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Abstract

This study presents a theoretical analysis examining whether abundant small municipalities increases efficiency. In recent years, large scale municipal consolidations have decreased the number of municipalities; however, some small municipalities have not consolidated and remain viable. This study examines the value of these circumstances. In the model analysis, this study uses the land capitalization.

This study found the following results. The optimal number of cities is larger than one only when a sufficient amount of land is needed for housing production. In this case, as the effect of local public good on preference and scale economies in producing the local public good are smaller, the optimal number of cities rises. When local public sector influence decreases, larger number of municipalities is more efficient and the economy has no need of consolidation.

JEL classification: R23, R53, H41, H73

Keywords: regional population; local public good; land capitalization; municipal consolidation

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1 Introduction

In developed countries, municipal consolidation is promoted for anticipating economies of scale and reducing public expenditure; however, some small municipalities do not consolidate, resulting in a larger number of municipalities. In Japan, Weese (2015) asserts that the current number of municipalities is larger than the optimal number. Nakagawa (2016) finds that the municipal consolidation in recent years has had a minimal effect on small municipalities. Avellaneda and Gomez (2014) note that the number of municipalities in developing countries has grown. This study analyzes whether the existence of many municipalities is desirable. To do so, this study theoretically examines the effect of local public sector on the optimal number of cities.

Previous studies indicate that municipal consolidation reduces local public expenditure through economies of scale; however, some studies such as Bless and Baskaran (2016) and Miyazaki (2018), show that municipal consolidation may increase local public expenditure. Although economies of scale are present, the effects of municipal consolidation appear elsewhere.

In this analysis, many factors other than public sector expenditure affect the optimal city number. Previous studies use land prices to evaluate consolidation (for example, Hu and Yinger (2016), Dumconbe, Yinger and Zhang (2016), Hayashi and Suzuki (2018)). Therefore, this study uses the land prices to examine the optimal number of cities. This study references Behrens, Kanemoto and Murata (2015), whose analysis uses the Henry George Theorem in which product differentiation and increasing returns cause agglomeration economies wherein second-best economies represent the modified Henry George Theorem.

In urban economies, optimal city size is achieved through a balance between agglomeration and dispersion forces. For the local public sector, local public goods have an agglomerative force, and local public goods congestion has a dispersive force. When the city population is larger, local public goods congestion is essential. However, because of the declining birthrate and an aging population, population growth has declined in some developed economies. In this economy, the congestion generated by local public goods consumption is not a significant problem; therefore, this study analyzes pure local public goods.

When a local public good is pure, it only exerts agglomerative force in urban economies. Intuitively, for the public sector, a smaller number of cities is optimal because the dispersive force (congestion) is not present. In other words, consolidation is always more desirable. This study analyzes whether the public sector increases the optimal number of cities.

The remainder of this paper is organized as follows. Section 2 presents the research model. Section 3 analyzes the optimal number of cities. Section 4 examines the effect of the public sector on the optimal number of cities and Section 5 concludes.

2 The Model

This study references Behrens, Kanemoto and Murata (2015), whose analysis uses the Henry George Theorem in which product differentiation and increasing returns cause agglomeration economies wherein second-best economies represent the modified Henry George Theorem. This study reconstructs Behrens et al.'s (2015) model by introducing absentee landlords and manufactured goods that can be transported between cities. Moreover, the study focuses the scale economies to analyze local public goods production.

In this study's model, many potential sites for city development are available, each site has a fixed amount of land ($\bar{X}_L$), and n cities are developed from these potential sites, where n is the number of cities and is endogenous to policies. One city receives a fixed amount of land on one site, and the total amount of land used in the economy is $n\bar{X}_L$. The model ignores the city's spatial structure, and no transport costs exist. For individuals, the place of residence is homogeneous within the city. Individuals are homogeneous and can migrate between cities without cost. Therefore, in equilibrium, utility is equal across individuals. The total population of the economy is $\bar{L}$. This study analyzes symmetric allocations; therefore, the city's population size is $l \equiv \bar{L}/n$. The city's land size is fixed at $\bar{X}_L$. When a new city is added to the economy, the total amount of land that is used in the economy increases, though the total population of the economy does not increase. Housing in each city is produced with land and labor as inputs, in addition to manufactured goods and local public goods.

This study analyzes the optimal number of cities by assuming that the number of cities is fixed to derive economic equilibrium conditions and determine the optimal number of cities.

