Chukyo University Institute of Economics

Discussion Paper Series

July 2025

No. 2502

Pollution, reproductive health capital, and fertility

Akira Ykita
Nagoya City University

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Nagoya City University

Abstract
Recent research has indicated that the human infertility rate is increasing. This study tests the hypothesis that this change results from increasing pollution stock, which consequently damages the reproductive health of individuals. Individuals can choose the number of their children without worrying about pollution effects when the pollution stock is small. Nevertheless, when the pollution stock becomes great relative to reproductive health capital, it starts constraining the number of children that individuals can bear and rear. Reduced childrearing time enables young workers to work longer and to earn more, consequently boosting economic growth, although such a constraint might retard economic growth temporarily. The accelerated physical capital accumulation in turn increases pollution stock, thereby lowering the fertility rate more. On a balanced growth path, the fertility rate remains constant over periods, but the fertility rate might be lower than the population replacement rate. Pollution abatement measures raise the fertility rate by reducing pollution damage to reproductive health. Although the long-term growth effect of the pollution tax changes is ambiguous, the balanced growth rate after the tax changes is lower than the balanced growth rate without fertility constraint binding.

Keywords: fertility, human infertility, pollution stock, reproductive health capital
JEL Classification: H23, J11, J13, Q54, Q56, Q58

Declaration of conflict of interest: The author has no competing interest or personal relation to this study

Data availability: The author does not analyze any dataset.

Funding information: Japan Society for the Promotion of Science (KAKENHI grant No. 25K05125)

Correspondence: A. Yakita, Graduate School of Economics, Nagoya City University, 1 Yamanohata, Mizuho-cho, Mizuho-ku, Nagoya 467-0802, Japan
E-mail: yakita@econ.nagoya-cu.ac.jp ORCID: 0009-0004-2728-1321

1. Introduction

Infertility has been recognized as an important health problem worldwide in these days (World Health Organization (WHO), 2023).[1] Recent research in medical science has found that pollution might exert significant effects on fertility by damaging reproductive health. Barreca et al. (2015) emphasize health effects of temperature fluctuation on the timing of childbirth. Ma et al. (2018) report that environmental chemicals such as heavy metals and pesticides can affect adversely fertility through human health capital: e.g., elementary mercury (Hg) increases spontaneous abortions. Human exposure to bisphenol A (BPA), a plasticizer, makes women sterile (Ziv-Gal and Flaws, 2016) and increases risks of erectile difficulty for male workers (Li et al., 2010). Stump and Szabo-Morvai (2025), using six years of data, examine widely various European regions and neighboring areas and report that particulate matter ($PM_{2.5}$ and PM_{10}) specifically has significant effects on birth rates. Nieuwenhuijsen et al. (2014) also report that exposure to air pollution reduces fertility significantly.[2] This study is intended to present an economic analysis of pollution effects on fertility behaviors of individuals.

Because of the complex causality linking pollution and fertility, analyses of their mutual interaction have not sufficiently assessed the overall magnitude of the problem either empirically or theoretically (Ma et al., 2018). An earlier work by Jöst and Quaas (2010) considers environmental externality and the external effects of the fertility decisions on environment simultaneously in a model with households of two types: micro-households and dynastic households. Checa Vizcaino et al. (2016) report that the mechanisms through which pollution impairs fertility remain unclear. Jöst et al. (2006) report that when human capital alone is needed for goods production, the optimal choice of the birth rate is unaffected by the state of economy and the environment. Recently, Casey et al. (2019) present analyses of effects of environmental changes on fertility behaviors of individuals by considering a quality-quantity tradeoff of children. Climate change affects the scarcity of agricultural goods, raising the wages and prices of the sector, thereby inducing labor reallocation to unskilled work and lowering education investment and increasing fertility.

By contrast, Gerlagh et al. (2023) report that a quarter of the total increase in CO_2 emissions is attributable to growth of emissions per capita, whereas three-quarters are

[1] Infertility, a disease of the male or female reproductive system, is defined by World Health Organization (2023) as failure to achieve pregnancy after 12 months or more of regular unprotected sexual intercourse.

[2] Many reports have been referenced in review articles such as those by Ma et al. (2018) and Checa Vizcaino et al. (2016).

attributable to population growth brought about by a quality-quantity tradeoff in terms of children. Haq et al. (2023) elucidate the effects of ecological footprints on fertility rates. In a theoretical report, Palokangas (2018) and Fodha and Yamagami (2025) among others consider the utility of an individual health measure that reflects damage by the pollution level, but the health measure does not directly affect fertility decision of the individual. By contrast, de la Croix and Gossaries (2012) report a case in which a shift of pollution rights from production to recreation will generate a demographic effect of capping emissions.

Nevertheless, most reports neglect explicit consideration of the role of the reproductive health capital of individuals in their reproductive capabilities. Endogenous responses of population to pollution have been neglected in environmental economics (de la Croix and Gosseries, 2012). Environmental changes might affect such reproductive health capital stock of individuals and might thereby affect fertility. For the present study, in the light of recent findings in medical sciences, the author assumes that the number of children depends on the reproductive health capital of individuals, which governs the capability of having children. If an individual has sufficient reproductive health capital, then the individual rears as many children as the individual wants, only subject to the economic constraint. By contrast, if the pollution stock damages reproductive health capital, then the damaged reproductive health capital constrains the fertility decisions of individuals. The pollution stock, which evolves with productive physical capital and mitigation measures, is a source of damage by reducing the reproductive health capital of individuals.[3] The explicit consideration of reproductive health capital of individuals in fertility behaviors is a salient feature of the analyses presented herein.

Figure 1 presents the fertility rates of Kumamoto, Nagasaki, and Kagoshima prefectures of Kyushu, Japan, during 1950–2000. The national average fertility rate is also presented. These prefectures are adjacent areas along the Ariake Sea on the western side of Kyushu Island. From the 1950s to the 1990s, Kumamoto prefecture experienced Minamata disease, a type of poisoning caused by industrial mercury (Hg) pollution.[4] Minamata city is located in the southwestern part of Kyushu, at the southern tip of

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[3] This formulation is inspired by Davin et al. (2025), who assume that the total factor productivity in goods production depends negatively on the pollution stock, whereas negative pollution effects might be dampened by government instruments.

[4] Kagoshima also had Minamata disease. Pollution prevention project was completed in 1990. Nevertheless, the number of patients was still more than 200 in Kumamoto and Kagoshima in 2022. Minamata City (2022) explains the history of the Minamata disease. It is not the case that Minamata disease had been overcome by the completion of pollution prevention project.

Kumamoto prefecture. Minamata disease was reported officially in 1956. The pollution prevention project was declared to be completed in 1990. The fertility rate of Kumamoto prefecture was lower than that of either Kagoshima or Nagasaki prefecture from 1955 to about 1990 although the fertility rate of Kagoshima was also slightly lower than Nagasaki during 1960–1975. Thereafter, the fertility rates of the three prefectures showed similar trends.

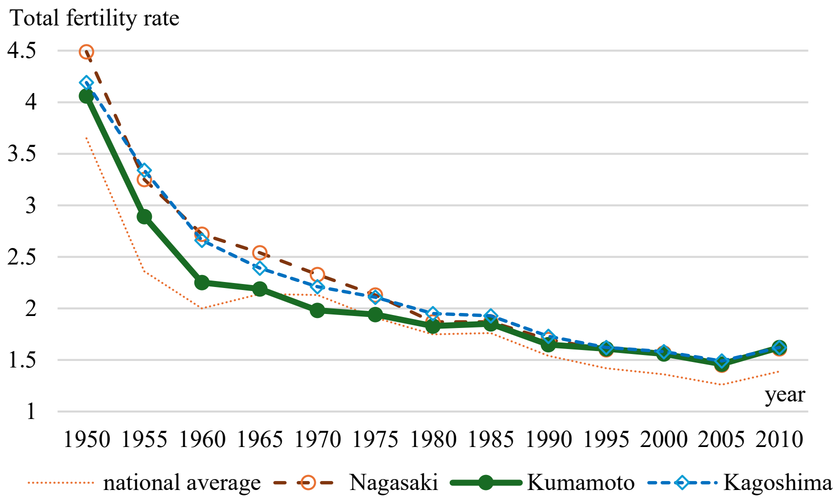

Figure 1 Fertility rates: Kumamoto, Nagasaki, Kagoshima.
(Source) National Institute of Population and Social Security Research 2025

This study is intended to present an analysis of pollution emission effects on fertility through changing individual reproductive health capital. The main finding is that when the pollution stock increases to a certain threshold level, fertility behaviors of individuals become constrained by the damaged reproductive health capital. The threshold level decreases with the pollution stock, thereby lowering the constrained fertility rate. The time freed from childrearing boosts economic growth. Pollution abatement policy might mitigate the damage to reproductive health capital, thereby positively affecting reproductive capabilities of individuals and the fertility rate.

The structure of this paper is the following. The next section introduces an overlapping generations model populated by three-period lived individuals. Section 3 presents the model dynamics. Section 4 considers effects of government abatement policy. Section 5 presents a numerical example. The last section concludes the paper.

