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Intergenerational transmission of preference for children, the fertility transition and economic growth

Akira Yakita
Nagoya City University

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Abstract
This paper presents an analysis of long-term effects of intergenerational transmission of fertility behavior as a proxy of culture in an overlapping generations setting. We assume that the fertility norm faced by a generation derives from the preceding generation's fertility behavior, as reflected in the number of siblings. The norm costs can be represented as the disutility from deviating from the norms. Fertility rate norms engender demographic transitions which converge to a long-term equilibrium fertility rate, whereas the norms affect only fertility transitions. It is noteworthy that, starting from a high fertility rate and low per-worker capital, the GDP growth rate transition can display an inverted U shape along the monotonic fertility transition. Because the long-term fertility rate coincides with the non-norms equilibrium, the child policy must be such that it can positively affect the long-term non-norm fertility rate.

JEL Classification: I30, J11, J13, O40
Keywords: fertility decisions, intergenerational transmission of preference for children, social norms

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Data availability statement
The author does not analyze or generate any datasets because my work proceeds within theoretical and mathematical approaches.

Correspondence to: Akira Yakita, Graduate School of Economics, Nagoya City University, 1 Yamanohata, Mizuho-cho, Mizuho-ku, Nagoya 467-0802, Japan.
Email: yakita@econ.nagoya-cu.ac.jp ORCID: 0000-0002-8966-9602

1. Introduction

The relation between culture and economic incentives has been analyzed intensively recently because of better empirical tools and expanded data (Guiso et al., 2006). Many reports present empirical analyses of transmissions of preference norms and cultural norms of parents who have immigrated from other economies. These studies include those by Giuliano (2007), Fernandez (2007), Fernandez and Fogli (2009), Guiso et al. (2006), Blau et al. (2013), Albanese et al. (2016), Gentili et al. (2017), Jergins (2021), and Grytten et al. (2024).[1] Specifically, Fernandez and Fogli (2009) describe that culture is proxied with past female labor force participation (LFP) and total fertility rates (TFR) from the woman's country of ancestry. These reports are mostly empirical. By contrast, this paper presents a theoretical analysis of long-term effects of intergenerational transmission of fertility rate norms as a proxy of culture and social environment in an overlapping generations setting of Diamond type (1965), assuming that the market labor supply and child rearing are perfectly substitutable for working generations.

Four theoretical contributions in the literature are related to this paper. Palivos (2001) reports that family size norms can engender a low fertility rate through coordination failures. Munshi and Myaux (2006) empirically show that fertility rate norms change in response to changes in the social environment in a Bayesian updating manner, affecting the fertility decisions of contemporaries in Bangladeshi religious groups. Bhattacharya and Chakraborty (2012) report that fertility rate norms might correct the decreased fertility rate brought about by infant mortality because individuals respond to a social pressure of not wanting to fall behind others. Barigozzi et al. (2020) extend the notion of social norms of Lindbeck et al. (1999, 2003) to demonstrate that the social norm costs faced by girls engender excess family elderly care provision over the social optimum because they suffer when providing less informal care than their peer. Nevertheless, these studies consider only horizontal transmissions of social norms, i.e., transmissions of social norms to the same cohort, involving multiple equilibria. Fertility rate norms (or preference for children) can also be transmitted vertically to the succeeding generations, consequently affecting the behaviors and economic performances of future generations.[2]

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[1] Miceli (2019) separates general transmission into vertical transmission and horizontal transmission. He presents a quantitative analysis of these transmission effects among second-generation immigrant women in U.S. Censuses from 1910 to 1970.

[2] Barigozzi et al. (2018) represents an exception, assuming that the norm is determined by the behavior of a majority of individuals in the preceding generation. Bisin and Verdier (2001) also describe that preferences of children are acquired through adaptation and imitation processes, which depend on their parents' socialization actions and on the cultural and social environment in which children live. We do not consider parents'

For the analyses explained herein, we assume that the number of siblings, i.e., the fertility rate of parents, affects the fertility decisions of a generation as a fertility rate norm.[3]

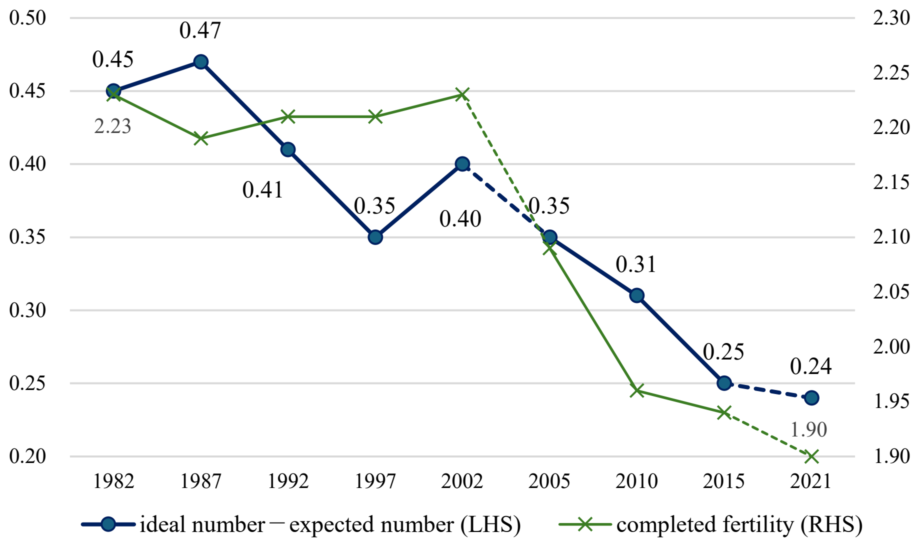

Fig. 1 Differences of the average ideal number of children in each wave from the average expected number of children of the preceding wave (LHS) and the average completed fertility (RHS) in each wave in Japan.

