-based and price-based regulation.

We compare the effectiveness of the cost-based and price-based regulation. Supposing that one percent change in each policy has the same social cost,<sup>8</sup> the price-based regulation is more effective than the cost-based regulation as shown in Table 3. In the price-based regulation, the indirect regulation always promotes economic growth along with welfare improvement whereas the direct regulation does not always promote the growth.

In the above discussion, we assume that the policymaker can change policies depending on the types of industries. If the policymaker cannot change policies depending on the types of industries and policy instruments are uniform changes in subsidies or markup ratios (i.e.  $s^\ell = s^k$  or  $\mu^\ell = \mu^k$ ), the price-based regulation is more effective than the cost-based regulation. This is because only price regulation can shrink the inequality with welfare improvement.

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<sup>8</sup>The social cost may include political costs and economic costs abstracted in our model.

## 5 Concluding remarks

To investigate the effects of regulation policies regarding automation, we have formulated an endogenous growth model with innovation and automation. In the model, there are two types of households: workers and capitalists. As regulation policies, we have considered price regulation and production subsidies in both automated and nonautomated industries. We have analytically shown the existence and uniqueness of the BGP, as well as some effects of the policies. Further, to evaluate all effects of the policies, we have conducted numerical analysis using US data.

A decrease in subsidies for production in nonautomated industries increases the share of automated industries and the economic growth rate, decreases the wage rate, enlarges inequality between workers and capitalists, and worsens the social welfare. A decrease in subsidies for production in automated industries has no effect on the share of automated industries, the economic growth rate, or the wage rate, shrinks inequality, and improves the welfare. A simultaneous decrease in both subsidies increases the share of automated industries and the economic growth rate, decreases the wage rate, shrinks inequality, and worsens the welfare. Strengthening price regulation in nonautomated industries increases the share of automated industries, decreases the wage rate and the growth rate, enlarges the inequality, and worsens the welfare. Strengthening price regulation in automated industries decreases the share of automated industries, increases the wage rate, shrinks inequality, and improves the welfare. Moreover, there exists an inverted-U relationship between price regulation in automated industries and the growth rate. If both price regulations are strengthened simultaneously, the share of automated industries and the growth rate decrease, the wage rate increases, inequality shrinks, and the welfare improves.

In reality, capitalists are richer than workers, and the expansion of automation may in-

crease inequality. Therefore, we consider regulation for automation to prevent the enlargement of inequality. In our model, not only strengthening the regulation in automated industries but also relaxing the regulation in nonautomated industries can be considered as direct and indirect regulation for automation. Regardless of direct and indirect, strengthening regulation for automation can reduce the inequality while improving the welfare. Further, there are two types of the policy instruments for both direct and indirect regulation: cost-based regulation (a cut in subsidies) and price-based regulation (a lower markup ratio). The price-based regulation is more effective than the cost-based regulation in terms of the magnitude of the welfare improvement for both direct and indirect regulation.

## **Acknowledgment**

We would like to thank the seminar participants at Chukyo university. Morimoto acknowledges the financial support from the Japan Society for the Promotion of Science (JSPS) through Grants-in-Aid for Early-Career Scientists Grant No. 25K16608. All remaining errors are our responsibility.

## **Declaration of competing interest**

We declare that no conflict of interest exists in relation to this study.

# Appendix

## A Proof of Proposition 1

On the BGP,  $\dot{v}_t^\ell/v_t^\ell = \dot{v}_t^k/v_t^k = g$  holds. Substituting this equation, (11), (22), and (24) into the no-arbitrage conditions for  $v_t^\ell$  and  $v_t^k$ , we have:

$$r = \varphi \frac{\mu^\ell - 1}{\mu^\ell} \frac{Y_t}{w_t} + g - (\alpha + \lambda), \quad (35)$$

$$r = (1 - \theta)\phi \frac{\mu^k - 1}{\mu^k} \frac{Y_t}{w_t} + g - \lambda. \quad (36)$$

Combining (35), and (36) yields:

$$\alpha = \frac{\pi^\ell}{v^\ell} - \frac{\pi^k}{v^k} = \left[ \varphi \frac{\mu^\ell - 1}{\mu^\ell} - \phi(1 - \theta) \frac{\mu^k - 1}{\mu^k} \right] \frac{Y_t}{w_t}. \quad (37)$$

To ensure  $\alpha > 0$ , we impose the following condition:

$$\phi(1 - \theta) \frac{\mu^k - 1}{\mu^k} < \varphi \frac{\mu^\ell - 1}{\mu^\ell}. \quad (38)$$

