Chukyo University Institute of Economics

Discussion Paper Series

August 2025

No. 2503

Time inconsistency, educational loan, and human capital accumulation

Daiki Maeda
Chukyo University

Kohei Okada
Osaka Sangyo University

Ryota Nakano
Kushiro Public University of Economics

Time inconsistency, educational loan, and human capital accumulation*

Daiki Maeda†, Kohei Okada‡, and Ryota Nakano§

August 5, 2025

Abstract

We develop an overlapping generations model that incorporates agents' time inconsistency, educational loans, and human capital accumulation. Using this model, we show that an increase in present bias raises the growth-maximizing educational subsidy.

Keywords: Hyperbolic discounting; Educational loans; Human capital accumulation

JEL *classification*: E70; I22; O41

1 Introduction

Models with agents' present bias yield different results from those without it. For example, Krusell et al. (2002) show that agents with hyperbolic discounting (i.e., present-biased preferences) have lower saving rates than those without such preferences, resulting in lower aggregate capital accumulation. This indicates that present bias affects other intertemporal decision-making as well. Therefore, in this paper, we focus on educational loans. In the US and Japan, many students face intertemporal decisions, such as whether to pursue education using student loans. This implies that

*This work was supported by JSPS KAKENHI Grant Number JP25K00184.

†School of Economics, Chukyo University, Address: 101-2 Yagoto-Honmachi, Showa-ku, Nagoya, Aichi, 466-8666, Japan. E-mail: d-maeda@mecl.chukyo-u.ac.jp

‡Faculty of Economics, Osaka Sangyo University. Address: 1-1-3 Nakagaito, Daito, Osaka 574-0013, Japan E-mail: koheiokada257@gmail.com

§Faculty of Economics, Kushiro Public University of Economics, Address: Ashino 4-1-1, Kushiro-shi, Hokkaido, Japan. E-mail:r.nakano1028@gmail.com

time inconsistency stemming from present bias may influence aggregate human capital accumulation and economic growth. To analyze these effects, we develop an overlapping generations model with agents' time inconsistency, as follows.

Agents in the model live for three periods and accumulate human capital by investing their time and goods—financed by educational loans—during the first period. In the second period, they earn labor income, save, and repay their loans. In the final period, they consume their savings. The government provides subsidies to reduce the agents' loan burdens. A key assumption we introduce is the presence of agents with hyperbolic discounting ($\beta\delta$) preferences, as in Laibson (1997). This extension allows us to analyze the effects of hyperbolic discounting on economic growth and the role of educational subsidies.

The main contribution of this paper is the finding that a stronger present bias (i.e., increased impatience) raises the growth-maximizing subsidy rate. This result arises because more impatient agents invest less in education and accumulate less human capital. Consequently, economic growth declines, requiring higher subsidies to restore it.

Next, we mention the related literature. The market structure in Boldrin and Montes (2005) closely resembles that in our model. They examine the case of time-consistent agents and focus on dynamic inefficiency. In contrast, we study the case of time-inconsistent agents and analyze the role of educational subsidies. Our study thus complements theirs. Hiraguchi (2016) also analyzes a model in which agents exhibit hyperbolic discounting, as in our setting. However, his model assumes that human capital is accumulated solely through time investment. In contrast, our model incorporates both time and goods (i.e., loans), allowing us to consider the role of educational loans more explicitly. Hence, our study complements his as well.

2 The model

2.1 Individuals

Time is discrete and denoted by $t = 0, 1, 2, \cdots$. Each individual lives for three periods (childhood, adulthood, and old age). In the first period (childhood), individuals decide how much to invest in their own education and borrow x_{t-1} units of funds necessary for that investment. Additionally, individuals decide how much time to devote to education. In the second period (adulthood), individuals supply efficient units of labor, pay a lump-sum tax, repay educational loans, consume final goods, and save any remaining income. In the final period (old age), individuals retire and consume final goods. Members of the cohort born in period $t-1$ become active workers in period t; thus, we call this cohort generation t. The population size is constant and normalized to one. In the first and second periods of life, individuals are endowed with one unit of time. In the first period, they spend e_t units of time on education and $1-e_t$ on leisure. In the second period, they devote l_t units of time to working in the labor market and spend $1-l_t$ on leisure. Individuals derive utility from leisure in the first and second periods, their own consumption during adulthood $c_{1,t}$, and their own consumption during old age $c_{2,t+1}$. The lifetime utility of individuals in generation t is expressed as

$$u_t = \gamma \log(1-e_{t-1}) + \beta\{\delta[\log c_t + \gamma \log(1-l_t)] + \delta^2 \log c_{t+1}\}. \tag{1}$$

where the positive parameter γ denotes the weight of leisure, and $\delta \in (0,1)$ is the long-term discount factor. Since we assume that $\beta \in (0,1]$, agents evaluate current utility more heavily than future utility. In other words, β is a parameter denoting present bias. Behavioral economics studies consider two types of agents. First, sophisticated agents understand that their preferences change over time. In this model, they know that their future selves will also have the present bias parameter β and plan accordingly. In other words, they recognize that their future selves will maximize $\log c_t + \gamma \log(1-l_t) + \beta\delta \log c_{t+1}$ in the second period. Second, naïve agents do not understand that their preferences change over time. In this model, they believe their future selves will not exhibit present bias, and therefore, they expect no deviation from their current plans. That is, they believe their future selves will maximize $\log c_t + \gamma \log(1-l_t) + \delta \log c_{t+1}$ in the second period. In this study, we consider the case where all agents are naïve.

Individuals divide their income $w_t h_t l_t$ between consumption, repayment of educational loans, payment of a lump sum tax, and saving s_t for old age. Here, w_t and τ_t are the wage rate for efficient units of labor and the lump-sum tax, respectively. R_t represents the gross interest rate in period t. Individuals must repay the borrowed funds plus interest, but the government subsidizes ρ percent of the repayment amount. In addition, they receive $R_{t+1}s_t$ in their old age, which is their savings plus interest. Thus, the budget and time constraints for individuals in generation t are expressed as follows:

$$c_t + s_t + (1-\rho)R_t x_{t-1} + \tau_t = w_t h_t l_t, \tag{2}$$

$$c_{t+1} = R_{t+1} s_t. \tag{3}$$

We assume that the human capital production function is given by the following expression:

$$h_t = \phi(e_{t-1}h_{t-1})^{\sigma} x_{t-1}^{1-\sigma}, \tag{4}$$

where ϕ and σ are parameters. h_{t-1} reflects externalities from the human capital stock of parents.