2. Model

We assume a simple overlapping generations economy consisting of homogeneous unisex individuals, pollution emitting goods producers, and a government engaging pollution abatement.

Individuals live for three periods: childhood, adulthood, and old age. Individuals are fed by their parents. They later enjoy retirement by dissaving during old age. Individuals make economic decisions related to consumption and childrearing during adulthood. Each allocates their personal time endowment to working and childrearing, where the time endowment is normalized to unity. Consumption during adulthood includes that of the person's children. Individuals obtain utility from lifetime consumption and from rearing children. Consumption goods are produced with physical capital and labor. During the production process, capital stock exerts positive externality to labor productivity and enables the economy to grow endogenously. Nevertheless, capital stock is also a source of pollution. It damages the reproductive health capital of individuals. Government spends on pollution abatement. Government imposes environmental taxes on physical capital stock as the source of pollution. The pollution stock increases with physical capital stock usage in goods production and decreases along with government abatement activities.

This study considers individuals' reproductive health capital, which enables individuals to bear and rear children. Reproductive health capital can be regarded as a kind of human capital which can be accumulated throughout the process of children growing up in the family. Nevertheless, pollution stocks that accumulate in the economy might damage the reproductive health capital, thereby undermining the reproductive capabilities of individuals. Therefore, fertility behaviors of individuals are constrained not only by the budget constraint but also possibly by reproductive health stock. When pollution stock is sufficiently low relative to the reproductive health capital of individuals, their fertility behaviors are not constrained by pollution-damaged reproductive capability. The number of children can be chosen by utility maximization subject only to the budget constraint. Nevertheless, if the pollution stock becomes great in a relative sense, then pollution stock erodes the reproductive health capital of individuals. Therefore, the number of children is constrained by their reproductive capability. For instance, the increased pollution stock increases risks of spontaneous abortion and of erectile difficulty of individuals. Consequently, the cumulative pollution stock presents dangers of lower fertility rates and infertility by eroding reproductive capabilities of individuals.

2.1. Individuals

Letting the working generation in period t be generation t, the budget constraint of a representative individual of generation t can be written as

$$(1 - zn_t)w_t = c_t^1 + \frac{c_{t+1}^2}{r_{t+1}}. \tag{1}$$

Therein, n_t stands for the number of children the individual has in period t. Symbols c_t^1 and c_{t+1}^2 represent respective consumption during (working) adulthood and during (retired) old age. The wage rate in period t and the (gross) interest rate in period $t+1$ are expressed respectively as w_t and r_{t+1}. The childrearing time per child is written as z, which is assumed to be constant for all children.

Reproductive health capital of an individual in period t is assumed to depend on the physical capital stock in that period. The amount of physical capital stock reflects the abundance of the working generation in the sense that higher income of the family engenders higher physical capital accumulation during childhood. Higher income during childhood is expected to bring about better health during adulthood. Therefore, their reproductive health capital can be regarded as depending on physical capital stock. The reproductive health capital can be designated as hK_t, where K_t represents the aggregate physical capital in period t and coefficient h is a positive constant. The constant h represents the efficiency of physical capital in forming reproductive health capital. By contrast, pollution stock P_t decreases reproductive capability by eroding reproductive health capital, i.e., hK_t / P_t. For study, it is assumed that the maximum number of children per capita in period t depends on the ratio of physical capital to pollution stock in that period, as

$$n_t \le \overline{n}(hK_t / P_t). \tag{2}$$

In that inequality, $\overline{n}_t = \overline{n}(hK_t / P_t)$ designates the maximum number of children an individual can have when the person's reproductive capability is defined as hK_t / P_t. Constraint (2) is designated as the fertility constraint for the following discussion.[5]

Although individuals cannot know the levels of reproductive health capital hK_t and pollution stock P_t directly, individuals can rationally recognize the level of $\overline{n}_t$, for example, through medical checkups and treatments of diseases at hospitals or other medical institutions. Consequently, we assume that $\overline{n}(.)$ satisfies $\overline{n}'(.) > 0$, $\overline{n}''(.) < 0$, and $\overline{n}(hK / P) \to 0$ as $hK / P \to 0$. If pollution stock is slight relative to reproductive health capital, then $\overline{n}_t = \overline{n}(hK_t / P_t)$ is great. In that case, the fertility constraint is not binding, i.e., $n_t < \overline{n}(hK_t / P_t)$. Therefore, the fertility constraint does not affect the fertility choices of individuals. By contrast, if pollution stock becomes greater in a relative

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[5] We assume representative individuals in this study, although reproductive capability might differ from person to person.

sense, then the fertility constraint governs the fertility behaviors of individuals, i.e., the number of children is constrained as $n_t = \overline{n}(hK_t / P_t)$. In this case, fertility choices are given by a corner solutions to the individual's utility maximization. When the reproductive capability becomes zero, i.e., when reproductive health capital approaches zero or when pollution stock goes infinite, then individuals can bear no children, i.e., individuals become infertile.

Research related to reproductive capability, i.e., effects of pollution stock on fertility, is in the stage at which effects of various forms of pollution on fertility behaviors of individuals are still being verified. No generally accepted hypothesis exists, apparently, as described in the *Introduction*. Therefore, following the hypothesis of Davin et al. (2025) in formulating pollution damage to total factor productivity (TFP), we assume the specification $\overline{n}(hK_t / P_t) = B(hK_t / P_t)/[1+(hK_t / P_t)]$ for the maximum number of children, where B is a positive constant. This formulation can be verified to satisfy the assumed properties of $\overline{n}(.)$.[6]

The lifetime utility function of a representative individual is assumed to be a log-linear function $u_t = \ln c_t^1 + \beta \ln c_{t+1}^2 + \varepsilon \ln n_t$, where $\beta \in (0,1)$ is the discount factor and where $\varepsilon > 0$ is the altruism factor toward children.[7] Our formulation of altruism is designated as "joy-of-giving" in the literature, not the case of dynastic altruism. Impure altruism is apparently an acceptable assumption as a means to obtain clear-cut analytical results for our purposes. The individual chooses consumption during working and retired periods and chooses the number of children to maximize the lifetime utility subject to the budget constraint (1) and the fertility constraint (2). The Lagrangian function can be written as

$$L_t = \ln c_t^1 + \beta \ln c_{t+1}^2 + \varepsilon \ln n_t + \lambda_t[(1-zn_t)w_t = c_t^1 + \frac{c_{t+1}^2}{r_{t+1}}] + \mu_t(\overline{n}_t - n_t). \quad (3)$$

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[6] The hypothesis presented by Davin et al. (2025) related to the damage caused to total factor productivity reflects the report by Burke et al. (2015), which describes that effects of a change in temperature on economic activity are slightly concave and by the report of work by Dasgupta et al. (2014) which describes that the health effects of global warming on labor productivity are nonlinear and concave, and especially that the number of hours worked decreases beyond a threshold. Nevertheless, Ma et al. (2018) conclude that the overall contribution of environmental exposure to infertility is unknown but that environmental exposure to containments may adversely affect fertility.

[7] Van Groezen et al. (2003) consider fertility choices of individuals in a Diamond (1965)-type overlapping generations model assuming the same lifetime utility function.

Herein, λ_t and μ_t are Lagrange multipliers attached respectively to constraints (1) and (2). The first-order conditions for utility maximization are the following.

$$\frac{\partial L_t}{\partial c_t^1} = \frac{1}{c_t^1} - \lambda_t = 0, \tag{4}$$

$$\frac{\partial L_t}{\partial c_{t+1}^2} = \frac{\beta}{c_{t+1}^2} - \frac{\lambda_t}{r_{t+1}} = 0, \tag{5}$$

$$\frac{\partial L_t}{\partial n_t} = \frac{\varepsilon}{n_t} - \lambda_t w_t z - \mu_t = 0. \tag{6}$$

Because constraint (2) is an inequality constraint, we have the following two cases: (i) $\mu_t = 0$ (i.e., $n_t < \overline{n}_t$) and (ii) $\mu_t > 0$ (i.e., $n_t = \overline{n}_t$). In the following, we explain these two cases in turn. Therein, lifecycle savings are defined as $s_t \equiv w_t(1 - zn_t) - c_t^1$.