Figure 1 presents the difference between the average ideal number of children of couples observed in a survey and the average expected number of children of couples in the preceding survey, which were survey data from the 8th wave in 1977 to the 16th wave in 2021 (National Institute of Population and Social Security Research, 2023). The difference is measured on the left-hand side ordinate. Although the differences do not correspond to those of the subsequent two generations, these differences apparently decrease over a long period. If the ideal number of children can be regarded as indicating the current social environment (norms) and if the expected number indicates the fertility behaviors of current generation, then one can consider that the decisions become close to the norms over a long period.[4] The completed fertility in each survey is also presented

socialization actions in the analyses described herein. Munshi and Myaux (2006) present empirical analyses of fertility transition as a process of changing social norms. Daido and Tabata (2013) present the analyses about the effects of the social norms on working hours and fertility choices of individuals in an overlapping setting.

[3] Canta and Pestieau (2013) assume a family norm concerning elderly care for parents to present an analysis of intergenerational transmission of the norm in an overlapping generations model.

[4] This finding is apparently consistent with the finding reported by Freedman et al. (1965) about American couples: changes in expectations tend to occur if a discrepancy exists

(measured on the right-hand side ordinate). They declined from 2.23 to 1.90 over the period 1982–2021.

For analytical simplicity, we consider that the fertility rate norm faced by a generation derives from the preceding generation's fertility decision, which is reflected in the number of siblings. Children grow as they look at the back of their parents and the social environment of their community during childhood even if parents do not take socialization actions toward their children.[5] Assuming homogeneous individuals, we are concerned only with vertical transmissions of norms.

The main findings are the following. First, fertility rate norms engender demographic dynamics. Second, the fertility rate converges to a long-term equilibrium whereas the norms affect only fertility transitions.[6] Third, for the long-term equilibrium, the norm costs disappear. These findings imply that an economy with a high fertility rate and a small capital stock, such as that of Japan after World War II, possibly experiences both declines in the fertility rate and economic growth rate deceleration if fertility norms affect the fertility decisions of generations.

The remainder of this paper is organized as explained hereinafter. The next section introduces a model. In Section 3, the system dynamics are analyzed. Section 4 presents a numerical example. Section 5 presents brief analyses of child subsidy policy. The last section concludes the paper.

2. Model

Individuals live for three periods: childhood in the first, young adulthood in the second,

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between the ideal and expected numbers of children. Moreover, change occurs in the direction of reducing the discrepancy. From that finding, they indicate that social norms about family size strongly affect the fertility behavior of American couples.

[5] Giavazzi et al. (2019) demonstrate that the persistence of cultural traits is strong to the next generation, but it becomes weaker for subsequent generations. We assume here that the preference is transmitted only to the next generation (Barigozzi et al., 2018). Bisin and Verdier (2001) consider parents' socialization actions when analyzing intergenerational transmission of cultural traits.

[6] Komamura (2011) described that the standard family for public livelihood protection has changed from those with 3 children in 1950 to those with 2 children in 1961, and then to those with a child in 1986. These numbers might reflect the average of those respective periods. The total fertility rates of these years were, respectively, 3.65, 1.96, and 1.72. The standard has apparently been regarded as a model case of a family in designing various institutions once it was defined. It is often recently reported that the ratios of unmarried and childless individuals have increased (e.g., Ministry of Health, Labour and Welfare in Japan, 2022).

and retired in the third. Individuals are assumed to be conformists. They respond to social norms of family size by minimizing their fertility distance from the norm. Assuming homogeneous individuals, the number of children of the parent can be regarded as the average fertility rate of the preceding generation.

2.1 Individual

We assume that individuals strive to conform to the behaviors of their parents and that they feel guilty if they have more or fewer children than the average number of children of their parent's generation, which is perceived as the fertility rate norm. Individuals incur a norm cost from the feeling of such an inability to measure up to societal and familial expectations. Denoting the fertility rate of generation t by n_t, the disutility from the norm costs is assumed in this paper as $-\upsilon(n_t - n_{t-1}) \equiv -\theta(n_t - n_{t-1})^2 / 2$ for a given fertility rate of the parents' generation n_{t-1}, i.e., the fertility rate norm in period t, where $\partial \upsilon / \partial n_t = -\theta(n_t - n_{t-1}) < 0$ and $\partial^2 \upsilon / \partial n_t \partial n_{t-1} = \theta > 0$. The disutility is an increasing and convex function of the discrepancy between one's own number of children and the fertility norm. Parameter $\theta \in (0,1]$ stands for the disutility weight of the cost of the deviation from the fertility rate norm. The disutility cost differs slightly from the costs of norm deviation described by Barigozzi et al. (2018) and Bhattachaya and Chakraborty (2012), who assume an increasing and *weakly* convex function in which the ideal level of fertility rate, i.e., the fertility rate norm, is influenced by the number of children of current members of the society.[7] By contrast, we assume that the fertility rate norm coincides with the average number of children of the parents' generation. It is independent of the fertility behaviors of contemporaries. There is no uncertainty about fertility rate norms. Therefore, we define the disutility from norm deviations as a quadratic function of distances from the fertility rate norm. The quadratic form is described as a special case by Akerlof (1997). In the case of a quadratic disutility function, there is only one equilibrium value (Akerlof, 1997). Albanese et al. (2016) use Italian population surveys conducted by the Bank of Italy to report that fertility is positively associated with an upbringing that emphasizes obedience to parents and teachers and family-oriented values. A greater θ might reflect a family-value-oriented upbringing of parents.[8]

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[7] Munshi and Myaux (2006) assume both Baynes' Rule and rational expectations about the belief changes of individuals. Under the assumption of a weakly convex function, social norms might engender multiple equilibria.