2.2 Firm

Assuming that there are many competitive producers with constant-returns-to-scale production technology, the aggregate production function of the economy is given by:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{5}$$

where Y_t, K_t, and L_t denote aggregate output, physical capital, and effective labor employed in period t, respectively. A_t is the productivity parameter and satisfies $A > 0$. The profit maximization conditions are:

$$w_t = (1-\alpha)A\left(\frac{K_t}{L_t}\right)^{\alpha}, \tag{6}$$

$$R_t = \alpha A\left(\frac{K_t}{L_t}\right)^{\alpha-1}. \tag{7}$$

2.3 Government

The government collects lump-sum taxes from individuals and subsidizes ρ percent of the repayment of educational loans. The subsidy is financed by a balanced budget. Therefore, the government's budget constraint is:

$$\tau_t = \rho R_t x_{t-1}. \tag{8}$$

3 The agent's optimization problem

In this study, we consider the case in which all agents are naïve. From (1), (2), (3), and (4), in the first period, the young agent solves the following utility maximization problem:

$$
\begin{aligned}
\max \quad & \gamma \log(1-e_{t-1}) + \beta\{\delta[\log c_t + \gamma \log(1-l_t)] + \delta^2 \log c_{t+1}\}, \\
\text{s.t.} \quad & c_t + s_t = w_t h_t l_t - R_t x_{t-1} - \tau_t, \\
& c_{t+1} = R_{t+1} s_t, \\
& h_t = \phi(e_{t-1} h_{t-1})^{\sigma} x_{t-1}^{1-\sigma}.
\end{aligned}
$$

Combining the first-order conditions and (8), we obtain the following solutions:

$$e_{t-1} = \frac{\beta\delta(1+\delta)(1-\rho)\sigma}{\beta\delta(1+\delta)(1-\rho)\sigma + \gamma(\sigma-\rho)} \equiv \bar{e}(\rho), \tag{9}$$

$$l_t = \frac{(1+\delta)(1-\rho)}{(1+\delta)(1-\rho) + \gamma(\sigma-\rho)} \equiv \hat{l}(\rho), \tag{10}$$

$$x_{t-1} = \frac{(1-\sigma)\hat{l}(\rho)\hat{w}_t \hat{h}_t}{(1-\rho)\hat{R}_t} \equiv \bar{x}_t, \tag{11}$$

$$s_t = \frac{\delta(\sigma-\rho)\hat{w}_t h_t}{(1+\delta)(1-\rho) + \gamma(\sigma-\rho)} \equiv \hat{s}_t, \tag{12}$$

$$c_t = \frac{1}{\delta}\hat{s}_t \equiv \hat{c}_t, \tag{13}$$

$$c_{t+1} = \hat{R}_t \hat{s}_t \equiv \hat{c}_{t+1}. \tag{14}$$

We define the right-hand sides of (9) to (14) as $\bar{e}(\rho)$, $\hat{l}(\rho)$, $\bar{x}_t$, $\hat{s}_t$, $\hat{c}_t$, and $\hat{c}_{t+1}$, respectively. Young agents of generation t solve the above optimization problem at time $t-1$. Therefore, they must predict future variables. The variables with a "$\hat{}$" represent the values predicted by young agents. As we will see later, the labor supply determined by the young agent differs from that determined by the adult agent. The naïve agent believes that the labor supply chosen in the first period will be the same as that chosen in the second period, so actual and predicted values differ.

In the second period, the naïve agent discounts the utility of the third period. The values of e_{t-1} and x_{t-1}, determined in the first period, cannot be changed in the second period. Therefore, from (2), (3), (9), and (11), the adult agent solves the following utility maximization problem with

e_{t-1} and x_{t-1} given:

$$\begin{aligned} \max \quad & \log c_t + \gamma \log(1 - l_t) + \beta\delta \log c_{t+1}, \\ \text{s.t.} \quad & c_t + s_t = w_t h_t l_t - (1-\rho) R_t x_{t-1} - \tau_t, \\ & c_{t+1} = R_{t+1} s_t, \\ & e_{t-1} = \bar{e}(\rho), \\ & x_{t-1} = \bar{x}_{t-1}. \end{aligned}$$

We obtain the following solutions:

$$l_t = \frac{1+\beta\delta}{1+\beta\delta+\gamma} + \frac{\gamma(1-\sigma)\hat{l}(\rho)\hat{w}_t\hat{h}_t R_t}{(1+\beta\delta+\gamma)(1-\rho)w_t h_t \hat{R}_t} \equiv \bar{l}_t(\rho), \tag{15}$$

$$s_t = \frac{\beta\delta}{1+\beta\delta+\gamma} w_t h_t - \frac{\beta\delta(1-\sigma)\hat{l}(\rho)\hat{w}_t\hat{h}_t R_t}{(1+\beta\delta+\gamma)(1-\rho)\hat{R}_t}. \tag{16}$$

We define the right-hand side of (15) as $\bar{l}_t(\rho)$. We make the following assumption to ensure positive equilibrium savings:

Assumption 1.

$$\rho < \sigma$$

4 Equilibrium

4.1 Market equilibrium

We first describe the labor market equilibrium condition. Effective labor is used for the production of final goods. A naïve agent determines a different labor supply in the first period from that in the second period. However, the naïve agent in the first period believes that the labor supply determined in that period will be the same as in the second period. Therefore, they predict that the following labor market equilibrium will hold in the second period at time $t-1$.

$$\hat{L}_t = \hat{l}(\rho)\hat{h}_t, \tag{17}$$

where $\hat{L}_t$ denotes the labor supply at time t predicted by the young at time $t-1$. This equation implies the following assumption: a naïve agent predicts that the other agents will also behave as naïve agents.[1] Hereafter, we will also adopt it. In reality, adults at time t solve the utility maximization problem again at time t. Therefore, the labor market equilibrium at time t is

$$L_t = \bar{l}_t(\rho)h_t. \tag{18}$$

Next, we describe the equilibrium condition of the asset market. The asset market equilibrium condition is:

$$s_t = K_{t+1} + x_t. \tag{19}$$

The left-hand side of (19) is the saving volume of adult individuals. The right-hand side is the sum of investment in capital and demand for education funds.

