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Old-age support policy effects on economic growth and fertility

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Abstract

Salient features of major economies around the world, specifically in high-income countries, are graying population and declining fertility. Incorporating the quantity–quality tradeoff of children into an overlapping generations model, this paper presents an analysis of the effects of formal old-age support on fertility and economic growth. Formal old-age support provision is assumed to involve management costs and labor costs. When the management costs are sufficiently small, i.e., cost-efficient, formal old-age support raises the balanced economic growth rate, involving smaller tax burden and freeing individual time from family old-age support, but it lowers fertility. The effect on the lifetime utility of individuals is indeterminate. By contrast, when management costs are high, the increased formal old-age support deters economic growth through a negative income effect caused by the greater tax burdens, but it also lowers the fertility rate. In this case, lifetime utility becomes lower.

Keywords: fertility, human capital, old-age support, quantity-quality tradeoff

JEL Classification: H51, J14, J22, J24, O40

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1. Introduction

Data for life expectancy at birth for 1950–2021, expressed in logarithmic values, are depicted for World Bank income groups of countries in Fig. 1. They increased for all countries during the period. Specifically, the growth rate in low-income countries after late 1990s is fairly high.[1] Although life expectancy after 2020 decreased because of the Covid-19 pandemic, long-term trends are positive for all country groups.

Data of the elderly dependency ratios, as expressed in logarithmic values, are depicted for World Bank income groups of countries in Fig. 2. The ratio is defined as the population size of people aged 70 and older relative to the population size of people aged 20–69 in countries. In contrast to longevity, the growth rate of the elderly dependency ratio has been higher for high-income countries. Therefore, in high-income countries, the young working generation must support more aged people per young worker now and perhaps in the future. Although the elderly dependency ratio remains low in middle-income countries, the growth rates of the ratio have become higher recently. Such might be the case even for low-income countries. This low dependence ratio might reflect high fertility of these countries. Therefore, in these countries, the needs for old-age support are expected to become great in the near future.

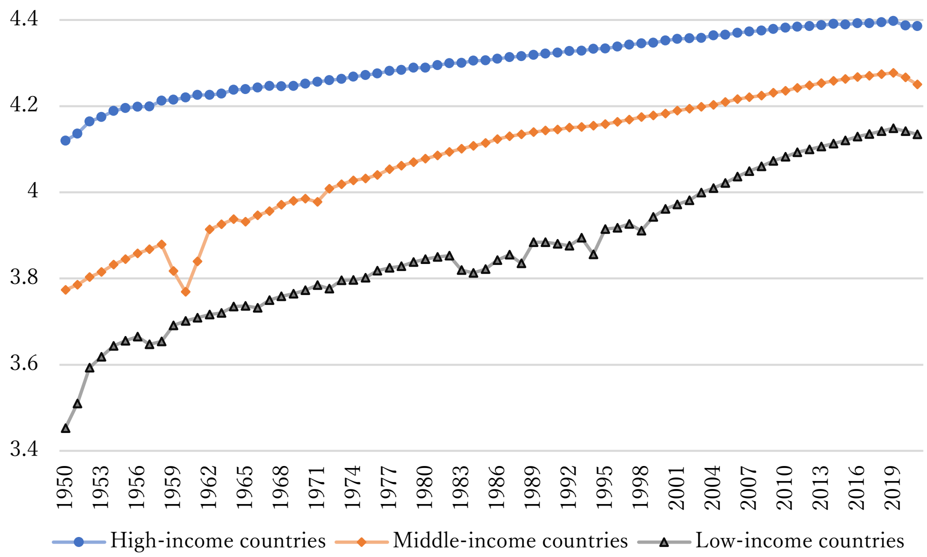

Figure 1 Life expectancy at birth of both sexes (in logarithm)

Source: United Nations: World Population Prospects 2022
https://population.un.org/wpp/Download/Standard/Mortality/

[1] Because life expectancy is measured in logarithmic terms, the slopes of the loci reflect their respective growth rates.