Case (i): $\mu_t = 0$ (i.e., $n_t < \overline{n}_t$)

From conditions (4)–(6) we obtain the individual's optimum plans as

$$c_t^1 = \frac{1}{1+\beta+\varepsilon} w_t, \tag{7}$$

$$s_t = \frac{\beta}{1+\beta+\varepsilon} w_t, \tag{8}$$

$$n_t = \frac{\varepsilon}{z(1+\beta+\varepsilon)}. \tag{9}$$

Case (ii) $\mu_t > 0$ (i.e., $n_t = \overline{n}_t$)

In this case, the number of children is constrained to $\overline{n}_t$. Therefore, from conditions (4)–(5) and from constraints (1) and (2), the optimum plans are given as

$$c_t^1 = \frac{1}{1+\beta}(1 - z\overline{n}_t) w_t, \tag{10}$$

$$s_t = \frac{\beta}{1+\beta}(1 - z\overline{n}_t) w_t, \tag{12}$$

$$n_t = \overline{n}(hK_t / P_t). \tag{13}$$

2.2. Production

Because the physical capital stock is the source of pollution, producers are imposed a capital tax according to the size of physical capital stock at tax rate $\tau \in (0,1)$. The production technology of representative goods producer i is assumed to be represented as a constant-returns-to-scale production function $Y_{it} = A_0 K_{it}^{\alpha}(A_t L_{it})^{1-\alpha}$, where $0 < \alpha < 1$. Herein, Y_{it}, K_{it}, and L_{it} represent the output, physical capital stock, and labor of producer i employed in period t. A_t stands for labor productivity during period t. Following Grossman and Yanagawa (1993), we assume that $A_t = K_t / aN_t$, where a is a positive constant reflecting the degree of capital externality to labor productivity and where K_t and N_t are the aggregate physical capital stock and the total number of workers in period t. Producer i chooses the physical capital stock and labor employment to maximize profit $Y_{it} - w_t L_{it} - (r_t + \tau)K_{it}$ during period t. The first-order conditions for profit maximization are

$$\alpha A_0 A_t^{1-\alpha}(K_{it} / L_{it})^{\alpha-1} = r_t + \tau \quad \text{and} \tag{14}$$

$$(1-\alpha) A_0 A_t^{1-\alpha}(K_{it} / L_{it})^{\alpha} = w_t . \tag{15}$$

Because factor prices are the same for all producers and because labor productivity is the same for all workers, $K_{it} / L_{it} = K_t / L_t$ for all i. The labor supply per worker in period t is given as $1 - zn_t$, such that the labor market clearing condition in period t is given as

$$L_t = (1 - zn_t)N_t . \tag{16}$$

Therefore, using (14) and (15), we have the factor market equilibrium conditions:

$$\alpha A(1 - zn_t)^{1-\alpha} = r_t + \tau \quad \text{and} \tag{17}$$

$$(1-\alpha) A(1 - zn_t)^{-\alpha}(K_t / N_t) = w_t , \tag{18}$$

where $A \equiv A_0 / a^{1-\alpha}$. The marginal productivity of factors is dependent on the childrearing time, i.e., fertility rates. The physical capital stock is assumed to depreciate completely after one period of use.

2.3. Government

Government imposes capital taxes on physical capital at tax rate τ and spends the tax revenue on pollution abatement activities. The tax rate is kept constant over periods. The government uses no revenue for other policy purposes: it runs a balanced budget. Denoting abatement spending as M_t, the government budget constraint is written as

$$M_t = \tau K_t . \tag{19}$$

The right-hand side of (19) is the capital tax revenue. Because physical capital is the source of pollution, it is rational to base the pollution tax on physical capital stock.[8]

2.4. Pollution

The stock of pollution increases with emission flow and decreases with public abatement activity. The pollution emission flow is assumed to be proportional to physical capital stock in the goods production process, i.e., $E_t = \theta K_t$, where the emission flow per unit of capital $\theta > 0$ is constant. For instance, Andersen (2017) demonstrates that credit constraints distort the composition of assets towards over-investment in tangible assets at the expense of intangible assets, thereby increasing the pollution emission intensity. Letting the efficiency of government abatement spending be $\psi > 0$, the pollution stock is reduced by ψM_t. In addition, some fraction of the pollution leaves the atmosphere through a natural process as $0 < m < 1$ to the next period. Consequently, the change of pollution stock is representable as

$$P_{t+1} = (1-m)P_t + E_t - \psi M_t . \tag{20}$$

Emphasizing the effects of pollution stock on reproductive health capital, this study does not consider the population change effects on pollution stock.[9]

2.5. Dynamics of the system

Aggregate savings and physical capital formation differ between case (i), in which the fertility constraint brought about by pollution stock is not binding, and case (ii), in which the fertility constraint is binding. This section explains the dynamics of the system for these two cases in turn.

2.5.1. Case (i) $\mu_t = 0$ (i.e., $n_t < \overline{n}_t$)

The clearing condition in the capital market is given as $K_{t+1} = S_t [\equiv s_t N_t]$. From (8), and (18), we can rewrite the condition as

$$K_{t+1} = \frac{\beta(1-\alpha)A}{1+\beta+\varepsilon}\left(\frac{1+\beta}{1+\beta+\varepsilon}\right)^{-\alpha} K_t \quad . \tag{21}$$

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[8] If some adaptation measure such as pollutant purifying facilities and water purifiers were provided by government in addition to pollution abatement activities, then the reproductive capabilities of individuals could be enhanced. The present study does not consider such adaptations because the policy efficiency of such measures is expected to be slight or only partial, for instance, for global air pollution and for global sea contamination.

[9] Most reports of the literature related to analyses of fertility and environment address effects of demographic changes or consumption changes on pollution, as do Gerlagh et al. (2023).

Letting the (gross) growth rate be $\gamma_{t+1} \equiv K_{t+1} / K_t$ and the pollution stock−physical capital stock ratio be $\pi_t \equiv P_t / K_t$, we have the dynamic system from (20) and (21) as[10]

$$\gamma_{t+1}\pi_{t+1} = (1-m)\pi_t + \theta - \psi\tau , \tag{22}$$

$$\gamma_{t+1} = \frac{\beta(1-\alpha)A}{1+\beta+\varepsilon}\left(\frac{1+\beta}{1+\beta+\varepsilon}\right)^{-\alpha} . \tag{23}$$

Growth rate (23) gives the gross balanced growth rate γ^*. For analytical purposes we assume that $\gamma^* > 1$. Inserting (23) into (22), one can obtain the rule of movement of the pollution stock−physical capital stock ratio as

$$\pi_{t+1} = \left(\frac{1+\beta}{1+\beta+\varepsilon}\right)^{\alpha}\left[\frac{\beta(1-\alpha)A}{1+\beta+\varepsilon}\right]^{-1}[(1-m)\pi_t + \theta - \psi\tau] \equiv \Omega(\pi_t) . \tag{24}$$

Equation (24) is a first-order difference equation of π_t. The conditions for the existence of a stable unique solution of this equation are (a) $0 < \Omega'(\pi^*) < 1$ where $\Omega'(\pi_t) = (1-m)\left(\frac{1+\beta}{1+\beta+\varepsilon}\right)^{\alpha}\left[\left(\frac{\beta(1-\alpha)A}{1+\beta+\varepsilon}\right]^{-1}$ and (b) $\theta - \psi\tau > 0$. [11] For analytical purposes, we assume that these conditions are satisfied. Denoting the stable stationary solution to (24) by π^*, we obtain

$$\pi^* = \frac{\theta - \psi\tau}{\frac{\beta(1-\alpha)A}{1+\beta+\varepsilon}\left(\frac{1+\beta}{1+\beta+\varepsilon}\right)^{-\alpha} - (1-m)} . \tag{25}$$

A phase diagram is presented in Figure 2. One can presume that the initial pollution stock−physical capital stock ratio is sufficiently low, e.g., π_0. As physical capital stock accumulates, the pollution stock increases faster than physical capital along with (24). After sufficient time passes, the pollution stock−physical capital stock ratio converges to the stationary value π^* because the fertility constraint does not become binding, i.e., because $n_t < \overline{n}(h / \pi_t)$ for $\pi_t \in [0, \pi^*]$, in case (i). The time path of π_t is shown in the upper panel of Figure 2 by a dotted arrow on the horizontal axis. In the meantime,

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[10] Reproductive capability can be re-expressed as $hK_t / P_t = h / \pi_t$.

[11] When $\theta - \psi\tau < 0$, the steady state equilibrium is unstable even if it exists. In that case, the balanced-growth pollution stock−physical capital stock ratio will be negative from (25). If $\theta - \psi\tau = 0$, then either a unique solution $\pi^* = 0$ is trivial when $\Omega'(\pi_t) \neq 1$ or the solution is indeterminate when $\Omega'(\pi_t) = 1$.

individuals choose the number of children they have such that the marginal utility of having children is equal to the marginal (opportunity) cost, i.e., condition (6) holds with $\mu_t = 0$. Denoting the fertility rate by n_0, the individual's fertility choice on the transition is represented by a dotted arrow in the lower panel of Figure 2.

The balanced growth equilibrium is depicted as point F in Figure 2. On the balanced growth path $\pi_t = \pi_{t+1} = \pi^*$, both the pollution stock and physical capital stock grow at the same rate. Because reproductive health capital is related positively to physical capital stock, this means that reproductive health capital grows at the same rate of pollution stock. The fertility rate, which is constant, is given as $n_0 = \varepsilon / [z(1+\beta+\varepsilon)]$ in this case because the fertility constraint is not binding, as shown by (9).