From (4) and (9), the predicted value of h_{t+1} for the young at time t is

$$\hat{h}_{t+1} = \phi(\bar{e}(\rho)h_t)^{\sigma}x_t^{1-\sigma}. \tag{20}$$

From (11) and (20), we obtain:

$$x_t = \left[\frac{\phi(1-\sigma)\hat{l}(\rho)\hat{w}_{t+1}}{(1-\rho)\hat{R}_{t+1}}\right]^{\frac{1}{\sigma}}\bar{e}(\rho)h_t. \tag{21}$$

From (6), (7), and (17), we obtain:

$$\frac{\hat{w}_{t+1}}{\hat{R}_{t+1}} = \frac{(1-\alpha)\hat{K}_{t+1}}{\alpha\hat{L}_{t+1}}. \tag{22}$$

From (10), (17), (21), and (22), we obtain:

$$x_t = \left[\frac{(1-\alpha)(1-\sigma)\phi\hat{K}_{t+1}}{\alpha(1-\rho)\hat{h}_{t+1}}\right]^{\frac{1}{\sigma}}\bar{e}(\rho)h_t. \tag{23}$$

From (20) and (23), we find that x_t is a function of $\hat{K}_{t+1}$ and h_t. Therefore, we denote $x_t =$

[1] This assumption adopted by Gabrieli and Ghosal (2013), Ojima (2017), and Futagami and Maeda (2023).

$x(\hat{K}_{t+1}, h_t)$. From (19), we obtain:

$$s_t = K_{t+1} + x(\hat{K}_{t+1}, h_t). \tag{24}$$

The value of s_t is determined by adults at time t, and is therefore given for the young. The value of x_t has already been determined at time t, so it cannot be changed at time $t+1$. From (24), the young predict K_{t+1} to satisfy $s_t = \hat{K}_{t+1} + x(\hat{K}_{t+1}, h_t)$ at time t. At the end of time t, s_t and x_t are determined, and $\hat{K}_{t+1} = K_{t+1}$ is established to satisfy (24). Therefore, the predicted value of K_{t+1} is the same as the realized value. When K_{t+1} is given, $\hat{h}_{t+1}$ is determined to be $\hat{h}_{t+1} = h_{t+1}$ using (20) and (23). Therefore, we rewrite (17) as follows:

$$\hat{L}_{t+1} = \hat{l}(\rho)h_{t+1}. \tag{25}$$

From (6), (7), (10), (15), (18), and (25), we obtain:

$$\bar{l}_t(\rho) = \frac{(1+\beta\delta)(1-\rho)}{(1+\beta\delta)(1-\rho) + \gamma(\sigma-\rho)} \equiv \bar{l}(\rho). \tag{26}$$

We define the right-hand side of (26) as $\bar{l}(\rho)$. From (6), (7), (16), (25), and (26), we obtain:

$$s_t = \frac{(1-\alpha)\beta\delta(\sigma-\rho)A}{(1+\beta\delta)(1-\rho) + \gamma(\sigma-\rho)} h_t \left[\frac{k_t}{\bar{l}(\rho)}\right]^{\alpha}, \tag{27}$$

where we define k_t as $k_t \equiv \frac{K_t}{h_t}$.

4.2 Dynamics

The dynamics of this economy are characterized by k_t. From (10), (20), (23), $\hat{K}_{t+1} = K_{t+1}$, $\hat{h}_{t+1} = h_{t+1}$, we obtain:

$$\frac{h_{t+1}}{h_t} = \phi\bar{e}(\rho) \left[\frac{(1-\alpha)(1-\sigma)\phi}{\alpha(1-\rho)}\right]^{\frac{1-\sigma}{\sigma}} k_{t+1}^{\frac{1-\sigma}{\sigma}} \equiv 1 + g_{h,t}. \tag{28}$$

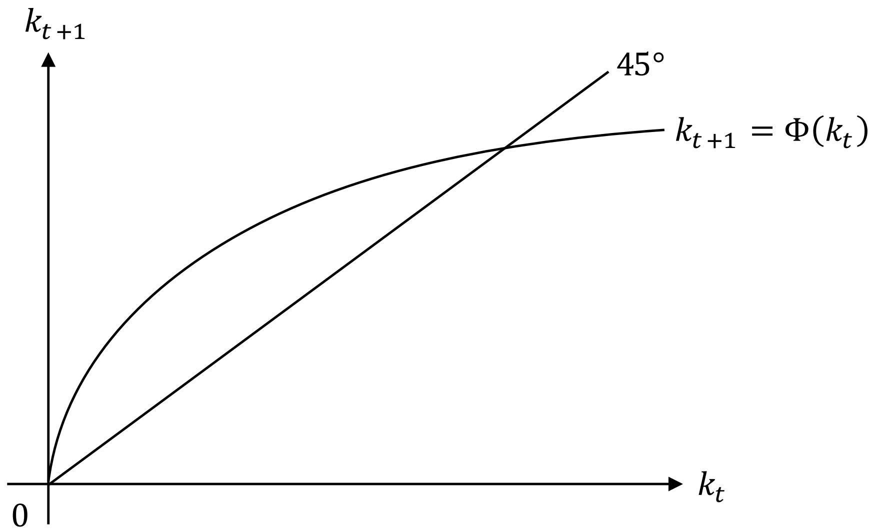


Figure 1: The dynamics of k_t

We define the right-hand side of (28) as $1 + g_{h,t}$. From (18), (19), (23), (25),(26), and (27), with $\hat{K}_{t+1} = K_{t+1}$, and $\hat{h}_{t+1} = h_{t+1}$, we obtain:

$$\frac{(1-\alpha)\beta\delta(\sigma-\rho)A}{(1+\beta\delta)(1-\rho)+\gamma(\sigma-\rho)}\left(\frac{k_t}{\bar{l}(\rho)}\right)^{\alpha} = k_{t+1}\frac{h_{t+1}}{h_t} + \left[\frac{(1-\alpha)(1-\sigma)\phi}{\alpha(1-\rho)}\right]^{\frac{1}{\sigma}} \bar{e}(\rho)k_{t+1}^{\frac{1}{\sigma}}. \qquad (29)$$