2.1 Model

Individuals obtain utility from manufactured goods, local public goods, and housing. Individuals in a city have the following utility:

$$u(x, g) = x + \alpha \log g$$

where x is manufactured goods, and g is local public goods. For simplicity, this study assumes that housing consumption ($\bar{x}_H$) is fixed when an individual relocates in a city; therefore, utility does not include this consumption. The individual's budget constraint is as follows:

$$w = P_x x + P_H \bar{x}_H + \frac{C(g)}{l}$$

where each individual in a city supplies one unit of labor and acquires wage w, and P_x and P_H are the prices of manufactured goods and housing, respectively. $C(g)$ is the cost function of local public goods, for which each individual must pay the per capita cost of this good, $C(g)/l$. In equilibrium, individuals have the same utility ($\bar{u}$) regardless of their location. The study determines the price of housing based on the bid price (P_H), which represents the maximum price that an individual is willing to pay at utility level $\bar{u}$. The bid price function is as follows:

$$P_H = \frac{1}{\bar{x}_H}\left[w - P_x\left\{\bar{u} - \alpha \log g\right\} - \frac{c(g)}{l}\right] \tag{1}$$

In each city, manufactured goods are produced under constant returns and offered on the exogenous national market. These goods are produced using intermediate goods as inputs, under increasing returns and monopolistic competition with labor as the input, and cannot be traded between cities. The production function of the manufactured good is as follows:

$$X = \left\{\int_0^N q_j^\rho dj\right\}^{\frac{1}{\rho}} \qquad 0 < \rho < 1$$

where q_j denotes intermediate good j ($j \in [0, N]$) and N is the variety of intermediate goods in a city. Each intermediate good is produced by one firm. The production function of each intermediate good is as follows:

$$L_{q_j} = f + bq_j$$

where L_{q_j} is labor input, f is fixed labor input and b is marginal labor input.

Housing is produced by land and labor, and is produced under constant returns. This sector is competitive with free entry. The total amount of land in a city is $\bar{X}_L$. Absentee landlords are assumed to own the land, and only housing producers use the land for production. In equilibrium, the housing producer pays to rent the land, which equals total revenue minus labor cost. The housing production function is as follows:

$$Y_H = F_H(Y_L^H, Y_O^H)$$

where Y_H is the total amount of housing and Y_L^H and Y_O^H are land and labor inputs. The housing producer's profit (Π) is determined as follows:

$$\Pi = P_H Y_H - p_L Y_L^H - w Y_O^H$$

where p_L is the land rent. Profit maximizing conditions are as follows:

$$F_H^L = \frac{\partial F_H}{\partial Y_L^H} = \frac{p_L}{P_H}, \qquad F_H^O = \frac{\partial F_H}{\partial Y_O^H} = \frac{w}{P_H}$$

Because of free entry, in equilibrium, $\Pi = 0$.

In each city, the local government provides local public goods, which are pure. Therefore, spillover effects on other cities do not occur and public goods do not have congestion effects. The local public good is produced using manufactured goods as inputs. The cost function $C(g)$ is as follows:

$$C(g) = P_x g^\gamma$$

where γ represents the scale economies producing the local public good. The government imposes the cost on individuals, which is equal among individuals.

2.2 Equilibrium

This subsection analyzes the equilibrium conditions, where the number of cities and the supply of local public goods are determined.

First, consider the housing market in one city. Because the individual's housing demand is fixed $(\bar{x}_H)$, the aggregate demand in the city is $l\bar{x}_H$. The market clearing condition for housing is as follows:

$$l\bar{x}_H = \frac{\bar{L}}{n}\bar{x}_H = Y_H = F_H(Y_L^H, Y_O^H)$$

Because the amount of land is fixed, $Y_L^H = \bar{X}_L$. Then, labor input (Y_O^H) is determined as a function of n, $Y_O^H(n)$.

Concerning the manufactured good producer's behavior, the following condition holds:

$$p_{mj} = P_x X^{1-\rho} q_j^{d\rho-1}$$

where p_{mj} is the price of intermediate good j and q_j^d is the demand for intermediate good j. Because intermediate goods are produced under monopolistic competition, the first order condition for profit maximization is as follows:

$$w = \frac{\rho}{b} p_{mj}$$

Moreover, based on monopolistic competition behavior, the output of goods j and labor input are obtained as follows:

$$q_j = \frac{\rho f}{b(1-\rho)}$$

$$L_{q_j} = \frac{f}{1-\rho}$$

The market clearing condition for intermediate goods and labor are as follows:

$$q_j^d = q_j$$
$$\frac{\bar{L}}{n} = l = N L_{q_j} + Y_O^H(n)$$

These equations obtain the following:

$$q_j^d = q_j = \frac{\rho f}{b(1-\rho)} \quad L_{q_j} = \frac{f}{1-\rho}$$

The variety of intermediate goods is obtained as follows:

$$N = \left(\frac{\bar{L}}{n} - Y_O^H(n)\right) \frac{1-\rho}{f}$$

The price of intermediate goods (p_{mj}) and associated wage (w) are as follows:

$$\begin{aligned} p_{mj} &= P_x \left\{ \frac{1-\rho}{f} \right\}^{\frac{1-\rho}{\rho}} \left[\frac{\bar{L}}{n} - Y_O^H(n) \right]^{\frac{1-\rho}{\rho}} \\ w &= P_x \frac{\rho}{b} \left\{ \frac{1-\rho}{f} \right\}^{\frac{1-\rho}{\rho}} \left[\frac{\bar{L}}{n} - Y_O^H(n) \right]^{\frac{1-\rho}{\rho}} \end{aligned}$$