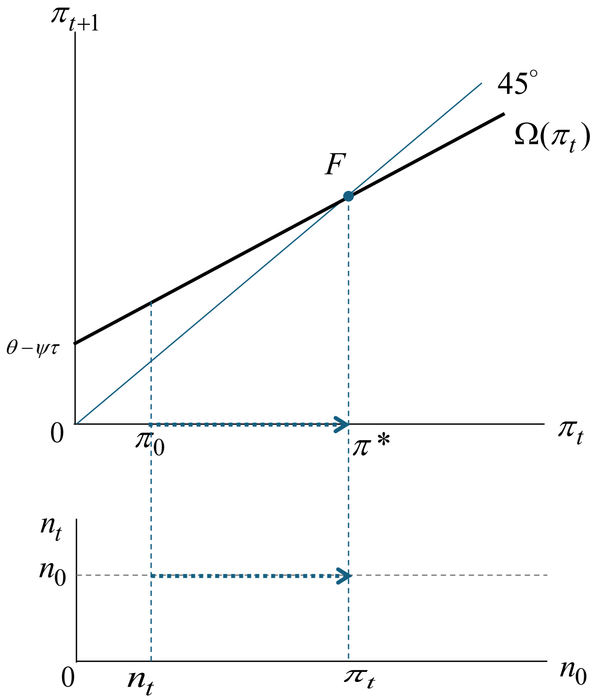

Figure 2 Dynamics: pollution capital ratio and fertility rate.

2.5.2. Case (ii) $\mu_t > 0$ (i.e., $n_t = \bar{n}_t$)

In this case, the clearing condition of the capital market $K_{t+1} = s_t N_t$ is rewritten, using (12), (18) and assumption $\bar{n}(hK_t / P_t) = B(hK_t / P_t) / [1 + (hK_t / P_t)]$, as

$$K_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)A(1-\frac{zBh}{\pi_t + h})^{1-\alpha} K_t . \tag{26}$$

The dynamic system can be written as the following equations:

$$\gamma_{t+1}\pi_{t+1} = (1-m)\pi_t + \theta - \psi\tau , \tag{27}$$

$$\gamma_{t+1} = \frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\pi_t + h})^{1-\alpha} . \tag{28}$$

Inserting (28) into (27), and rearranging terms, we obtain a nonlinear difference equation of pollution stock–physical capital stock ratio π_t as

$$\pi_{t+1} = \frac{(1-m)\pi_t + \theta - \psi\tau}{\frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\pi_t + h})^{1-\alpha}} \equiv \Sigma(\pi_t) . \tag{29}$$

The right-hand side of (29), which is defined as $\Sigma(\pi_t)$, can be shown as a convex curve (see Appendix A1). Appendix A1 also shows that there exists a stationary equilibrium Q as presented in Figure 3. The stationary equilibrium pollution stock–physical capital stock ratio π^{**} satisfies $\pi^{**} = \Sigma(\pi^{**})$ in (29). For analytical purposes, we assume that the equilibrium is stable. The stability condition is given as $-1 < \Sigma'(\pi^{**}) < 1$.[12]

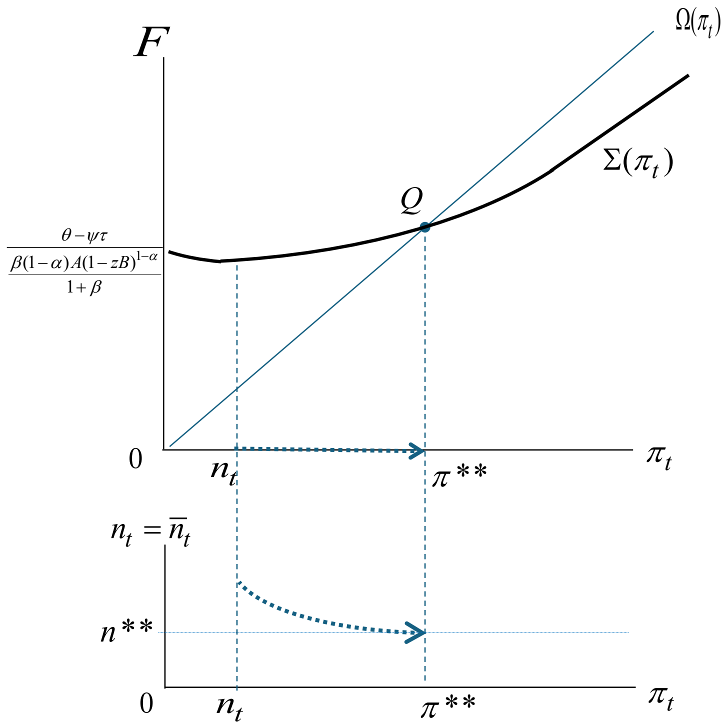

Figure 3 Dynamics: pollution capital ratio and fertility rate.

[12] We cannot exclude the possibility that $\Sigma'(\pi^{**}) \leq 0$ (Appendix A1). Nevertheless, the results would not be altered in that case. We assume that $\Sigma'(\pi^{**}) > 0$ in the following.

On the balanced growth path, from (28), the gross balanced growth rate is given as

$$\gamma^{**} = \frac{\beta(1-\alpha)A}{1+\beta}\left(1-\frac{zBh}{\pi^{**}+h}\right)^{1-\alpha}. \tag{30}$$

From (13) it follows that $n_t = \bar{n}(hK_t / P_t) \equiv Bh/(\pi_t + h)$. The stationary fertility rate is presented as $n^{**} = Bh/(\pi^{**}+h)$ in Figure 3.

Starting from the initial pollution stock−physical capital stock ratio π_0 which is sufficiently small, the movement of $\pi(t)$ is represented by a dotted arrow on the horizontal line in the upper panel of Figure 3. In this case, individuals' fertility decisions are constrained by the fertility constraint (2). The fertility choice of individuals is represented by a dotted curve in the lower panel of Figure 3. Therefore, the constrained fertility rate decreases with the pollution stock − physical capital stock ratio and approaches the new stationary level n^{**}.

3. Development, pollution, and fertility dynamics

3.1. Pollution and fertility

The preceding section presents analyses of the time-paths of the pollution stock−physical capital stock ratio and the fertility rate assuming either that the fertility constraint is binding or not. Based on the analyses in the preceding section, this section considers development time-paths of the pollution stock−physical capital stock ratio and the fertility rate, assuming that the economy starts from a sufficiently low pollution stock−physical capital stock ratio being unaffected by the fertility constraint. As physical capital formation proceeds, i.e., along with economic development, the pollution stock−physical capital stock ratio becomes greater. For instance, during the high economic growth period, Japanese economy was characterized by heavy and chemical industrialization rather than light industry. During these periods, industrial waste caused heavy pollution, especially in industrialized areas.

Consequently, pollution-stock-related negative effects on individuals' fertility decisions might overwhelm the positive effects of reproductive health capital of individuals on fertility, damaging the reproductive capabilities of individuals. When the pollution stock–physical capital stock ratio exceeds a threshold, the fertility constraint becomes binding. In other words, the phase of transition paths changes from case (i) of Section 2.1 to case (ii) of Section 2.2 at the threshold pollution stock–physical capital stock ratio.[13] Denoting the threshold ratio by $\hat{\pi}$, the time paths of the pollution stock–physical capital stock ratio and the fertility rate can be presented in Figure 4, where the threshold ratio is given from the fertility constraint as $\hat{\pi} = [(1+\beta+\varepsilon)zB/\varepsilon - 1]h$.[14] Thereafter, the increased pollution stock − physical capital stock ratio lowers the constrained fertility rate. It is noteworthy that the threshold pollution stock−physical capital stock ratio is independent of the tax rate.

The movement of the pollution stock–physical capital stock ratio π_t and the fertility rate n_t along the economic development is presented in Figure 4. Starting from an initial pollution stock–physical capital stock ratio π_0, the system moves up and to the left along with line $\Omega(\pi_t)$, increasing $\pi(t)$, when the fertility constraint is unbinding. Beyond the threshold of the pollution stock–physical capital stock ratio $\hat{\pi}$, the fertility constraint becomes binding. Then, the system shifts onto curve $\Sigma(\pi_t)$ and moves upward along with the curve until the system reaches a new stationary equilibrium Q. The movement of $\pi(t)$ is shown by a dotted arrow on the horizontal line of the upper panel of Figure 4. The fertility rate remains constant until the pollution stock–physical capital stock ratio reaches threshold $\hat{\pi}$. Nevertheless, after the pollution stock–physical capital stock ratio becomes greater than the threshold value, it becomes lower and lower as the pollution

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[13] The possibility exists that such a phase change does not occur. For instance, if $\pi^* \geq \pi^{**}$, then individuals will not be subject to the fertility constraint overall periods, i.e., we have case (i) only. For our purposes, we assume that $\pi^* > \pi^{**}$. We also assume that $\beta(1-\alpha)A(1-zB)/(1+\beta) < 1$, which suffices for the existence of both cases (i) and (ii) along with economic development paths.