By substituting (28) into (29), we obtain:

$$k_{t+1} = \frac{\alpha(1-\rho)}{(1-\alpha)(1-\sigma)\phi}\left\{\frac{(1-\alpha)^2\beta\delta(\sigma-\rho)(1-\sigma)A}{[(1+\beta\delta)(1-\rho)+\gamma(\sigma-\rho)][1-\sigma+\alpha(\sigma-\rho))]\bar{e}(\rho)\bar{l}(\rho)^{\alpha}}\right\}^{\sigma} k_t^{\alpha\sigma} \equiv \Phi(k_t). \qquad (30)$$

We define the right-hand side of (30) as $\Phi(k_t)$. We find that $\Phi(0) = 0$, $\Phi'(k_t) > 0$, $\Phi''(k_t) < 0$, $\lim_{k_t \to 0} \Phi'(k_t) = \infty$, and $\lim_{k_t} \Phi'(k_t) \to 0$. Therefore, the phase diagram is shown in Figure 1. From Figure 1, the economy converges monotonically to the unique steady state. We define the steady-state value of k_t as k^*. From (30), we obtain:

$$k^* = \left(\frac{\alpha(1-\rho)}{(1-\alpha)(1-\sigma)\phi}\left\{\frac{(1-\alpha)^2\beta\delta(\sigma-\rho)(1-\sigma)A}{[(1+\beta\delta)(1-\rho)+\gamma(\sigma-\rho)][1-\sigma+\alpha(\sigma-\rho)]\bar{e}(\rho)\bar{l}(\rho)^{\alpha}}\right\}^{\sigma}\right)^{\frac{1}{1-\alpha\sigma}} \qquad (31)$$

From (14), we obtain:

$$1 + g_{Y,t} = (1 + g_{k,t})^{\alpha}(1 + g_{h,t}). \tag{32}$$

where we define $1 + g_{Y,t}$ and $1 + g_{k,t}$ as $1 + g_{Y,t} \equiv \frac{Y_{t+1}}{Y_t}$ and $1 + g_{k,t} \equiv \frac{k_{t+1}}{k_t}$. In the steady state, $g_{k,t} = 0$ holds. Therefore, from (33), we obtain:

$$g_{Y,t} = g_{h,t}. \tag{33}$$

From (28), (31), and (33), the steady-state growth rate g_Y is given by:

$$1 + g_Y = \phi\Omega \left[\bar{e}(\rho)^{\frac{(1-\alpha)\sigma}{1-\sigma}} \Lambda(\rho) \right]^{\frac{1-\sigma}{1-\alpha\sigma}}, \tag{34}$$

where $\Omega \equiv \left\{ (1-\alpha)^2 \beta\delta(1-\sigma) A \left[\frac{\alpha}{(1-\alpha)(1+\beta\delta)(1-\sigma)\phi} \right]^{\alpha} \right\}^{\frac{1-\sigma}{1-\alpha-\sigma}}$ and $\Lambda(\rho) \equiv \frac{\sigma-\rho}{[(1+\beta\delta)(1-\rho)+\gamma(\sigma-\rho)]^{1-\alpha}[1-\sigma+\alpha(\sigma-\rho)]}$. We obtain Proposition 1.

Proposition 1. *We define $\Psi(\rho, \sigma)$ and $\Upsilon(\sigma)$ as $\Psi(\rho, \sigma) \equiv -\alpha(1-\alpha)(1+\beta\delta+\gamma)(\rho-\sigma)^2 - \alpha(1+\beta\delta+\gamma)(1-\sigma)(\rho-\sigma) + (1+\beta\delta)(1-\sigma)^2$ and $\Upsilon(\sigma) \equiv \frac{(1-\alpha)\gamma\sigma(1+\beta\delta+\gamma\sigma)[1-(1-\alpha)\sigma]}{\gamma+\beta\delta(1+\delta)}$, respectively. We also define $\bar{\sigma}$ as the value of σ such that $\Psi(0, \sigma) = \Upsilon(\sigma)$ holds. If $\sigma > \bar{\sigma}$, there exists a value of ρ that maximizes the steady-state growth rate g_Y.*

Proof. See Appendix A. □

If ρ increases, government expenditure on education increases. Therefore, human capital accumulation is promoted, and the growth rate increases. However, a higher ρ also reduces disposable income because taxes are increased to fund government education spending. Therefore, individuals' savings decrease, investment in physical capital declines, and the growth rate falls. Hence, an increase in ρ has an inverted U-shaped effect on the steady-state growth rate g_Y.

We obtain Proposition 2.

Proposition 2. *As β increases, the scholarship subsidy rate $\bar{\rho}$ that maximizes the growth rate decreases.*

Proof. See Appendix B. □

As β rises (i.e., present bias decreases), the level of effort put into education, e_t, increases. Due to the complementarity of the education production function, investment in education through goods, x_t, also increases. Therefore, goods tend to be used excessively for education. In this case, the growth rate would increase if the proportion of scholarship subsidies were reduced, the amount of investment in education x_t were moderated, and more resources were allocated to physical capital investment.

5 Numerical Analysis

In this section, we demonstrate how a scholarship subsidy affects welfare and growth quantitatively. First, we consider the impact on social welfare, defined as the discounted sum of each generation's welfare, as in Grossmann (2007). We numerically show that the subsidy rate that maximizes economic growth does not necessarily coincide with the subsidy rate that maximizes social welfare. Second, we demonstrate how changes in the subsidy rate affect each generation's welfare over time.