From the Euler equations,  $g = r - \rho$  holds on the BGP. Combining this equation and (36) yields:

$$\lambda = \phi(1 - \theta) \frac{\mu^k - 1}{\mu^k} \frac{Y_t}{w_t} - \rho \quad (39)$$

Combining (37), (39), and  $\dot{\theta}_t = 0$ , we have:

$$\frac{w_t}{Y_t} = \omega(\theta; \mu^\ell, \mu^k) \equiv \frac{\phi \frac{\mu^k - 1}{\mu^k} - \varphi \frac{\mu^\ell - 1}{\mu^\ell}}{\rho} \frac{1 - \theta}{\theta},$$

which is (32). Substituting (21), (23), (37), and (39) into (27), we have

$$\frac{w_t}{Y_t} = \Omega(\theta; \mu^\ell, \mu^k, s^\ell) \equiv \frac{1}{\bar{L} + \frac{\rho}{\phi}} \left\{ \left[ \frac{1}{\mu^\ell(1-s^\ell)} + \left( \frac{\phi}{\phi} - 1 \right) \frac{\mu^k - 1}{\mu^k} \right] (1-\theta) + \frac{\phi \mu^\ell - 1}{\phi \mu^\ell} \right\}$$

which is (33). In (32) and (33), there are two BGP variables,  $w_t/Y_t$  and  $\theta$ .  $\omega(\theta; \mu^\ell, \mu^k)$  is monotonically decreasing from positive infinity to zero as  $\theta$  increases from zero to one.  $\Omega(\theta; \mu^\ell, \mu^k, s^\ell)$  is linear in  $\theta$  and  $\Omega(1; \mu^\ell, \mu^k, s^\ell)$  is positive. Hence, (32) and (33) have only one intersect with  $\theta \in (0, 1)$  and  $w_t/Y_t > 0$ .

Because  $r$  is constant on the BGP,  $R = r + \delta$  also becomes constant. From (8), (10), (14), and (15), we obtain the following the aggregate production function and the unit cost function ( $UC$ ):

$$Y_t = \left( \frac{Z_t L_t}{1-\theta} \right)^{1-\theta} \left( \frac{A K_t}{\theta} \right)^\theta, \quad (40)$$

$$UC = \left[ \frac{\mu^\ell(1-s^\ell)w_t}{Z_t} \right]^{1-\theta} \left[ \frac{\mu^k(1-s^k)R}{A} \right]^\theta. \quad (41)$$

Since the final good is numéraire and  $UC = 1$ ,  $w_t/Z_t$  is constant. From this and the fact that  $w_t/Y_t$  is constant,  $Z_t/Y_t$  is also constant. Eq. (40) brings about constant  $K_t/Y_t$ . From (29), we have:

$$\frac{\dot{K}_t}{K_t} = \frac{Y_t}{K_t} - \frac{C_t}{K_t} - \delta = g, \quad (42)$$

where we implicitly assume that the growth rate of  $K_t$  is the same as that of  $C_t$ . We can confirm the consistency of this assumption because  $C_t/K_t$  is constant in (42). From (22), (24) and (30),  $v_t^\ell$ ,  $v_t^k$  and  $a_t$  grow at same rate of  $w_t$ , that is,  $g$ . Hence, all variables which grow on

the BGP have the same growth rate,  $g$ , which is given by:

$$g = \frac{\dot{Z}_t}{Z_t} = \lambda \ln z = \left[ \phi(1 - \theta) \frac{\mu^k - 1}{\mu^k} \frac{Y_t}{w_t} - \rho \right] \ln z. \quad (43)$$

From (32), we obtain:

$$(1 - \theta) \frac{Y_t}{w_t} = \frac{\rho}{\phi(\mu^k - 1)/\mu^k - \varphi(\mu^\ell - 1)/\mu^\ell} \theta \quad (44)$$

Substituting this into (43) yields:

$$g = \left[ \phi(1 - \theta) \frac{\mu^k - 1}{\mu^k} \frac{Y_t}{w_t} - \rho \right] \ln z = \left[ \frac{\theta}{1 - \frac{\varphi(\mu^\ell - 1)/\mu^\ell}{\phi(\mu^k - 1)/\mu^k}} - 1 \right] \rho \ln z$$

which is (34). Therefore, we show that there exists unique BGP and obtain the growth rate.