Mayhew (2011) reports that the gap separating life expectancy and healthy life expectancy has expanded in the past decade. Moreover, it is expected to expand in the future. The Ministry of Health, Labour and Welfare in Japan (2022) reports that, in Japan, the gaps between life expectancy and healthy life expectancy were $8.73(=81.41-72.68)$ years for men and $12.06(=87.45-75.38)$ years for women in 2019, whereas those were $8.67(=78.07-69.40)$ years and $12.28(=84.93-72.65)$ years in 2001. Extended longevity might engender a longer old-age period during which individuals need to be care-dependent. In that case, young adult children might have to provide old-age care for their parents in addition to caring for their children simultaneously. If individuals are altruistic toward their children, then they might also invest in children's human capital through education. Human capital accumulation boosts economic growth, increasing the children's affordability. Therefore, extensions of life expectancy might make the time allocation of young adult children between working and caring for both parents and children severer.

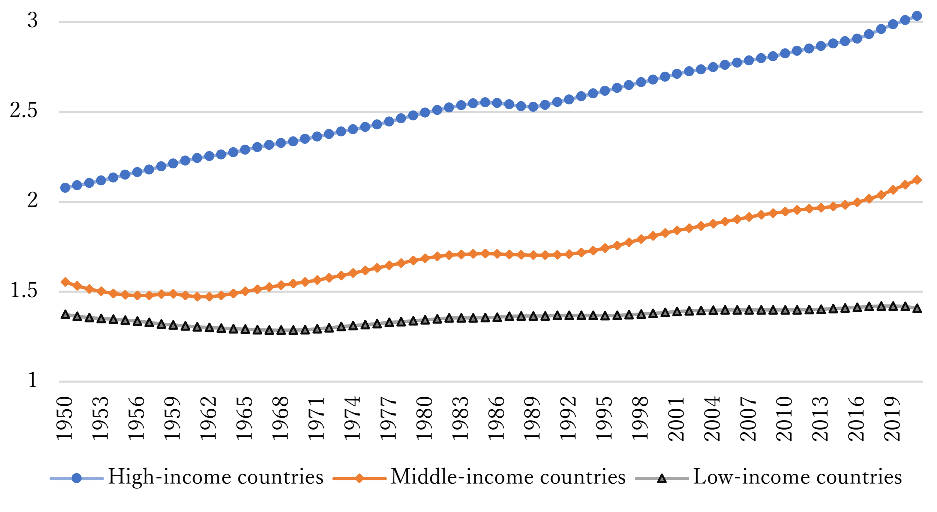

Figure 2 Elderly dependency ratio (70+/20–69) (in logarithm)

Source: United Nations: World Population Prospects 2022
https://population.un.org/wpp/Download/Standard/Population/

For each, whereas the number of children and the amount of care for them are fairly predictable, the risks related to the parents' health condition are unpredictable because health conditions during old age vary from person to person. What degree of care becomes necessary and how long the care is necessary are all uncertain. The risk of caring for parents might be severer for each family member than caring for children. In fact, many developed countries directly provide formal or public elderly long-term care services.[2] Nevertheless, individuals come to make decisions related to family or informal old-age support and on child care simultaneously during the young adulthood period.[3] These young-adult working generations are called sandwich generations.

As pointed out by Carney (2023), few theoretical studies examine sandwich generations. Therefore, few studies have assessed their simultaneous decisions about child care and elderly care.[4] Nevertheless, among others, Suh (2016) empirically reports for 2012 that about half of people aged between 47–59 cared for older parents aged 65 and older and simultaneously for children under age 18 or provided financial support to children older than 18 in the US. Yamashita and Soma (2020) find, based on a 2012–2018 sample survey of parents who have children of university student age and younger, that about 30% of Japanese people have experienced caring for both parents and children. The analyses described in this paper are intended to fill the gap.