We choose the parameters of the model such that the growth rate fits empirical observations for advanced countries. There are seven structural parameters: $\{\alpha, \beta, \gamma, \delta, \sigma, \phi, A\}$. We set the capital share α to 0.33. As explained earlier, β is the parameter denoting present bias. As β approaches 1, the bias diminishes, implying that the agent becomes more time-consistent. To evaluate how the magnitude of present bias quantitatively changes the results, we conduct numerical analyses using various values—specifically 0.6 and 0.7—based on the behavioral economics literature. In line with Cipriani and Fioroni (2024), we set the weight of leisure in youth and adulthood, γ, equal to 0.2. We regard one period in this overlapping-generations economy as 25 years and set $\delta = (0.98)^{25}$. A large number of studies on economic growth in OLG frameworks employ a Cobb-Douglas-type human capital production function. As most set the elasticity parameter between 0.6 and 1 in numerical analyses, we adopt an intermediate value of $\sigma = 0.8$. Following Cardak (2004), we set ϕ at 1.6. The remaining parameter, A, is calibrated to match the observed growth rate in advanced countries. Assuming an annual growth rate of 2%, the corresponding targeted growth rate over 25 years is approximately 1.64.

5.1 Social welfare

Following Grossmann (2007), we define the social welfare function as

$$W = \sum_{t=0}^{\infty} \varepsilon^t V_t, \tag{35}$$

where $\varepsilon \in (0, 1)$ is the time preference rate of the social planner. In the following numerical analyses, we adopt an intermediate value, setting $\varepsilon = 0.5$. Substituting solutions (9), (10), (13), and (14) into (1), we obtain the indirect utility of the t-th generation as follows:

$$V_t \equiv \gamma \log[1 - \bar{e}(\rho)] + \beta\gamma \log[1 - \hat{l}(\rho)] - \beta\delta \log \delta + \beta\delta(1 + \delta) \log \hat{s}_t + \beta\delta^2 \log \hat{R}_{t+1}. \tag{36}$$

Substituting (36) into (35) and rearranging the equation, we derive

$$\begin{aligned} W = &\frac{1}{1-\varepsilon} \Big\{ \gamma \log[1 - \bar{e}(\rho)] + \beta\gamma \log[1 - \hat{l}(\rho)] - \beta\delta \log \delta + \beta\delta^2 \log \alpha A \\ &+ \beta\delta(1+\delta) \log \frac{(1-\alpha)\delta(\sigma-\rho)Ah_0}{(1+\delta)(1-\rho) + \gamma(\sigma-\rho)} + \beta\delta[\alpha(1+2\delta) - \delta] \log \frac{\hat{k}^*}{\hat{l}(\rho)} \Big\} \\ &+ \frac{\beta\delta(1+\delta)\varepsilon}{(1-\varepsilon)^2} \log(1 + \hat{g}_h), \end{aligned} \tag{37}$$

where h_0 denotes the initial value of human capital, and we set $h_0 = 1$.

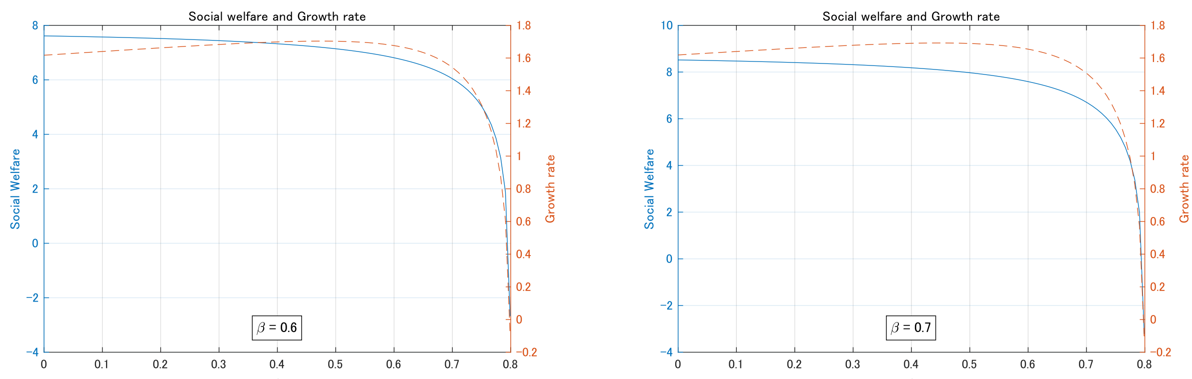


Figure 2: Numerical examples of social welfare and the growth rate. The horizontal axis represents the scholarship subsidy $\rho \in [0, \sigma)$. The left and right y-axes represent social welfare and the growth rate, respectively. The solid line shows social welfare; the dashed line indicates the growth rate.

Figure 2 compares the scholarship subsidy rate that maximizes social welfare with the one that maximizes the growth rate. In all three cases, the growth rate exhibits an inverted U-shape, indicating that a positive subsidy rate maximizes growth. However, social welfare decreases monotonically with ρ. Therefore, we confirm that the subsidy rate that maximizes economic growth does not necessarily coincide with the rate that maximizes social welfare.

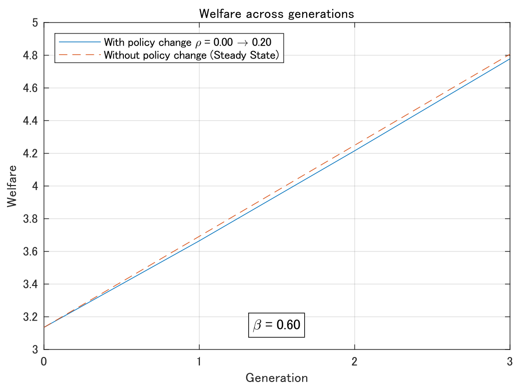
5.2 Generation's welfare

We examine the effect of policy changes on each generation's welfare in this subsection. To evaluate each generation's welfare, we take the period of their birth as the reference point. In other words, the t-th generation's welfare is measured from the perspective of period $t-1$ (young). Therefore, all variables from period t onward are predicted variables. Substituting (13) and (14) into (36), the t-th generation's welfare can be rewritten as follows:

$$
\begin{aligned}
V_t = {} & \gamma \log[1-\bar{e}(\rho)] + \beta\gamma \log[1-\hat{l}(\rho)] - \beta\delta \log\delta \\
& + \beta\delta(1+\delta)\log\left[\frac{(1-\alpha)\delta(\sigma-\rho)A}{(1+\delta)(1-\rho)+\gamma(\sigma-\rho)}\right]\hat{h}_t\left[\frac{\hat{k}_t}{\hat{l}(\rho)}\right]^{\alpha} \\
& + \beta\delta^2 \log\left[\alpha A \hat{l}(\rho)^{1-\alpha}\hat{k}_{t+1}^{\alpha-1}\right].
\end{aligned} \tag{38}
$$