[8] Guiso et al. (2006) also assume that cultural upbringing can affect the parameters of an

An individual of generation t, defined as working in period t, chooses the number of children, consumption in young adulthood, and savings for the next retired period in period t.[9] That individual allocates the personal time endowment, which is normalized to unity, between market labor supply and child rearing. The individual also consumes the fruits of savings during the third retired period. Denoting consumption in the second and third periods and savings for retirement respectively by c_t^1, c_{t+1}^2, and s_t, the budget constraints in the second and third periods are written respectively as

$$w_t(1-\phi n_t) = c_t^1 + s_t \text{, and } c_{t+1}^2 = (1+r_{t+1})s_t . \tag{1}$$

Herein, w_t denotes the wage rate in period t and r_{t+1} denotes the interest rate in period $t+1$. Parameter $\phi > 0$ stands for the rearing time per child, which is assumed to be constant. The lifetime utility of the individual can be written as

$$u^t = \ln c_t^1 + \varepsilon \ln n_t + \rho \ln c_{t+1}^2 - \upsilon(n_t - n_{t-1}),$$

which depends not only on the intrinsic utility that the individual derives from consumption and children but also from the social pressure or sanctions associated with it (Kandori,1992). The optimal conditions for the lifetime utility maximization are given as

$$\frac{1}{c_t^1} - \lambda_t = 0, \tag{2}$$

$$\frac{\varepsilon}{n_t} - \theta(n_t - n_{t-1}) - \lambda_t w_t \phi = 0, \tag{3}$$

$$\frac{\rho}{c_{t+1}^2} - \lambda_t \frac{1}{1+r_{t+1}} = 0 \text{, and} \tag{4}$$

$$w_t(1-\phi n_t) = c_t^1 + \frac{c_{t+1}^2}{1+r_{t+1}}, \tag{5}$$

where λ_t is the Lagrange multiplier attached to the lifetime budget constraint (5) in lifetime utility maximization and where $\upsilon(n_t - n_{t-1}) = \theta(n_t - n_{t-1})^2/2$. From (2)–(5), we obtain

$$\varepsilon - \theta n_t(n_t - n_{t-1}) = \frac{1+\rho}{1-\phi n_t}\phi n_t . \tag{6}$$

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individual's utility.

[9] We neglect infant mortality for simplicity.

Condition (6) gives a unique solution of the number of children for a given fertility rate norm, $n_t = n(n_{t-1})$, where $dn_t / dn_{t-1} > 0$. Proof of the existence and uniqueness of the optimal number of children is given in Appendix A.

Because the fertility rate is determined uniquely, we have from (2) and (3)

$$c_t^1 = \frac{w_t}{1+\rho}(1-\phi n_t)\text{, and} \tag{7}$$

$$c_{t+1}^2 = \frac{\rho}{1+\rho}(1+r_{t+1})w_t(1-\phi n_t). \tag{8}$$

Using $n_t = n(n_{t-1})$, equations (7) and (8) become $c_t^1 = c^1(n_{t-1})$ and $c_{t+1}^2 = c^2(n_{t-1})$. Therefore, inserting these into the lifetime utility function, we obtain

$$u^t = \ln c^1(n_{t-1}) + \varepsilon \ln n(n_{t-1}) + \rho \ln c^2(n_{t-1}) - \theta(n(n_{t-1}) - n_{t-1})^2 / 2.$$

$$= u^t(n_{t-1}) \tag{9}$$

The lifetime utility of individuals depends on the fertility rate norm.

2.2 Production

The aggregate production technology is assumed as

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} = AK_t^{\alpha}(1-\phi n_t)^{1-\alpha} N_t^{1-\alpha}. \tag{10}$$

where $L_t = (1-\phi n_t)N_t$ represents the aggregate labor supply of generation t and where K_t denotes the aggregate capital stock in period t. For simplicity of exposition, we assume no technological progress, i.e., a constant $A > 0$. Assuming that the production factor markets are perfectly competitive, we have

$$1 + r_t = \frac{\partial Y_t}{\partial K_t} = \alpha A(1-\phi n_t)^{1-\alpha} k_t^{\alpha-1}\text{, and} \tag{11}$$

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)Ak_t^{\alpha}(1-\phi n_t)^{-\alpha}, \tag{12}$$

where $k_t = K_t / N_t$ represents the per-worker capital stock. The factor prices are equal to the respective marginal productivities.