[14] We obtain that from $n_t = \varepsilon/[z(1+\beta+\varepsilon)] = Bh/(\hat{\pi}+h) = \bar{n}(h/\hat{\pi})$, where the equal on the left-hand side is the unconstrained fertility in (9).

stock–physical capital stock ratio increases, i.e., as the reproductive capability decreases. The time path of fertility rate is kinked at $\hat{\pi}$, as shown by a dotted curve in the lower panel of Figure 4.

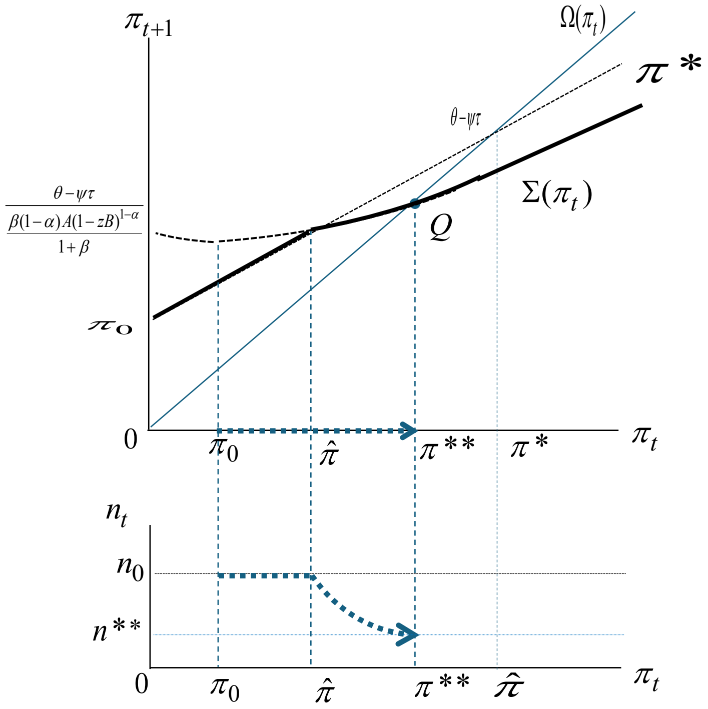

Figure 4 Time paths: pollution capital ratio and fertility rate.

3.2. Economic development

The growth rate in each period is given by (23) in case (i), where the fertility constraint is not binding, and by (28) in case (ii), in which the fertility constraint is binding. Although the growth rate in case (i) is constant over periods, the growth rate in case (ii) increases with the pollution stock–physical capital stock ratio. Nevertheless, as the pollution stock–physical capital stock ratio increases and progresses to exceed threshold $\hat{\pi}$, the growth rate of π_t becomes lower temporarily because $\Omega'(\pi_t) > \Sigma'(\pi_t)$ at the threshold $\pi_t = \hat{\pi}$, as depicted in the upper panel of Figure 4. At the threshold pollution stock–physical capital stock ratio $\pi_t = \hat{\pi}$, we have

$\gamma_{t+1}\big|_{\pi_t=\hat{\pi}} = \frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\hat{\pi}+h})^{1-\alpha} = \frac{\beta(1-\alpha)A}{1+\beta+\varepsilon}(\frac{1+\beta}{1+\beta+\varepsilon})^{-\alpha} = \gamma_{t+1}\big|_{case(i)}$ from (23) and (28), where $\pi\big|_{\pi_t=\hat{\pi}}$ and $\gamma_{t+1}\big|_{case(i)}$ respectively designate the balanced growth rate at the threshold and in case (i). Therefore, $\Omega'(\pi_t) > \Sigma'(\pi_t)$ at $\pi_t = \hat{\pi}$ implies that, immediately after the pollution stock–physical capital stock ratio exceeds threshold $\hat{\pi}$, the growth rate under the fertility constraint might become lower than the growth rate $\gamma_{t+1}\big|_{case(i)}$ temporarily.[15]

Nevertheless, from (28), one can obtain

$$\frac{d\gamma_{t+1}}{d\pi_t} = \frac{\beta(1-\alpha)^2 A}{1+\beta}(1-\frac{zBh}{\pi_t+h})^{-\alpha}\frac{zBh}{(\pi_t+h)^2} > 0. \tag{31}$$

Increases in the pollution stock–physical capital stock ratio erode reproductive capabilities of individuals, thereby reducing child-rearing time and prolonging working time in the market. The increased labor income increases savings and thereby accelerates physical capital formation and economic growth, as shown in (31). Because the pollution stock–physical capital stock ratio continues to increase after passing the threshold $\hat{\pi}$, the system approaches the stationary equilibrium Q in which the pollution stock–physical capital stock ratio remains constant at π^{**} and the balanced growth rate is γ^{**}.[16] The balanced growth rate in stationary equilibrium Q under the fertility constraint is higher than the balanced growth rate in case (i) where the fertility constraint is not binding (Appendix A2). It is noteworthy that the fertility constraint retards growth temporarily at the threshold of pollution stock–physical capital stock ratio $\hat{\pi}$. The stationary growth rate is higher under the fertility constraint than without the fertility constraint.

From the arguments presented above, we have the following proposition.

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[15] Because the transitional adjustments are discrete, the growth rate does not show the decrease necessarily.

[16] It is noteworthy that we do not consider damages of pollution to the labor productivity of workers in this study. Bosi and Desmarchelier (2013) consider effects of pollution on labor productivity through effects on health. Zivin and Niedell (2012) present an empirical report related to effects of pollution on agricultural labor productivity.

Proposition 1

Consider an economy starting with the initial state of sufficiently low pollution stock−physical capital stock ratio. Assume also that there exists a threshold pollution stock–physical capital stock ratio beyond which the fertility constraint is binding. The economic development process can be described as follows:

(1) The fertility rate is chosen optimally by individuals, i.e., being unconstrained by pollution-damaged reproductive health capital in earlier stages of economic development. The economy grows at a constant rate in earlier stages of economic development.

(2) The pollution stock−physical capital stock ratio, or the pollution stock–reproductive health capital ratio, grows to exceed the threshold beyond which the fertility behaviors of individuals become constrained by pollution-damaged reproductive capability.

(3) The constrained fertility choices exert positive effects on savings and physical capital accumulation, accelerating economic growth. The increased physical capital stock leads to greater pollution stock, damaging reproductive capabilities of individuals, thereby lowering the fertility rate, and consequently accelerating economic growth further.

4. Environmental policy change: tax increase for abatement spending

This section presents analyses of policy effects of public abatement activity on fertility and economic growth. If the pollution stock−physical capital stock ratio is too great to prevent the fertility rate from going below the population replacement rate, then the government might increase the tax rate financing abatement spending to prevent the fertility rate from going down further. It is rational for the government to prevent the pollution stock−physical capital stock ratio from increasing further by undertaking an environmental policy of a capital tax increase.

Because such a situation can occur in case (ii) where the fertility constraint is binding, we consider case (ii) in this section. Assuming that the economy is initially at stationary equilibrium Q of $(\pi, n) = (\pi^{**}, n^{**})$ in Figure 4, we consider effects of an increase in the capital tax rate. From (29) we obtain

$$\frac{d\pi}{d\tau}=\frac{1}{1-\Sigma'(\pi^{**})}\frac{-\psi}{\frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\pi^{**}+h})^{1-\alpha}}<0. \tag{32}$$

In that equation, $1>\Sigma'(\pi^{**})$ is satisfied by the stability condition. The increased tax rate reduces the pollution stock–physical capital stock ratio. Because the constrained fertility rate decreases with the pollution stock–physical capital stock ratio ($\partial\overline{n}_t/\partial\pi_t<0$), equation (32) implies that the fertility rate increases with the tax hike. These are explainable by using Figure 5 as follows. The policy effects of the tax increase from τ to $\tau^{\dagger}(>\tau)$ on the pollution stock–physical capital stock ratio ($\pi^{\dagger}$) and the fertility rate ($n^{\dagger}$) are presented in Figure 5. The tax increase shifts both curves $\Omega(\pi_t;\tau)$ and $\Sigma(\pi_t;\tau)$ downward. From (24) and (29), one can obtain

$$\frac{\partial}{\partial\tau}\Omega(\pi_t;\tau)=-\psi(\frac{1+\beta}{1+\beta+\varepsilon})^{\alpha}/(\frac{\beta(1-\alpha)A}{1+\beta+\varepsilon})<0, \tag{33}$$

and

$$\frac{\partial}{\partial\tau}\Sigma(\pi_t;\tau)=-\psi/[\frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\pi_t+h})^{1-\alpha}]<0. \tag{34}$$

Therefore, the stable stationary equilibrium shifts to the lower-left along the 45° line from Q to $Q^{\dagger}$. The tax hike lowers the pollution stock–physical capital stock ratio. When the tax rate increases from τ to $\tau^{\dagger}(>\tau)$, the system jumps from $\Sigma(\pi^{**};\tau)$ on curve $\Sigma(\pi_t;\tau)$ to $\Sigma(\pi^{**};\tau^{\dagger})$ on curve $\Sigma(\pi_t;\tau^{\dagger})$, where $\Sigma(\pi^{**};\tau)>\Sigma(\pi^{**},\tau^{\dagger})$ from (34). Then, the pollution stock–physical capital stock ratio decreases along curve $\Sigma(\pi_t;\tau^{\dagger})$ until it reaches the new equilibrium $Q^{\dagger}$. This transition path is depicted as a dotted arrow on the horizontal line of the upper panel in Figure 5. The changes in the pollution stock–physical capital stock ratio mitigate the fertility constraint, consequently increasing the constrained fertility rate. The constrained fertility rate $n_t=\overline{n}(h/\pi_t)$ is independent of the tax rate in case (ii). Therefore, the fertility rate curve in the lower panel of Figure 5 does not shift. As the pollution stock–physical capital stock ratio decreases, the constrained fertility rate becomes higher and higher as represented in Figure 5 by a dotted curve in the lower panel. The fertility rate increases from n^{**} to $n^{\dagger}$ because of the tax increase.