Because the predicted value of human capital h_t appears in the fourth term of (38), the welfare level varies across generations. Changes in the subsidy rate ρ influence a generation's welfare through three channels: First, they affect the optimal choices of educational effort $\bar{e}(\rho)$ and predicted labor supply $\hat{l}(\rho)$. Second, they alter the predicted value of capital per unit of human capital, $\hat{k}_t^*$. Third, they impact the growth rate of human capital, g_h.

Suppose the economy is in a steady state at time zero. The government raises the scholarship subsidy from $\rho = 0$ to $\rho = 0.2$ at time 1. Figure 3 shows the welfare levels of each generation with and without the policy change. Regardless of the degree of present bias, increasing the subsidy rate slightly reduces welfare for the next generation. However, the policy promotes human capital accumulation, resulting in a higher growth rate. Due to this increased growth rate, welfare eventually exceeds the level observed without the policy change.



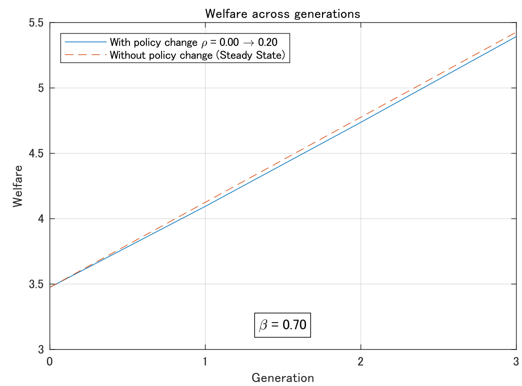


Figure 3: Response to a change in the subsidy rate.

6 Conclusion

We constructed an overlapping-generations model with endogenous education choice and hyperbolic discounting preferences. We showed that there exists a growth-maximizing subsidy rate for educational loans. In addition, a stronger present bias raises the growth-maximizing subsidy rate. This result arises because more impatient agents invest less in education and accumulate less human capital. Consequently, economic growth declines, requiring higher subsidies to restore it.

Appendix

Appendix A: Proof of Proposition 1

From (34), we obtain:

$$\frac{dg_Y}{d\rho} = \frac{(1-\sigma)\phi\Omega}{1-\alpha\sigma}\left[\bar{e}(\rho)^{\frac{(1-\alpha)\sigma}{1-\sigma}}\Lambda(\rho)\right]^{\frac{1-\sigma}{1-\alpha\sigma}}\left\{\frac{(1-\alpha)\sigma}{1-\sigma}\frac{\bar{e}'(\rho)}{\bar{e}(\rho)} - \left[-\frac{\Lambda'(\rho)}{\Lambda(\rho)}\right]\right\}. \tag{A1}$$

From (28), we obtain:

$$\frac{(1-\alpha)\sigma}{1-\sigma}\frac{\bar{e}'(\rho)}{\bar{e}(\rho)} = \frac{(1-\alpha)\gamma\sigma}{(1-\rho)[(\gamma(\sigma-\rho)+\beta\delta(1+\delta)(1-\rho)\sigma]}. \tag{A2}$$

From (A2), the right-hand side becomes $\frac{(1-\alpha)\gamma}{\beta\delta(1+\delta)(1-\sigma)^2}$ when $\rho = \sigma$. In addition, the right-hand side of (A2) increases monotonically with ρ for $0 < \rho < \sigma$. From $\Lambda(\rho) \equiv \frac{\sigma-\rho}{[(1+\beta\delta)(1-\rho)+\gamma(\sigma-\rho)]^{1-\alpha}[1-\sigma+\alpha(\sigma-\rho)]}$ and $\Psi(\rho,\sigma) \equiv -\alpha(1-\alpha)(1+\beta\delta+\gamma)(\rho-\sigma)^2 - \alpha(1+\beta\delta+\gamma)(1-\sigma)(\rho-\sigma) + (1+\beta\delta)(1-\sigma)^2$, we

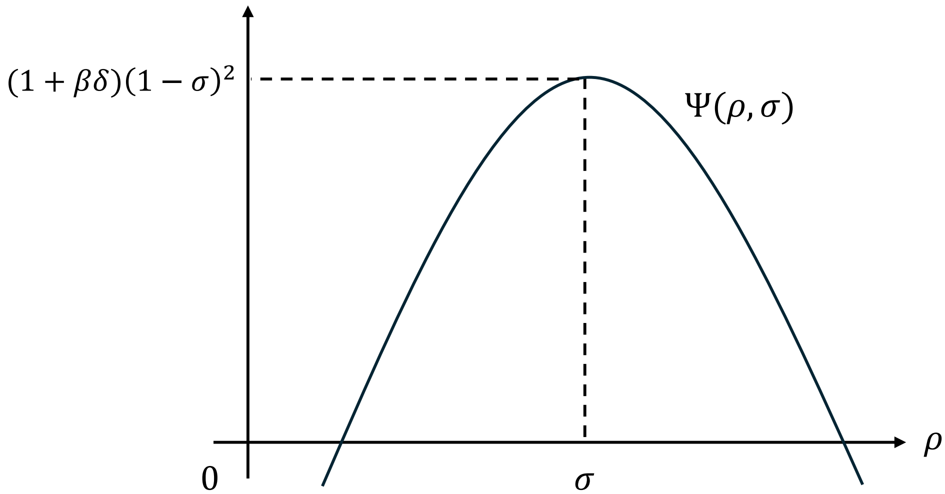


Figure 4: The shape of $\Psi(\rho, \sigma)$

obtain:

$$-\frac{\Lambda'(\rho)}{\Lambda(\rho)} = \frac{\Psi(\rho, \sigma)}{(\sigma - \rho)[(1 + \beta\delta)(1 - \rho) + \gamma(\sigma - \rho)][1 - \sigma + \alpha(\sigma - \rho)]}. \tag{A3}$$