2.3 Capital market

The clearing condition in the capital market is given as $K_{t+1} = s_t N_t$, where we assume that capital stock depreciates perfectly after one-period use. Using (1) and (7), the capital market clearing condition can be rewritten as

$$k_{t+1} = \frac{\rho(1-\alpha)}{(1+\rho)n_t}(1-\phi n_t)^{1-\alpha} A k_t^{\alpha}. \tag{13}$$

3. Dynamics

For a given initial condition (k_1, n_0) where $n_0 = N_1 / N_0$, the dynamic system is given by the two equations (6) and (13).[10] As described above, equation (6) is an autonomous difference equation of n_t. If a steady state fertility rate n exists, then from (6) we have

$$n = \frac{\varepsilon}{\phi} \frac{1}{1+\varepsilon+\rho}. \tag{14}$$

The steady state fertility rate satisfies the stability condition of (6) as

$$0 < \frac{dn_t}{dn_{t-1}} = [1 + \frac{(1+\varepsilon+\rho)^3 \phi^2}{(1+\rho)\theta\varepsilon}]^{-1} < 1. \tag{15}$$

Hereafter, we are concerned with such stable dynamic paths.[11] Because $n_t = n_{t-1} = n$ in the long-term equilibrium, the fertility norm costs disappear in the long-term equilibrium. On the transition converging to the long-term equilibrium, the disutility of the fertility rate norm deviation becomes smaller. When the disutility weight of the cost of the deviation from the fertility rate norm θ is great, individuals choose the fertility rate close to the fertility norm to make the cost of deviations from the fertility rate norm smaller. Therefore, a greater disutility weight of norm costs tends to slow the convergence speed of fertility to the long-term equilibrium fertility rate down. It is noteworthy that the long-term fertility rate coincides with the non-norms equilibrium, i.e., the equilibrium at which $\theta = 0$.[12] Then, from (13), the system is stable. It has a unique steady state capital stock per worker as[13]

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[10] The dynamic mechanism is as follows: equation (6) determines n_t for given n_{t-1}, and k_{t+1} is determined for such n_t and given k_t in equation (13).

[11] The stability condition of the dynamic system can be obtained from equations (6) and (13). We can demonstrate that the two eigen values are positive and smaller than unity. The derivation is available from the author upon request.

[12] Without fertility rate norms, the fertility rate chosen by individuals remains constant over time in such a simple model that considers only time costs of child rearing and that assumes a loglinear utility function (e.g., Yakita, 2001). Spolaore and Wacziarg (2021) call it the intrinsically optimal level of fertility.

[13] In a more general model that also involves horizontal transmissions of preferences,

$$k=[\frac{\rho(1-\alpha)A}{(1+\rho)n}]^{\frac{1}{1-\alpha}}(1-\phi n)\,. \tag{16}$$

Therefore, from (15) and (16) we can rewrite the long-term per-worker capital as

$$k=(\frac{1+\rho}{1+\varepsilon+\rho})^{-\alpha/(1-\alpha)}[\rho\phi(1-\alpha)A]^{\frac{1}{1-\alpha}}\,. \tag{17}$$

From the arguments presented above, we obtain the following result.

Proposition 1 *Intergenerational transmissions of fertility preference, i.e., fertility rate norms, engender the fertility rate transition. Nevertheless, the fertility rate norms do not affect the long-term equilibrium fertility rate. The disutility weight of fertility norm costs affects only the convergence speed of the fertility rate transitions.*

Condition (15) presents that a greater disutility weight θ decreases the left-hand side of (15), *ceteris paribus*.
In addition, the following Corollary is obtainable.

Corollary 1 *The economy converges to the non-norm long-term equilibrium after having transition.*

It is noteworthy that the long-term equilibrium is a social optimal because the 'inefficiency' caused by fertility rate norm is absent. This result contrasts to the reports assuming strategic complementarity (substitutability) such as that is presented by Palivos (2001). The fertility rate norm is independent of other contemporaries' behaviors in this paper. Nevertheless, it is noteworthy that the lifetime utility function changes from generation to generation on the transition because it depends on the fertility rate of the preceding generation.

4. Numerical Example

The analyses described in the preceding section suggest that a fertility rate norm can affect the fertility dynamics, while the non-norms equilibrium fertility rate remains constant

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variations of transitions might introduce multiple equilibria, reflecting a slow response to changes of the social environment and the differential response to the same environmental changes (Munshi and Myaux, 2006).

over time in our simple model. To shed light on the matter further, we consider a numerical example in this section.

We set parameters $(\varepsilon, \phi, \rho, \theta)$ for the consumption sector and (A, α) for the production sector as follows. The utility weight on children $\varepsilon = 0.23$ is slightly lower than the value 0.271 which is assumed by de la Croix and Doepke (2003). The per-child rearing time $\phi = 0.15$ is higher and twice that of 0.075 assumed by de la Croix and Doepke (2003), but we consider a couple as an individual unit. The discount factor $\rho = 0.99^{120}$ is commonly assumed in the literature. These parameters are chosen so as to engender the long-term non-norms equilibrium fertility rate around unity. Disutility of the deviation from the fertility norm θ only affects the convergence speed to the long-term fertility rate, but it does not affect the long-term equilibrium fertility rate. Therefore, we set $\theta = 0.5$ arbitrarily. The elasticity of capital in goods production is set $\alpha = 0.33$ as commonly assumed in the literature. Finally, the scale parameter of production function is set as $A = 3$ arbitrarily.

In setting the initial values, we follow the fact that the total fertility rate of Japan in the early 1950s was about 3.[14] Therefore, it is assumed that $n_1 = 3$. The per-worker capital in the production sector was set as $k_1 = 0.06$ for the sake of analytical convenience.[15] With these parameters, the stable long-term fertility rate is $n_\infty = 1.0026$. The steady state per-worker capital is $k_\infty = 0.2683$. The lifetime utility of individuals is $u^1 = -0.6833$ for generation 1 and $u^\infty = -0.2715$ for individuals in the long-term equilibrium.