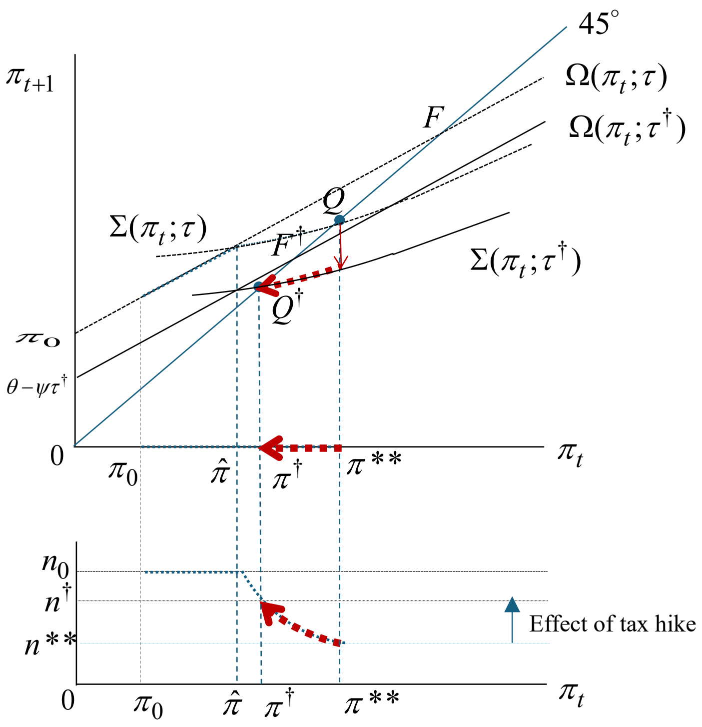

Figure 5 Policy effect of a tax hike.

The stationary pollution stock–physical capital stock ratio might be lowered by the abatement policy change. As shown by (31), the growth rate decreases with the pollution stock–physical capital stock ratio decreases when the tax rate increases. Nevertheless, whether the new balanced growth rate is higher than, equal to, or lower than the growth rate before the abatement policy change is ambiguous *a priori*. To explain the policy effects on the balanced growth rate, we rewrite the balanced growth rate. From (29) evaluated at the balanced growth path Q, and inserting it into (28), the balanced growth rate γ^{**} can be rewritten as

$$\gamma^{**} = (1-m) + \frac{\theta - \psi\tau}{\pi^{**}}. \tag{35}$$

If the effect of the tax change on pollution emission, i.e., $\theta - \psi\tau$, is great (small) relative to the effect on the pollution stock–physical capital stock ratio π^{**}, then the balanced growth rate is lower (higher) than the growth rate before policy change, as might be apparent in (A6) of Appendix A3.[17]

Up to this point in the discussion, we have considered an economy that is initially in stationary balanced growth equilibrium. Nevertheless, even if the economy is on the transition to stationary equilibrium, tax increases lower the pollution stock−physical capital stock ratio and thereby increase the fertility rate. It is noteworthy that if the decrease in the pollution stock–physical capital stock ratio by the tax hike is sufficiently great to make the ratio lower than the threshold ratio (i.e., $\pi_t < \hat{\pi}$), then the fertility constraint becomes unbinding, i.e., case (i) applies. In this case, the fertility rate becomes determined by individual's utility maximization behaviors in the absence of the fertility constraint.

Summing up the arguments presented above, we obtain the following proposition.

Proposition 2

An increase in pollution tax on physical capital stock decreases the stationary pollution stock–reproductive health capital ratio when the fertility choices of individuals are constrained by the pollution-damaged reproductive capability. The policy change increases the constrained fertility rate. Although the growth effect of the tax changes is ambiguous, the balanced growth rate after the tax changes might be lower than the balanced growth rate without fertility constraint binding.

Remark: It might be readily apparent from (23) that an abatement tax change does not affect the balanced growth rate in case (i) when the fertility constraint is not binding. Nevertheless, the tax increase shifts curve $\Omega(\pi_t, \tau)$ downward, thereby lowering the

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[17] For expositional purposes, we assume here that the policy change does not occur immediately after the period in which the fertility constraint becomes valid. If this is the case, then the economy reverts to case (i). Although the growth rate becomes lower at the threshold, the balanced growth rate is retained when the threshold becomes a stationary equilibrium.

balanced-growth pollution stock–physical capital stock ratio (corresponding equilibrium F in Figure 2).

5. Numerical example

This section presents a simple numerical example. Model parameters are set fictitiously as presented in Table 1.

Table 1. Parameters

variable	value
β : discount factor	0.3
ε : utility weight of children	0.7
z : rearing time per child	0.075
α : capital elasticity	1/3
A : scale parameter	9
m : natural resilience rate	0.2
θ : pollution flow per unit of physical capital	0.4
B : positive constant	8
h : health efficiency of asset	1.1
ψ : efficiency of abatement	1.6
τ : capital tax rate	0.1

Herein, β and z are the same as those presented by de la Croix and Doepke (2003), although ε here is slightly greater than theirs is.[18] The parameter value of α is common in the literature of macroeconomics. Nevertheless, other parameters are set ad hoc to obtain the solutions.

The stationary equilibrium in case (i) is characterized as $(n^*, \pi^*, \gamma^*) = (4.6667, 1.0043, 1.0390)$. Assuming that a period lasts 30 years, the average annual fertility rate is $n^*_{annual} = 1.0527$; the average balanced growth rate is $\gamma^*_{annual} = 1.0013$. The stationary equilibrium in case (ii) is

[18] De la Croix and Gosseries (2012) have $\varepsilon = 0.471$ in their model with quality-quantity tradeoff whereas de la Croix and Doepke (2003) set $\varepsilon = 0.271$.

$(n^{**}, \pi^{**}, \gamma^{**}) = (4.3695, 0.91410, 1.0626)$. The average annual fertility and balanced growth rates are $(n^{**}_{annual}, \gamma^{**}_{annual}) = (1.0504, 1.0559)$. The fertility rate is lower in case (ii) than in case (i), although the balanced growth rate is higher in case (ii) than in case (i).

The development paths starting from the initial pollution stock–physical capital stock ratio $\pi_0 = 0.5$ are presented in Figure 6. Threshold pollution stock-physical capital stock ratio is $\hat{\pi} = 0.7857$. The system moves from case (i) to case (ii) in period 5. The upper-left panel shows the path of the pollution stock–physical capital stock ratio, whereas the upper-right panel presents the growth rate in terms of average annual growth rates. The pollution stock–physical capital stock ratio increases monotonically along the path. By contrast, the growth rate starts to increase with the pollution stock-physical capital stock ratio beyond the threshold ratio, although the growth rate remains constant below the threshold pollution stock–physical capital stock ratio. The lower panel shows the time path of fertility rates in terms of average annual rates. Beyond the threshold pollution stock–physical capital stock ratio, the fertility rate decreases and converges to $n^{**} = 1.0504$.