Then, the resulting amount of the manufactured good is as follows:

$$X^* = \frac{\rho}{b} \left\{ \frac{1-\rho}{f} \right\}^{\frac{1-\rho}{\rho}} \left[\frac{\bar{L}}{n} - Y_O^H(n) \right]^{\frac{1}{\rho}} \tag{2}$$

In the following section, this study analyzes the optimal number of cities to maximize the aggregate land rent. In equilibrium, we assume that utility ($\bar{u}$) is given. Then, the bid price of housing is as follows:

$$P_H = \frac{P_x}{\bar{x}_H} \left[\frac{\rho}{b} \left\{ \frac{1-\rho}{f} \right\}^{\frac{1-\rho}{\rho}} \left\{ \frac{\bar{L}}{n} - Y_O^H(n) \right\}^{\frac{1-\rho}{\rho}} - \{\bar{u} - \alpha \log g\} - \frac{ng^{\gamma}}{\bar{L}} \right]$$

The aggregate land rent in the economy is as follows:

$$\begin{aligned} &AR = n \left[P_H \bar{x}_H \frac{\bar{L}}{n} - w Y_O^H(n) \right] \\ &= P_x \left[\frac{\rho}{b} \left\{ \frac{1-\rho}{f} \right\}^{\frac{1-\rho}{\rho}} \left\{ \frac{\bar{L}}{n} - Y_O^H(n) \right\}^{\frac{1}{\rho}} n + \left\{ \alpha \log g - \bar{u} - \frac{ng^{\gamma}}{\bar{L}} \right\} \bar{L} \right] \end{aligned} \tag{3}$$

Before focusing the optimal number of cities, this study analyzes the effect of this number on the public sector. When the number is given, the optimal amount of local public goods is as follows:

$$g^* = \left(\frac{\alpha \bar{L}}{\gamma n}\right)^{\frac{1}{\gamma}}$$

Total public spending in the economy is as follows:

$$nC(g^*) = nP_x g^\gamma = P_x \frac{\alpha}{\gamma} \bar{L}$$

When the number of cities (n) increases, the amount of local public goods decreases, as total public spending does not change. In the public sector context, it is optimal for the number of cities to be smaller; that is, $n = 1$ is optimal. In other words, consolidation is always desirable from the public sector perspective. The next section examines a case in which multiple cities are more valuable than a single city.

3 Social optimum

This section examines the optimal number of cities, considering the problem of maximizing the aggregate land rent that incurs all individual's fixed utility. In the social optimum, the social planner maximizes land rent with respect to the number of cities and local public goods.

Because this study analyzes symmetric allocations, the social planner must solve the following problem:

$$\max_{g,n} AR \tag{4}$$

First-order conditions for g, n are as follows:

$$\frac{\partial AR}{\partial g} = P_x \bar{L} \left[\frac{\alpha}{g} - \frac{n}{\bar{L}} \gamma g^{\gamma-1} \right] = 0 \tag{5}$$

$$\frac{\partial AR}{\partial n} = P_x \frac{\rho}{b} \left\{ \frac{1-\rho}{f} \right\}^{\frac{1-\rho}{\rho}} \times$$
$$\left[\frac{1}{\rho} \left\{ \frac{\bar{L}}{n} - Y_O^H(n) \right\}^{\frac{1-\rho}{\rho}} \left\{ -\frac{\bar{L}}{n^2} - Y_O^H(n)' \right\} n + \left\{ \frac{\bar{L}}{n} - Y_O^H(n) \right\}^{\frac{1}{\rho}} \right]$$
$$- P_x g^{\gamma} = 0 \tag{6}$$

where $Y_O^H(n)'$ is $(dY_O^H)/(dn)$. Equation (5) presents the optimal allocation of local public goods in each city, and equation (6) indicates the optimal number of cities. In the following analysis, the model assumes that second-order conditions are satisfied, from which, the following condition holds:

$$\begin{vmatrix} \frac{\partial^2 AR}{\partial g^2} & \frac{\partial^2 AR}{\partial g \partial n} \\ \frac{\partial^2 AR}{\partial n \partial g} & \frac{\partial^2 AR}{\partial n^2} \end{vmatrix} = |D| > 0 \tag{7}$$

This study next examines the condition in which the optimal number of cities is larger than one and it is not optimal for multiple cities to consolidate and only one city carries the economy as a result.