The fertility rate declines monotonically to the long-term equilibrium, starting a high fertility rate, as proposed in the preceding theoretical analyses (Fig. 2(a)). The lifetime utility of individuals increases monotonically to the long-term equilibrium level (Fig. 2(c)). Figure 2(c) also presents the intrinsic utility levels. The lifetime utility function includes the fertility rate of the preceding generation as a parameter although the intrinsic utility function does not. The growth rates of GDP declines monotonically although per-

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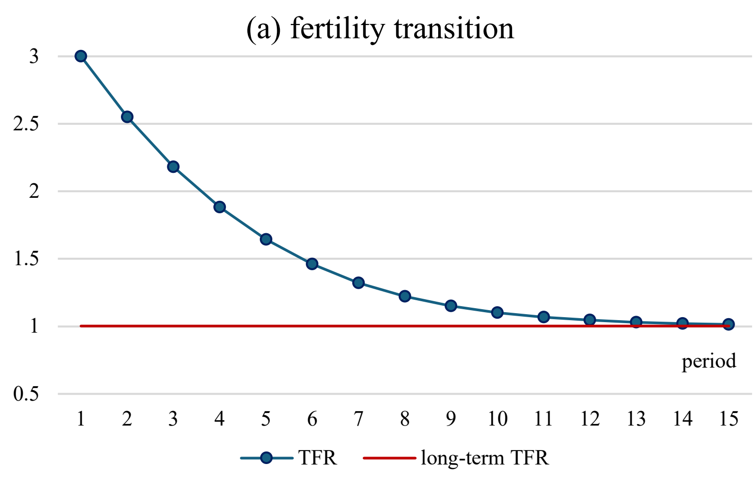
[14] The total fertility rate was 3.65 in 1950 and 2.37 in 1955 (Ministry of Health, Labour and Welfare, 2022).

[15] For other initial values of per-worker capital, the transition of per-worker growth rates might also be monotonically decreasing (e.g., $k_1 = 0.05$, other things being equal). However, Godo and Hayami (2002) report that capital labor ratios did not change before and after World War II, i.e., 4.4 in 1930, 6.3 in 1940, 6.6 in 1955, 7.9 in 1960, and 19.5 in 1970.

worker GDP show inverted U-shaped (Fig. 2(b)).[16]

As time passes, the fertility rate decreases; the population growth rate decelerates. The GDP growth rate declines because of the decreases in the population and worker size. By contrast, the per-worker capital stock increases (not shown). Therefore, in earlier periods, the positive effect of increasing capital accumulation on per-worker GDP growth dominates the negative effect of declining population growth rate. Nevertheless, as periods proceed, the declining rate of population growth becomes predominate. Consequently, the per-worker GDP growth rate decreases, converging to the long-term equilibrium. The lifetime utility of individuals increases because of physical capital accumulation, which increases the wage income of individuals. Because the time endowment of individuals is allocated between market labor and child rearing in this paper, fertility declines increase labor supply. As the fertility rate declines from generation to generation, the fertility rate norm becomes close to the non-norm fertility rate, decreasing the disutility of norm costs (i.e., the difference between lifetime utility and intrinsic utility). This result of the fertility transition in the numerical example apparently replicates the observed relation between the ideal number of children (i.e., fertility rate norm) and the expected number (i.e., the number of children chosen by parents) in Japan, as presented in the *Introduction*.



[16] The growth rate is defined as $g_t = (y_{t+1} - y_t) / y_t$. In Fig. 2(b), we set $n_0 = 3.347$ using (6) and $n_1 = 3$.

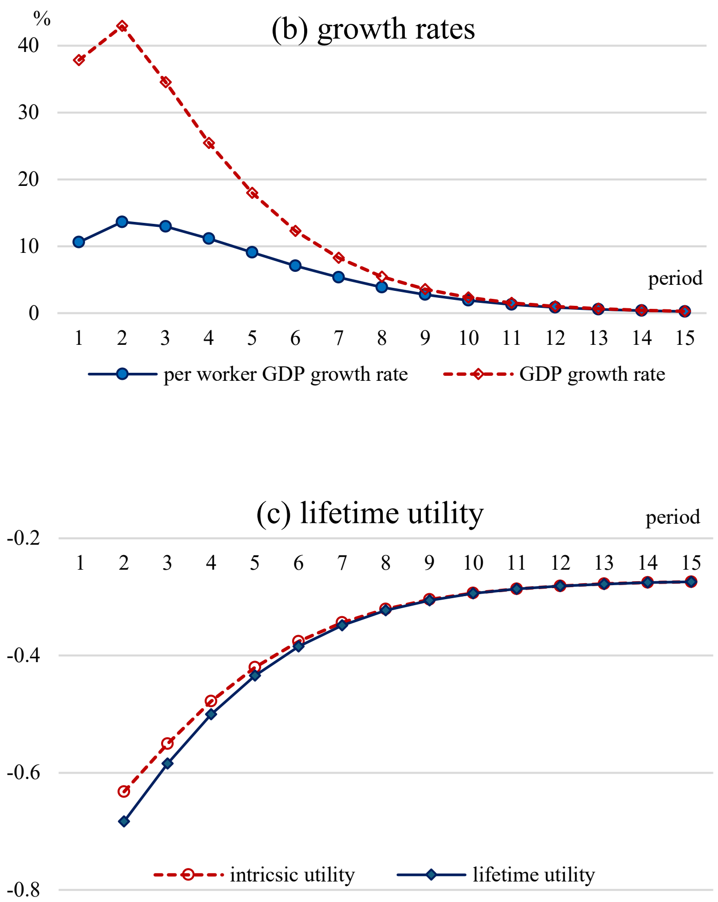


Fig. 2 Transition paths: Fertility rate, growth rates, and lifetime utility.