The shape of $\Psi(\rho, \sigma)$ is shown in Figure 4. From the figure, we find that $\lim_{\rho\to\sigma} \Psi(\rho, \sigma) = (1 + \beta\delta)(1-\sigma)^2$. As ρ to σ, the denominator of (A3) approaches zero. Therefore, $-\frac{\Lambda'(\rho)}{\Lambda(\rho)} \to \infty$ as $\rho \to \sigma$. In addition, the denominator of (A3) decreases monotonically with ρ for $0 < \rho < \sigma$, and from Figure 4, the numerator increases monotonically. Therefore, the entire expression in (A3) increases monotonically with ρ in that range. The relationship between (A2) and (A3) is shown in Figure 5. From the figure, if $-\frac{\Lambda'(0)}{\Lambda(0)} < \frac{(1-\alpha)\sigma}{1-\sigma} \frac{\bar{e}'(0)}{\bar{e}(0)}$, then $-\frac{\Lambda'(\rho)}{\Lambda(\rho)}$ crosses $\frac{(1-\alpha)\sigma}{1-\sigma} \frac{\bar{e}'(\rho)}{\bar{e}(\rho)}$ from below. From (A2), we obtain:

$$\frac{(1-\alpha)\sigma}{1-\sigma} \frac{\bar{e}'(0)}{\bar{e}(0)} = \frac{(1-\alpha)\gamma}{\gamma + \beta\delta(1+\delta)}. \tag{A4}$$

From (A3), we obtain:

$$-\frac{\Lambda'(0)}{\Lambda(0)} = \frac{\Psi(\rho, \sigma)}{\sigma(1 + \beta\delta + \gamma\sigma)[1 - (1-\alpha)\sigma]}. \tag{A5}$$

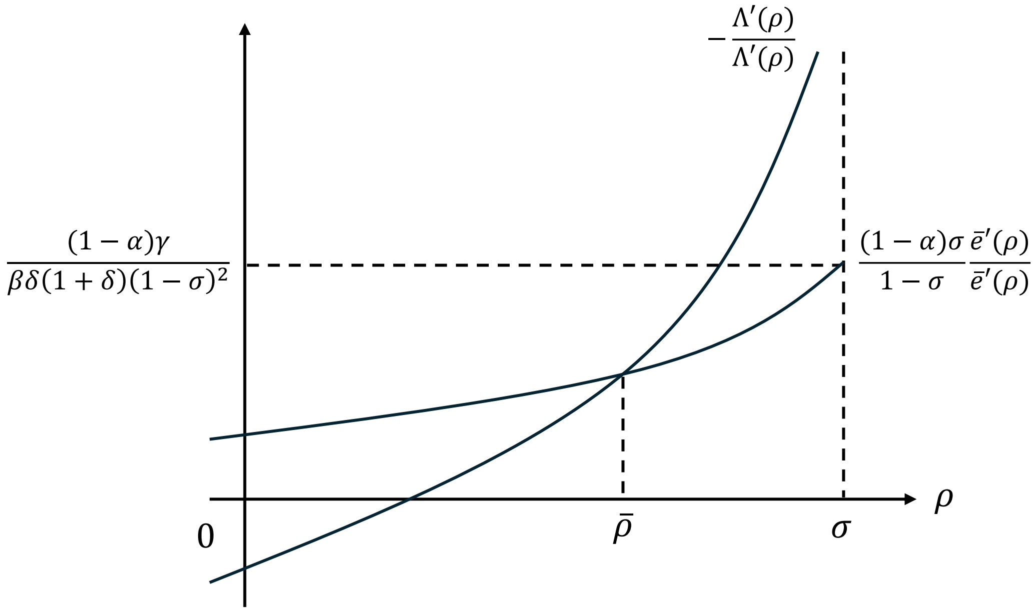


Figure 5: The shapes of $\frac{(1-\alpha)\sigma}{1-\sigma}\frac{\bar{e}'(\rho)}{\bar{e}(\rho)}$ and $-\frac{\Lambda'(\rho)}{\Lambda(\rho)}$

From (A4) and (A5), we obtain:

$$-\frac{\Lambda'(0)}{\Lambda(0)} < \frac{(1-\alpha)\sigma}{1-\sigma}\frac{\bar{e}'(0)}{\bar{e}(0)},$$

$$\Psi(0,\sigma) < \Upsilon(\sigma), \tag{A6}$$

where $\Psi(0,\sigma) = -\alpha(1-\alpha)(1+\beta\gamma+\gamma)\sigma^2 + \alpha(1+\beta\delta+\gamma)\sigma(1-\sigma) + (1+\beta\delta)(1-\sigma)^2$ and $\Upsilon(\sigma) \equiv \frac{(1-\alpha)\gamma\sigma(1+\beta\delta+\gamma\sigma)[1-(1-\alpha)\sigma]}{\gamma+\beta\delta(1+\delta)}$. The shapes of $\Psi(0,\sigma)$ and $\Upsilon(\sigma)$ are shown in Figure 6. From the figure, we see that $\Psi(0,\sigma) = \Upsilon(\sigma)$ has a unique solution with respect to σ. We define this solution as $\bar{\sigma}$. If $\sigma > \bar{\sigma}$, then there exists a value of ρ that maximizes the steady-state growth rate g_Y.