If $n = 1$, $\partial AR/\partial n$ is expressed as follows:

$$\left.\frac{\partial AR}{\partial n}\right|_{n=1} = P_x \frac{\rho}{b}\left\{\frac{1-\rho}{f}\right\}^{\frac{1-\rho}{\rho}} \times \left\{\bar{L} - Y_O^H(1)\right\}^{\frac{1-\rho}{\rho}} \left[\frac{\rho-1}{\rho}\bar{L} - Y_O^H(1) - \frac{1}{\rho}Y_O^H(1)'\right] - P_x g^{\gamma} \quad (8)$$

where $Y_O^H(1)' = -F_H(\bar{X}_L, Y_O^H)/F_H^O$. The condition in which the optimal number of cities (n^*) is larger than one is obtained as (8) > 0. This condition is satisfied only if $|Y_O^H(1)'|$ is larger, indicating that the labor input for housing production drastically decreases with respect to the number of cities. This condition holds when the marginal product of labor in housing (F_H^O) is sufficiently small and the total amount of production ($F_H(\bar{X}_L, Y_O^H)$) is sufficiently large.

It is worth examining a case in which the labor input is unnecessary for housing production. In this case, $\partial AR/\partial n$ is as follows:

$$\left.\frac{\partial AR}{\partial n}\right|_{n=1} = P_x \frac{\rho}{b}\left\{\frac{1-\rho}{f}\right\}^{\frac{1-\rho}{\rho}} \left[\left\{\frac{\bar{L}}{n}\right\}^{\frac{1}{\rho}} \frac{\rho-1}{\rho}\right] - P_x g^{\gamma} < 0$$

The number of cities should be smaller to maximize the aggregate land rent; that is, $n^* = 1$. This means that the optimal number of cities is larger than one only when labor input is necessary for housing production.

To summarize these results, the following proposition holds:

Proposition 1 The optimal number of cities is larger than one only if the marginal product of labor in housing is sufficiently small and the total amount of housing production is sufficiently large; otherwise, one city is optimal.

When the demand for housing is larger, the economy should provide sufficient input for subsequent production. In the case of low labor productivity, a sufficient amount of land should be provided. If the number of cities is smaller, a sufficient amount of labor is required for housing production, although it decreases the production of manufactured goods that promotes agglomeration economies. Thus, multiple cities must provide a sufficient amount of land.

4 The public sector's effect on the number of cities

This section analyzes the effect of the public sector on the number of cities. The model examines the effect of local public goods on preference (α) and scale economies in producing local public good (γ) . One object of this study is to examine a case in which the existence of many cities is desirable.

This section analyzes the impact of the public sector on these cities.

Before analyzing the public sector, the study examines the effect of total population ($\bar{L}$) on the optimal number of cities as follows:

$$\frac{dn^*}{d\bar{L}} = \frac{n}{\bar{L}} > 0$$

where an increase in the total population ($\bar{L}$) increases the optimal number of cities. Supporting many individuals requires many cities that provide housing and local public goods.

The study next analyzes the public sector. First, the effect of the preference for local public good α is as follows:

$$\frac{dn^*}{d\alpha} = \frac{-{P_x}^2 g^{\gamma-2}\gamma\bar{L}}{|D|} < 0 \tag{9}$$

where the preference for the local public good decreases the optimal number of cities because cities need a larger population to provide adequate local public goods.

The effect of scale economies in producing local public good γ is as follows:

$$\frac{dn^*}{d\gamma} = \frac{{P_x}^2 g^{\gamma-2}\alpha\bar{L}}{|D|} > 0 \tag{10}$$

An increase in γ indicates a decrease in scale economies in the production of the local public good, which leads to an increase in the optimal number of cities. When public sector productivity decreases, a city should reduce the production of local public goods and stimulate the private sector. The optimal number of cities should be increased to promote the private sector because it will raise the labor input for private production.

These results derive the following proposition.

Proposition 2 If public sector influence decreases, the optimal number of cities increases.

Proposition 2 posits that multiple cities are more desirable when public sector influence decreases. In this case, the private sector is more important than the public sector. Stimulating the private sector requires a larger number of cities to increase the input for private production.

5 Conclusion

This study examines the effect of the local public sector on the optimal number of cities. In each city, the local government provides a pure local public good that has agglomerative force. It is optimal for the public sector that the number of cities is smaller; therefore, consolidation should be promoted. In this circumstance, the study demonstrates a case in which some small municipalities do not consolidate and the number of municipalities is larger, which is desirable.

This study found the following results. The optimal number of cities is larger than one only when a sufficient amount of land is needed for housing production. In this case, as the effect of local public good on preference and scale economies in producing the local public good are smaller, the optimal number of cities rises. When local public sector influence decreases, larger number of municipalities is more efficient and the economy has no need of consolidation.

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