From the findings presented above, the following proposition is obtainable:

Proposition 2 *Supposing that the fertility rate of the initial generation is sufficiently high, then the fertility rate monotonically decreasing from generation to generation, converging to the long-term non-norm equilibrium. The transition of economic growth rate is not necessarily monotonic. The lifetime utility, i.e., the intrinsic utility plus norm costs, monotonically increases, converging to the long-term intrinsic utility level.*

5. Policy Effects of Child Subsidies

Fertility transitions caused by fertility rate norm bring about norm costs to individuals. Although the fertility rate eventually converges to a long-term equilibrium, most economically developed countries have fertility rates lower than the replacement rate in the past decades.[17] Considering policy effects on fertility transitions, we simply assume a child subsidy at rate μ, financed by lump-sum taxes, for the expositional purposes. Such a policy is expected to increase fertility rates. The budget constraint of the policy authority can be written as

$$T_t = \mu w_t n_t . \tag{18}$$

In that equation, T_t stands for a per-worker lump-sum tax. The lifetime budget constraint of an individual is given as

$$w_t(1-\phi n_t) + \mu w_t n_t - T_t = c_t^1 + \frac{c_{t+1}^2}{1+r_{t+1}} . \tag{19}$$

The second term on the left-hand side represents child subsidies. The fourth stands for a lump-sum tax. We can safely assume that the child subsidy does not cover all child rearing cost, i.e., $\phi > \mu$.

Assuming the same lifetime utility function, we obtain the optimal fertility rate implicitly as

$$\varepsilon - \theta n_t(n_t - n_{t-1}) = \frac{1+\rho}{1-\phi n_t} n_t(\phi - \mu) . \tag{20}$$

Although the left-hand side of (20) is the same as (6) in the text, the right-hand side is modified to $\hat{\Lambda}(n_t;\mu)$, where $\partial\hat{\Lambda}/\partial\mu < 0$ for $n_t > 0$. As the child subsidy rate increases, the locus $\hat{\Lambda}(n_t;\mu)$ rotates clockwise in Fig. A1. That is, the increased subsidy rate raises fertility rate n_t for given n_{t-1}.

In the long-term equilibrium, i.e., when $n = n_t = n_{t-1}$, one obtains

$$n = \frac{\varepsilon}{1+\varepsilon+\rho}\left(\phi - \frac{1+\rho}{1+\varepsilon+\rho}\mu\right)^{-1} > \frac{\varepsilon}{1+\varepsilon+\rho}\phi^{-1} . \tag{21}$$

The long-term fertility rate is higher with a child subsidy policy than without such a policy.

Based on the arguments described above, we conclude that a child subsidy might raise the fertility rate not only in the long term but also in the short term. Because the stability condition is satisfied, the fertility rate converges to the long-term fertility rate although it is higher than that without any such a policy.

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[17] This issue will become the one for all countries. The medium scenario of the UN population projections assumes that the world approach sub-replacement fertility (i.e., below 2.07), which will be reached in 2055, after which the global total fertility rate will converge to 1.84 in 2100 (UN, 2022).

Nevertheless, it is noteworthy that the child subsidy policy lowers lifetime utility of individuals both on the transition and in the long-term equilibrium. Assuming the social welfare based on individual lifetime utility, the long-term non-norm equilibrium is the first best. The policy intervention might worse off welfare of the economy despite increases in the fertility rate.

Using the numerical example in the preceding sector, we present the effects of the child subsidy policy. For exposition, we assume such child subsidy policy is introduced at initial period. Nevertheless, the timing of introduction of the policy does not affect the conclusion qualitatively.

Setting $\mu = 0.003$ in the numerical example in the preceding section, new transition paths are presented in Fig. 3. With such a child subsidy policy, we have $n_{\infty}^{cp} = 1.0199$, $k_{\infty}^{cp} = 0.2607$, and $u^{cp\infty} = -0.2774$, where superscript cp designates the values under the policy. Figure 3(a) presents the effects of the policy on the fertility transition. As expected, the fertility rate with the policy is higher than that without such a policy at each period. Figure 3(b) presents the difference between the fertility rate with the policy and without such a policy. The difference is not equal in each period. It increases in earlier periods and then decreases. In the long term, it approaches a constant, $n_{\infty}^{cp} - n_{\infty} = 0.0173$. Figure 3(c) shows lifetime utility without child subsidy policy (WO) and with such a policy (W). The policy-derived increases in the fertility rate are great in earlier periods and then become smaller later. The great increased fertility rate lowers lifetime utility of individuals largely, whereas it decreases the per-worker capital slightly less (not shown). In later periods, the declines in per-worker capital become great and the increases in fertility rates become smaller. The welfare effect of the child subsidy policy is negative in terms of long-term lifetime utility of individual, i.e., $u^{cp\infty} - u^{\infty} = -0.0059$.

From the numerical example, we have the following result.