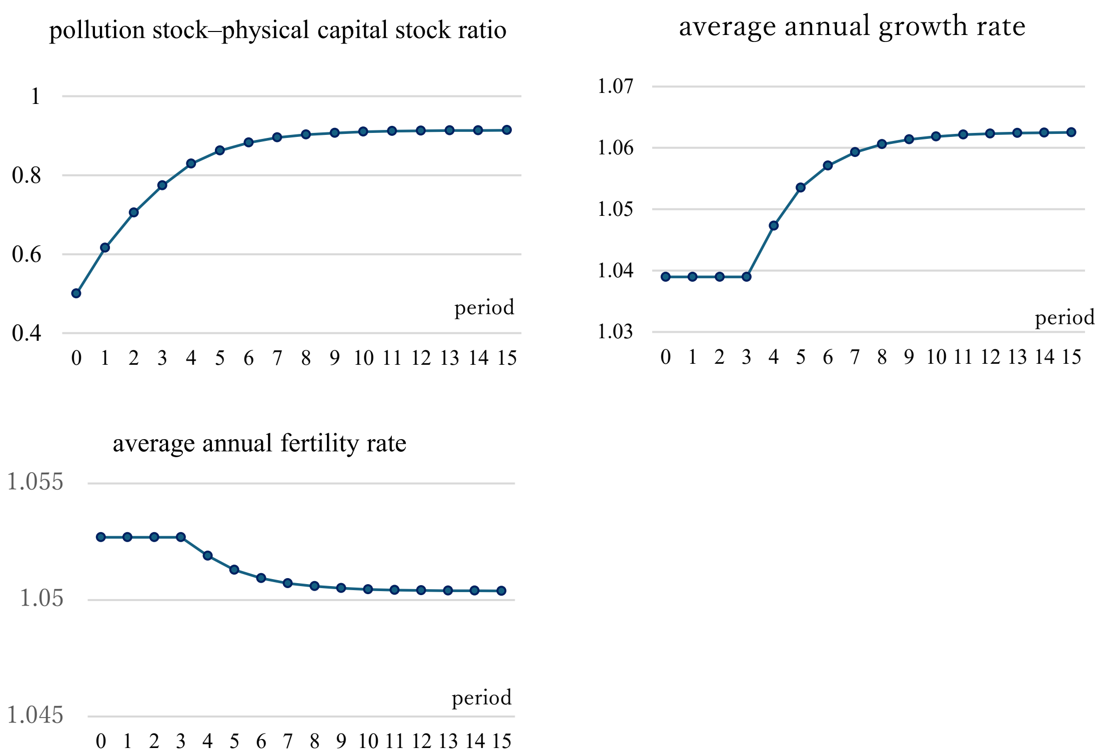

Figure 6 Transition paths: pollution stock–physical capital stock ratio, growth rate, and fertility rate.

Next, we assume that government increases the tax rate from $\tau = 0.1$ to $\tau^{\dagger} = 0.11$ at the case (ii) stationary equilibrium $(n^{**}, \pi^{**}, \gamma^{**}) = (4.3695, 0.91410, 1.0626)$. After the tax increase, the system reaches a new stationary equilibrium which is characterized as $(n^{\dagger}, \pi^{\dagger}, \gamma^{\dagger}) = (4.4546, 0.8755, 1.0559)$. The average annual growth rates are $(n^{\dagger}{}_{annual}, \gamma^{\dagger}{}_{annual}) = (1.0511, 1.0018)$. The tax hike for pollution abatement increases the fertility rate and lowers the balanced growth rate. Relabeling the period of the tax change as 0, the time paths of the pollution stock–physical capital stock ratio, the transition paths of the average annual growth rate, and the average annual fertility rate are presented in Figure 7. The pollution stock–physical capital stock ratio decreases. The fertility rate increases. In this example, because $\dfrac{\theta - \psi\tau}{\pi^{**}} = 0.2626$ and $\dfrac{\theta - \psi\tau^{\dagger}}{\pi^{\dagger}} = 0.2599$, the balanced growth rate becomes lower after the tax increase (Appendix A3).

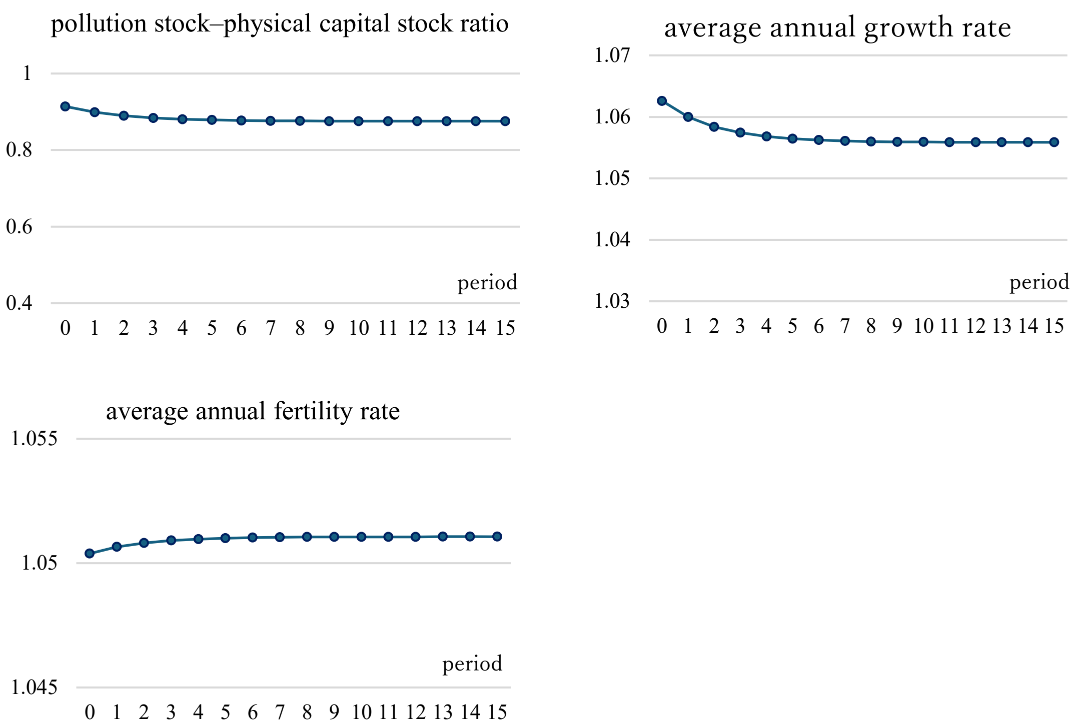

Figure 7. Effects of a tax increase on pollution stock–physical capital stock ratio, growth rate, and fertility rate.

It is noteworthy that the tax increase lowers the pollution stock–physical capital stock ratio considerably in earlier periods. Then the fertility rate increases greatly in correspondence. Changes in the pollution stock–physical capital stock ratio are greater in earlier periods of transition.

6. Concluding remarks

Introducing the notion of reproductive health capital and reproductive capabilities of individuals into a model, this study presents an analysis of effects of the magnitude of pollution stock relative to reproductive health capital on fertility and economic growth. Because the magnitude of pollution stock increases more than reproductive health capital of individuals along with an economic development path, fertility behaviors of individuals might become constrained by their damaged reproductive capabilities. The damaged reproductive capability causes individuals to rear less children, lowering the fertility rate. The decreased childrearing time can be reallocated to market labor, leading to higher physical capital accumulation and thereby higher economic growth.

Public abatement activity functions to lower the pollution stock−physical capital stock ratio, thereby increasing the constrained fertility rate, although policy effects on the balanced growth rate are ambiguous. If the pollution stock−physical capital stock ratio is sufficiently lowered by public abatement policy, then the constraint on fertility choices might be removed. In such a case, the fertility rate can be chosen according to the utility maximization behaviors of individuals, not being constrained by the pollution-damaged reproductive health capital.

This study proposes another formulation of individuals' responses to pollution, differing from earlier reports of the literature such as that by de la Croix and Gosseries (2012).[19] Nevertheless, the simple first-step model presented herein has some limitations. First, by emphasizing the pollution damage to the reproductive health capital of individuals, we have not considered other aspects of pollution damage. Many reports in the literature consider the utility of environmental quality or the utility of pollutant stock,

[19] De la Croix and Gosseries (2012) propose the traditional mechanism based on quality-quantity tradeoff: taxing pollution and thereby income lowers the opportunity cost of rearing children and thereby raises the fertility rate. Consequently, capping the population might become necessary to avoid deterioration of environment and decreases in production per capita.

e.g., Jöst et al. (2006). Other reports have presented consideration of the negative effects of pollution stock on labor productivity through health capital, as have Bosi and Desmarchelier (2013). Second, we have not assumed various government policies but only pollution abatement activity. Government might finance abatement spending through income taxes, e.g., Davin et al. (2025). In addition, the government might take adaptation measures against pollution, e.g., Davin et al. (2025) and Fodha and Yamagami (2025). Third, intergenerational transfers and intergenerational optimality have not been considered. The pollution stock persists over multiple periods and exerts effects on multiple generations. The optimality of policy might be analyzed well assuming infinite time horizons of agents, as posited by Gerlagh et al. (2023).

Acknowledgements
Financial support from the Japan Society of the Promotion of Science (KAKENHI grant No. 25K05125) is gratefully acknowledged.

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Appendices

Appendix A1. Equilibrium in case (ii)

Let the numerator and the denominator of (29) respectively represent $\Gamma(\pi_t)$ and $\Lambda(\pi_t)$. As might be readily apparent, $\Gamma(\pi_t)$ is a line with slope of $0<1-m<1$ and the intercept of $\theta-\psi\tau$. The properties of $\Lambda(\pi_t)$ are such that $\Lambda(0)=\beta(1-\alpha)A(1-zB)^{1-\alpha}/(1+\beta)>0$, $\Lambda(\pi_t)\to\beta(1-\alpha)A/(1+\beta)>0$ as $\pi_t\to+\infty$, $\Lambda'(\pi_t)=\dfrac{\beta(1-\alpha)^2 AzBh}{(1+\beta)(\pi_t+h)^2}(1-\dfrac{zBh}{\pi_t+h})^{-\alpha}>0$, and

$\Lambda''(\pi_t) = -(1-\frac{zBh}{\pi_t+h})^{-\alpha-1}\frac{zBh}{(\pi_t+h)^3}[\frac{\alpha zBh}{\pi_t+h}+2(1-\frac{zBh}{\pi_t+h})]<0$. Therefore, $\Lambda(\pi_t)$ is an increasing concave curve with intercept $\frac{\beta(1-\alpha)A}{1+\beta}(1-zB)^{1-\alpha}$ and an upper bound of $\frac{\beta(1-\alpha)A}{1+\beta}>0$. When $\frac{\beta(1-\alpha)A}{1+\beta}(1-zB)^{1-\alpha}>\theta-\psi\tau$, two curves can be depicted as presented in Figure A1.