Appendix B: Proof of Proposition 2

From (A1), we obtain:

$$\frac{d}{d\beta}\frac{(1-\alpha)\sigma}{1-\sigma}\frac{\bar{e}'(\rho)}{\bar{e}(\rho)} = -\frac{(1-\alpha)\gamma\delta(1+\delta)\sigma^2}{[\gamma(\sigma-\rho)+\beta\delta(1+\delta)(1-\rho)\sigma]^2} < 0, \tag{39}$$

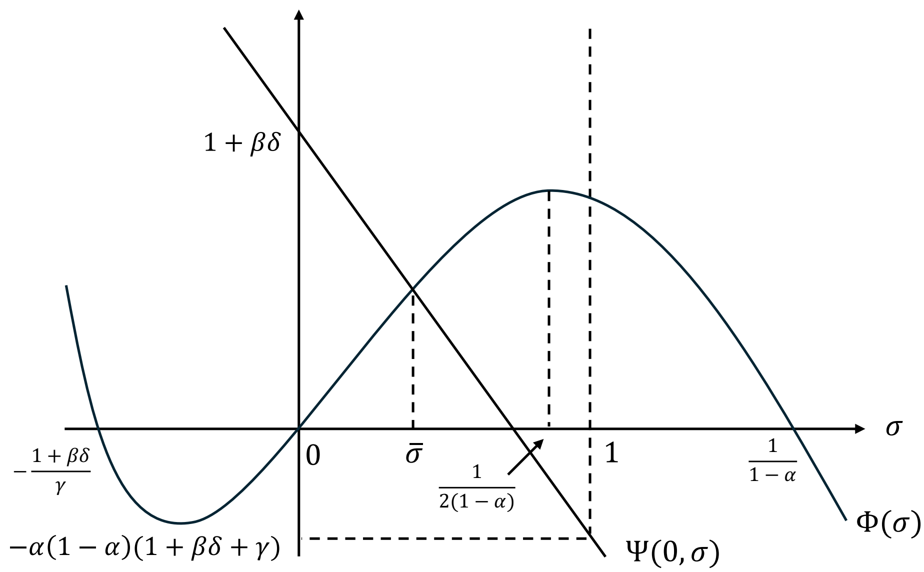


Figure 6: The shapes of $\Psi(0,\sigma)$ and $\Upsilon(\sigma)$

From (A3), we obtain:

$$\frac{d}{d\beta}\left[-\frac{\Lambda'(\rho)}{\Lambda(\rho)}\right] = \frac{(1+\alpha)\gamma\delta(1-\sigma)}{[(1+\beta\delta)(1-\rho)+\gamma(\sigma-\rho)]^2} > 0. \tag{40}$$

From (39) and (40), we obtain Figure 7. From Figure 7, as β increases, $\bar{\rho}$ decreases.

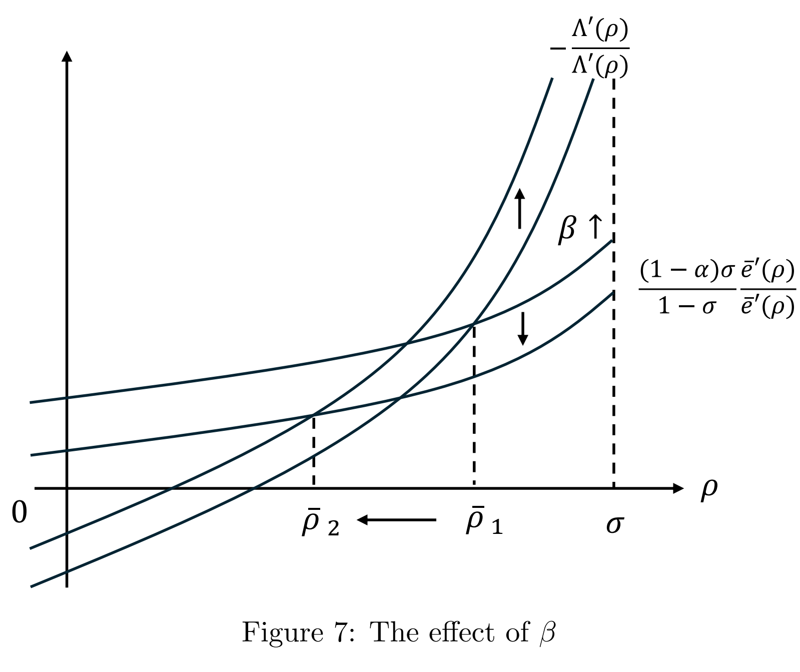


Figure 7: The effect of β

References

[1] Boldrin, M., Montes, A. (2005) "The Intergenerational State Education and Pensions". *Review of Economic Studies*, 72, 651-664.

[2] Cardak, B. A. (2004). Education choice, endogenous growth and income distribution. *Economica*, 71(281), 57-81.

[3] Cipriani, G. P., Fioroni, T. (2024). "Human capital and pensions with endogenous fertility and retirement". *Macroeconomic Dynamics*, 28(2), 478-494.

[4] Grossmann, V. (2007). How to promote R&D-based growth? Public education expenditure on scientists and engineers versus R&D subsidies. *Journal of Macroeconomics*, 29(4), 891-911.

[5] Futagami, K., Maeda, D. (2023) "Naive agents with non-unitary discounting rate in a monetary economy." *Journal of Macroeconomics*, 78, 103550.

[6] Hiraguchi, R. (2016) "On a two-sector endogenous growth model with quasi-geometric discounting". *Journal of Mathematical Economics*, 65, 26-35.

[7] Gabrieli, T., Ghosal, S. (2013) "Non-existence of competitive equilibria with dynamically inconsistent preferences." *Economic Theory*, 52, 299-313.

[8] Hori, T., Futagami, K. (2019) "A Non-unitary Discount Rate Model" *Economica*, 86(341), 139-165.

[9] Krusell, P., Kuruscu, B., Smith A. (2002) "Equilibrium welfare and government policy with quasi-geometric discounting". *Journal of Economic Theory*, 105, 42-72.

[10] Laibson, I. D. (1996) "Hyperbolic discounting functions, Undersaving and savings policy". *National Bureau of Economic Research Working Paper*, No. 5635

[11] Ojima, T. (2017). "General equilibrium dynamics with naïve and sophisticated hyperbolic consumers in an overlapping generations economy." *Economica*, 85(338), 281-304.