Proposition 3 *Child policy such as child subsidies can only reduce the decline in the fertility rate on the transition caused by fertility rate norms. Nevertheless, it cannot prevent the downward trend of the fertility rate until it reaches the long-term equilibrium.*

This result has important policy implications. In the presence of fertility norms, child policy might not be able to reverse the fertility trend on the transition path.

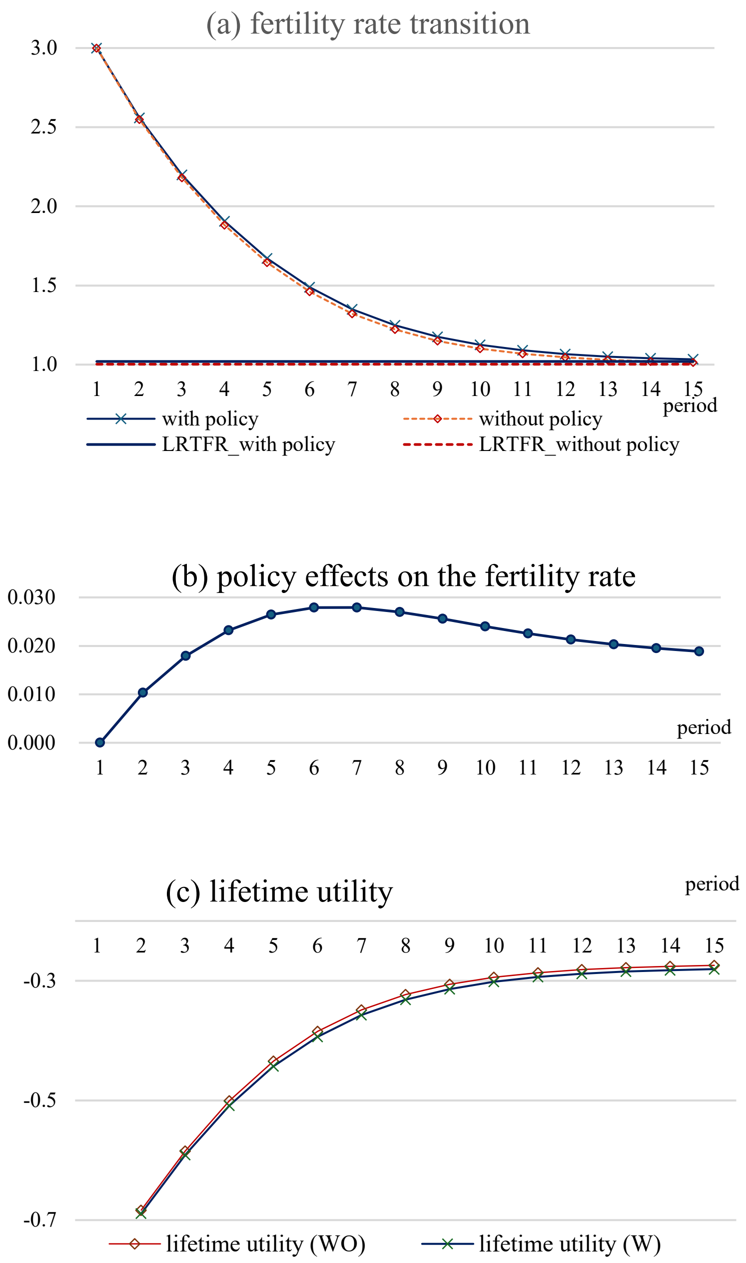

Fig. 3 Policy effects of child subsidies.

6. Concluding Remarks

The fact about Japan presented in the *Introduction*. is apparently consistent with the prediction about the existence of fertility rate norm. With that, we have demonstrated that fertility rate norm does not affect the long-term equilibrium fertility rate. The long-term equilibrium fertility rate coincides with the non-norm equilibrium fertility rate which is obtainable in the absence of fertility norms. When the deviation of fertility rate from that of parent's fertility rate brings about disutility, individuals might not immediately choose the intrinsically optimal fertility rate, thereby generating fertility rate transition. The intergenerational fertility rate norm costs only affect the convergence speed of the fertility rate to the long-term equilibrium level. The process of changing fertility rate norms which we assume in this paper is consistent with that considered by Munshi and Myaux (2006), although they consider intertemporal changes in the individuals' beliefs whereas we consider intergenerational changes of norms. They conclude that many societies have put norms into place to regulate fertility and the fertility transition might be characterized as a process of changing social norms in the community.[18]

In a sense, we present a theoretical analysis of long-term consequences of such a process in an overlapping generations setting. It is also noteworthy that, starting from a high fertility rate and low per-worker capital, the GDP growth rate transition can display an inverted U shape along the monotonic fertility transition.[19]

Most countries, specifically economically developed countries, have been adversely affected by declining fertility rates for the past several decades. One important implication of our analyses is that if the authorities aim to increase the fertility rate in the presence of fertility rate norms, then the optimal policy must be such that it can positively affect the long-term non-norm fertility rate. Fertility preference transmission works to converge the fertility rate to the long-term equilibrium rate, i.e., non-norms fertility rate, both sooner and later. Nevertheless, the magnitude of the policy effect on the transition might not be constant, probably depending on the policy instrument and the economic structure, although the numerical example shows that they are always positive. More importantly, such a policy intervention might low the welfare of individuals.

Appendix

A. Optimal number of children

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[18] They conclude that individuals respond strongly to external interventions in the individual's religious group, although cross-religion effects are entirely absent. Spolaore and Wacziarg (2021) also report that the transition from higher traditional fertility to lower modern fertility is the outcome of a process of social innovation and social influence, which depends on the social distance between early adopters and late adopters.