Curve $\Sigma(\pi_t)=\Gamma(\pi_t)/\Lambda(\pi_t)$ is convex as depicted in Figure 3 of the text. The difference equation (29) has multiple solutions when $\Gamma(\pi_t)=\Lambda(\pi_t)$ at $\pi_t>1$. In this case, curve $\Sigma(\pi_t)$ is below the 45° line, i.e., $\Sigma(\pi_t)=\Gamma(\pi_t)/\Lambda(\pi_t)=1<\pi_{t+1}$, at $\pi_t>1$. By contrast, if $\theta-\psi\tau\geq\beta(1-\alpha)A/(1+\beta)$, i.e., if $\Gamma(\pi_t)>\Lambda(\pi_t)$ for any π_t, then there might be no interception of these two curves $\Gamma(\pi_t)$ and $\Lambda(\pi_t)$. Nevertheless, curve $\Sigma(\pi_t)$ is concave even in this case.

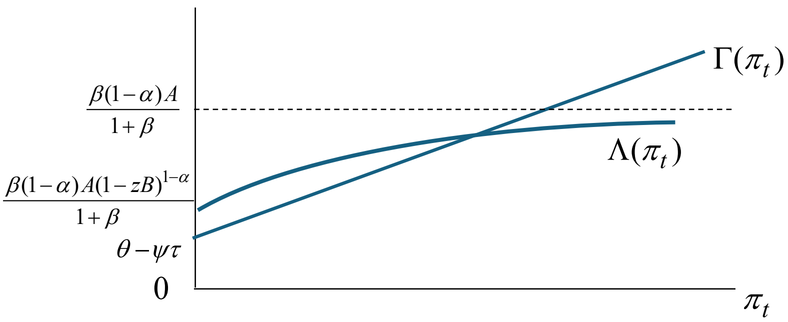

Figure A1 $\Gamma(\pi_t)$ and $\Lambda(\pi_t)$.

Curve $\Sigma(\pi_t)$ has the following properties: $\Sigma(0)=(\theta-\psi\tau)(1+\beta)/[\beta(1-\alpha)A(1-zB)^{1-\alpha}]>0$, $\Sigma(\pi_t)\to\infty$ as $\pi_t\to\infty$, and $\Sigma'(\pi_t)=\Gamma'(\pi_t)/\Lambda(\pi_t)-\Sigma(\pi_t)[\Lambda'(\pi_t)/\Lambda(\pi_t)]$. Therefore, we have $\Sigma'(\pi_t)\to(1-m)(1+\beta)/\beta(1-\alpha)A$ as $\pi_t\to\infty$. Assuming that the gross balanced growth rate γ^{**} is greater than one, we have $(1+\beta)/\beta(1-\alpha)A<1$ from (28). Both curves $\Lambda(\pi_t)$ and $\Gamma(\pi_t)$ monotonically increase with π_t. The slope of $\Sigma(\pi_t)$ is smaller than one as the pollution stock–physical capital stock ratio becomes infinite, i.e., $\lim_{\pi_t\to\infty}\Sigma'(\pi_t)<1$. Therefore, curve $\Sigma(\pi_t)$ has an interception with the $\Omega(\pi_t)$ line only once.

The stability condition of π^{**} is given by $-1<\Sigma'(\pi^{**})<1$, where

$$\Sigma'(\pi^{**}) = \frac{1-m}{\frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\pi^{**}+h})^{1-\alpha}}$$

$$-(1-\alpha)\frac{[(1-m)\pi^{**}+\theta-\psi\tau]}{\frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\pi^{**}+h})^{2-\alpha}}\frac{zBh}{(\pi^{**}+h)^2}$$

$$= \frac{1}{1+\frac{\theta-\psi\tau}{(1-m)\pi^{**}}} - \frac{1-\alpha}{(1+\frac{h}{\pi^{**}})(\frac{\pi^{**}+h}{zBh}-1)}. \qquad \text{(A1)}$$

Herein, we use $\pi^{**} = \Sigma(\pi^{**})$ from (29). We assume that the stability condition is satisfied, as presented in Figure 3.

Appendix A2. Balanced growth rates

From (29), the stationary pollution stock–physical capital stock ratio is given as

$$\pi^{**} = \frac{(1-m)\pi^{**}+\theta-\psi\tau}{\frac{\beta(1-\alpha)A}{1+\beta}(1-\frac{zBh}{\pi^{**}+h})^{1-\alpha}}. \qquad \text{(A2)}$$

Letting the gross balanced growth rate in case (ii) as γ^{**} and using (27), we obtain

$$\pi^{**} = \frac{(1-m)\pi^{**}+\theta-\psi\tau}{\gamma^{**}},$$

from which we have

$$\gamma^{**}\Big|_{case(ii)} = (1-m) + \frac{\theta-\psi\tau}{\pi^{**}}. \qquad \text{(A3)}$$

For case (i), in which the growth rate is constant, we obtain the balanced growth rate in case (i) from (22) and (25)

$$\gamma^{*}\Big|_{case(i)} = (1-m) + \frac{\theta-\psi\tau}{\pi^{*}} = \frac{\beta(1-\alpha)}{1+\beta+\varepsilon}(\frac{1+\beta}{1+\beta+\varepsilon})^{-\alpha}. \qquad \text{(A4)}$$

From (23) the balanced growth path is constant for $\pi_t \in (0,\hat{\pi}]$. Because $\pi^{*} > \pi^{**}$ from Figure 4, we have $\gamma^{*}\big|_{case(i)} < \gamma^{**}\big|_{case(ii)}$. The balanced growth rate when the fertility constraint is binding is higher than the balanced growth rate without fertility constraint.

Nevertheless, because $\Omega'(\hat{\pi}) > \Sigma'(\hat{\pi})$ at threshold pollution stock–physical capital stock ratio $\pi_t = \hat{\pi}$, π_{t+1} on curve $\Sigma(\pi_t)$ is lower than that on curve $\Omega(\pi_t)$. If the pollution stock–physical capital stock ratio is given by (24) at $\pi_t = \hat{\pi}$, i.e., if π_{t+1} is on curve $\Omega(\pi_t)$ when $\pi_t = \hat{\pi}$, then the growth rate given by (28) is the same as that given by (23). This means that the economic growth rate decreases when the system moves from $\Omega(\pi_t)$ in case (i) to $\Sigma(\pi_t)$ in case (ii). Therefore, the economic growth rate temporarily decreases immediately after the fertility constraint becomes binding, although the balanced growth rate is higher when the fertility rate is binding than otherwise.

Appendix A3. Tax effects

With a tax hike for government abatement policy, the balanced growth rate becomes

$$\gamma^{\dagger}\Big|_{case(ii)} = (1-m) + \frac{\theta - \psi\tau^{\dagger}}{\pi^{\dagger}} \tag{A5}$$

from (A2). Therefore, the balanced growth rate with the policy change $\gamma^{\dagger}$ is higher than, equal to, or lower than the balanced growth rate without the policy change γ^{**} depends on the following equation:

$$\frac{\theta - \psi\tau^{\dagger}}{\pi^{\dagger}} \gtreqless \frac{\theta - \psi\tau}{\pi^{**}}. \tag{A6}$$

If the tax hike increases the pollution stock–physical capital stock ratio largely, i.e., if $(\theta - \psi\tau)/\pi^{**} < (\theta - \psi\tau^{\dagger})/\pi^{\dagger}$, then the balanced growth rate is higher with the policy change, and vice versa.

A tax increase immediately lowers the growth rate because curve $\Sigma(\pi_t;\tau)$ shifts downward, lowering π_{t+1} for predetermined π^{**}. After then, the pollution stock–physical capital stock converges to a steady state $\pi^{\dagger}$, thereby the growth rate decreasing. Nevertheless, if $\pi^{\dagger}$ is sufficiently small (great), then the balanced growth rate is higher (lower) than the before-tax-change growth rate.

If the pollution stock–physical capital stock is reduced to the threshold level $\hat{\pi}$, then the balanced growth rate becomes $(1-m) + \dfrac{\theta - \psi\tau^{\dagger}}{\hat{\pi}}$. Therefore, the balanced growth rate with the tax hike from τ to $\tau^{\dagger}$ is lower than the balanced growth rate that would obtain in the absence of the fertility constraint.