## B Calibration of the benchmark parameters

From (14), we obtain the benchmark share of the automated industries as follows:

$$\theta_b = 1 - \mu^\ell(1 - s_b^\ell) \frac{1}{1 - (R\&D/GDP)} \left( \frac{w\bar{L}}{GDP} - \frac{R\&D}{GDP} \right). \quad (45)$$

Substituting (32) into (34), we have:

$$\frac{\varphi}{\phi} = \frac{\mu_b^k - 1}{\mu_b^k} \frac{\mu_b^\ell}{\mu_b^\ell - 1} \left( 1 - \frac{\theta_b}{\rho \log z + 1} \right). \quad (46)$$

From (32) and (33), we obtain:

$$\phi = \frac{\rho}{\bar{L}} \left\{ \frac{\frac{1}{1-\theta_b} - 1}{\frac{\mu_b^k - 1}{\mu_b^k} - \frac{\varphi \mu_b^\ell - 1}{\phi \mu_b^\ell}} \left\{ \left[ \frac{1}{\mu_b^\ell (1 - s_b^\ell)} + \left( \frac{\phi}{\varphi} - 1 \right) \frac{\mu_b^k - 1}{\mu_b^k} \right] (1 - \theta_b) + \frac{\varphi \mu_b^\ell - 1}{\phi \mu_b^\ell} \right\} - \frac{\phi}{\varphi} \right\}. \quad (47)$$

Eqs. (46) and (47) determine the values of  $\varphi$  and  $\phi$ . From (6) and (7), we have the benchmark rental price of capital as  $R_b = g_b + \rho + \delta$ . From (18), (41) and  $UC = 1$ , we obtain:

$$1 < \frac{A}{\mu_b^\ell (\mu_b^k)^{\frac{\theta_b}{1-\theta_b}} R_b} \left[ \frac{A}{\mu_b^k (1 - s_b^k) R_b} \right]^{\frac{\theta_b}{1-\theta_b}} < z. \quad (48)$$

We set  $A$  to the average of the upper and lower bounds of (48), that is,

$$A = (1 - s_b^k) R_b \left( \frac{1 + z}{2} \right)^{1-\theta_b} (\mu_b^\ell)^{1-\theta_b} (\mu_b^k)^{\theta_b}. \quad (49)$$

Next, we derive the ratio of per capita consumption between the capitalist and the workers on the BGP. From the asset clearing condition (30), we derive:

$$\frac{a_t}{Y_t} = \left[ \frac{1}{\varphi} (1 - \theta) + \frac{1}{\phi} \frac{\theta}{1 - \theta} \right] \frac{w_t}{Y_t}. \quad (50)$$

Combining (14), (26), and  $R = g + \rho + \delta$  yields:

$$\frac{\tau_t}{Y_t} = \frac{1}{1 + \bar{L}} \left[ \frac{(1 - \theta) s^\ell}{\mu^\ell (1 - s^\ell)} + s^k (g + \rho + \delta) \frac{K_t}{Y_t} \right]. \quad (51)$$

From the budget constraints of households (2) and (5), we have:

$$\frac{c_t^\ell}{Y_t} = \rho \left( 1 - \frac{a_0^k}{a_0} \right) \frac{a_t}{Y_t} + \frac{w_t}{Y_t} - \frac{\tau_t}{Y_t}, \quad (52)$$

$$\frac{c_t^k}{Y_t} = \rho \left( \frac{a_0^k}{a_0} \frac{a_t}{Y_t} + \frac{k_t}{Y_t} \right) - \frac{\tau_t}{Y_t}. \quad (53)$$

Substituting (28) and (51) – (53) into (29) yields:

$$\frac{k_t}{Y_t} = \frac{1}{(1 - s^k)R} \left[ 1 - \rho \frac{a_t}{Y_t} - \frac{w_t \bar{L}}{Y_t} + \frac{(1 - \theta)s^\ell}{\mu^\ell(1 - s^\ell)} \right]. \quad (54)$$

Eqs. (50) – (54) determine the per capita consumption on the BGP.

We define the social welfare as follows:

$$U^\ell \bar{L} + U^k. \quad (55)$$

On the BGP, the utilities of the worker and the capitalist are given by

$$U^i = \frac{1}{\rho} \left( \ln c_0^i + \frac{\rho}{g} \right), \quad (56)$$

where  $i \in \{\ell, k\}$ . Combining (14) and (40), we obtain the initial value of the final good as follows:

$$Y_0 = \left[ \frac{Z_0}{\mu^\ell(1 - s^\ell)} \frac{Y_0}{w_0} \right]^{1-\theta} \left( \frac{AK_0}{\theta} \right)^\theta. \quad (57)$$

Using (51) – (53) and (57) at  $t = 0$ , we obtain the initial values of consumption and the value of the social welfare.

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