[19] Whether the growth rate transition displays such an inverted U shape depends on the initial fertility rate in relation to the magnitude of the initial capital stock. If the initial capital stock is sufficiently small, then the transition paths might be monotonic. Figure A2 in Appendix B presents the 3-year moving averages of real GDP growth rate and the total fertility rate in Japan. We use Cabinet Office (2022) data for growth rates after 2014.

Letting both sides of (6) be $\Phi = \varepsilon - \theta n_t(n_t - n_{t-1}) = \Phi(n_t; n_{t-1})$ and $\Lambda = (1+\rho)[\phi n_t / (1-\phi n_t)] = \Lambda(n_t)$ respectively, then we have

$$\frac{d\Phi}{dn_t} = 2\theta\left(\frac{n_{t-1}}{2} - n_t\right) \overset{>}{\underset{<}{=}} 0 \quad \text{as} \quad n_t \overset{<}{\underset{>}{=}} \frac{n_{t-1}}{2}, \tag{A1}$$

$$\frac{d\Lambda}{dn_t} = \frac{\phi}{(1-\phi n_t)^2} > 0. \tag{A2}$$

Therein, $\Phi(n_t; n_{t-1})$ represents the marginal utility of children less norm costs. In addition, $\Lambda(n_t)$ is the marginal cost in utility, where $\Phi(0, n_{t-1}) = \varepsilon > 0$ and $\Lambda(0) = 0$. Functions $\Phi(n_t; n_{t-1})$ and $\Lambda(n_t)$ are depicted in Fig. A1. The former locus is inversely U shaped, whereas the latter is monotonically increasing. Therefore, it is readily apparent that there is a unique intersection between these two loci, i.e., $n_t = n(n_{t-1})$.

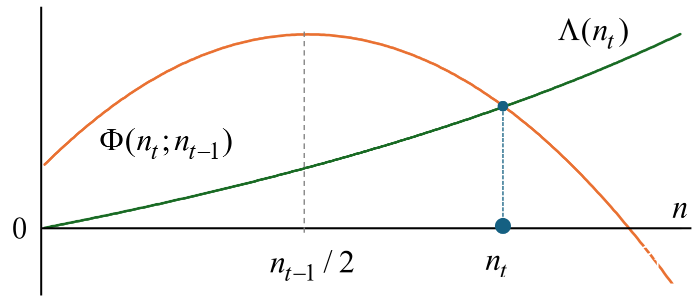

Fig. A1 Fertility decision of generation t.

B. Japan's experience: fertility rates and growth rates

The transitions of real GDP growth rate and the total fertility rate (TFR) of Japan during 1950–2021 are presented in terms of the 3-year moving average in Fig. A2. The former is measured on the left-hand side ordinate; the latter is measured on the right-hand side ordinate. The baby boom after World War II apparently initiated the fertility transition afterward.

There is a caveat on the comparison of the model prediction with the Japanese experience. Assuming the length of one period as 30 years, the real GDP growth rate of 42.937% in the numerical example corresponds to an (average) annual growth rate of 1.198% during the period. Although Japan has experienced very high economic growth of above 10% of annual growth during about 1960–1970, the high growth era was not so long and many generations were involved actually. Therefore, we might not be able to find a clear correspondence between our model predictions and the evidence although the shapes of the time paths are qualitatively analogous.

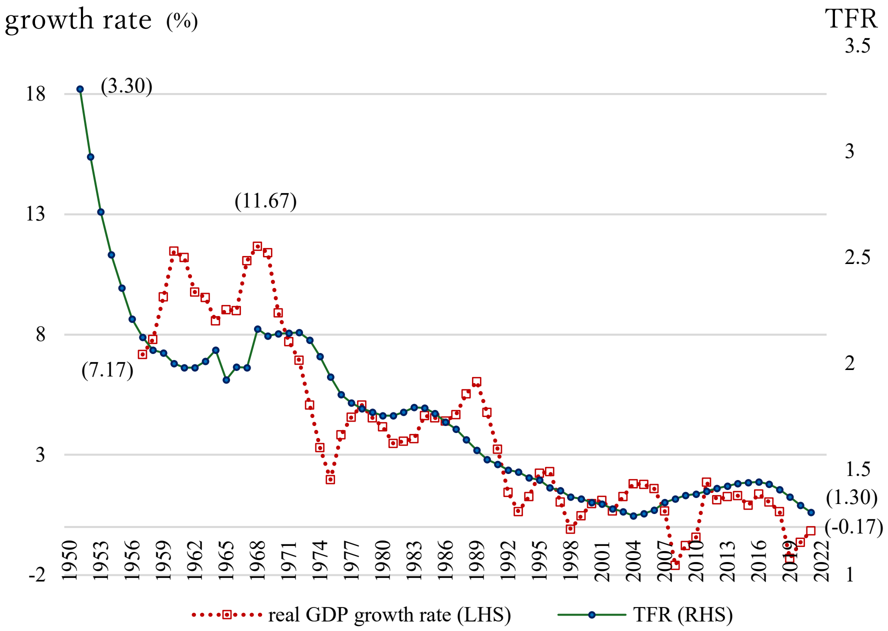

Fig. A2 Japanese experience: real GDP growth rate and TFR (3-year moving averages).
(Sources) Cabinet Office in Japan (2016, 2022), Ministry of Health, Labour and Welfare (2022)

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