Miyazawa (2010) shows that public old-age support promotes economic growth more than public cash transfers do because the valuation of the formal old-age support generates an additional benefit of parental education. Nevertheless, he does not consider the fertility decisions of individuals. d'Albis et al. (2018) report that a rise in human capital pushes the economy toward fewer births. Moreover, when the human capital level becomes sufficiently high, it engenders the so-called modern regime of the decline in fertility associated with birth postponement. However, in that study, they do not consider family elderly care provision. Yakita (2023) presents an analysis of effects of increases in formal long-term care provision and of increases in the labor productivity on fertility when formal elderly long-term care provision involves management costs and labor employment costs. He demonstrates that increases in long-term care provision lower fertility when formal long-term care is less cost-efficient than informal care. Nevertheless,

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[2] They are Japan, Germany, Korea, Luxembourg, Austria, Canada, and the UK. Those listed as the last three countries have means-tested public programs. Public long-term care support systems established in the US provide supplemental or substitute informal care.

[3] Klimaviciute et al. (2017) describe motivations of three main types for family long-term care provision: altruism, exchange, and family norm (forced altruism). This study presumes the last motive. Those hypotheses are tested by Alessie et al. (2014) and Klimaviciute et al. (2017) among others. The empirical results are mixed.

[4] For instance, van Houtven et al. (2013) and Skira (2015) present empirical analyses of the effects of formal long-term care provision on the labor supply of working generation and the wage rates. Many theoretical studies have examined elderly long-term care, e.g., investigations by Pestieau and Sato (2008), Cremer and Roeder (2013), Ponthiere (2014), Cremer et al. (2017), and Yakita (2020).

he does not consider parental education of the children or human capital accumulation. This paper extends Yakita's (2023) analyses by also considering the quantity–quality tradeoff of children to present an analysis of the effects of formal old-age support in kind on fertility and economic growth of the economy through these sandwich generation's decisions.[5]

The salient findings are the following. The effects of formal old-age support provision on labor employment in consumption goods production, the physical capital–human capital ratio, and the balanced economic growth rate all depend on the magnitude of management costs other than labor costs in formal old-age support provision. We assume that formal old-age support is financed by lump-sum taxes. The effect on fertility is negative. The more cost-inefficient is formal old-age support, the decline of the fertility rate is greater. If the management cost factor of formal old-age support provision is sufficiently small (large), then the balanced growth rate and the physical capital–human capital ratio becomes higher (lower) with greater formal old-age support provision. These effects derive from the fact that the management costs affect worker behaviors through the negative income effect caused by changes in the tax burden. The effects on lifetime utility of individuals might be positive if the management costs are sufficiently small, and indeterminate otherwise.

The next section introduces the model used for the analyses. Section 3 analyzes the long-term equilibrium of the model. Section 4 presents an analysis of the effects of old-age support on the long-term equilibrium. The last section concludes the paper.

2. Model

We consider an overlapping generations model populated by individuals with three life periods. Individuals are unisex and are homogeneous, except for their ages. The time horizon of the economy is infinite. Letting the population size of the working generation in period t be N_t, then the population of their children is $N_{t+1} = N_t n_t$, where n_t denotes the number of children per working individual in period t, i.e., the fertility rate in period t. Each individual receives educational expenditures by the parent during the

[5] We consider elderly support in kind, which requires some time input. Becker et al. (2016) point out that many elderly parents do not rely much on pecuniary support from their children in countries with government-financed social security systems. Fu et al. (2017) point out that the fact the introduction of Japanese Long-Term Care Insurance exerted positive spillovers on the caregivers' labor force participation can be explained by the benefits in kind, unlike other countries such as Germany and Korea, whose systems have cash allowances.

first childhood period. During the second adulthood period, the child allocates the time endowment between market labor supply and care time for the parent and for the children. With human capital accumulated during childhood, the individual earns wage income by supplying labor in the labor market. After paying taxes, the individual allocates the earned wage income among adulthood consumption, education payments for children, and savings for the individual's own old age. The second adulthood period consumption includes the individual's children's consumption. The young adulthood generation is employed by the consumption goods production sector and the government. Consumption goods are produced under constant-returns-to-scale production technology using physical capital and effective labor. The human capital of a child in period $t+1$ is produced by parental education expenditure and the average human capital stock of the parent's generation.

As described in this paper, especially emphasizing effects of formal old-age support on fertility and human capital accumulation, we assume that all old-age individuals receive a minimum level of old-age support $\overline{z}$ from the children and the government.[6] The minimum level of support is assumed to remain constant along growth paths because such support is considered necessary for life irrespective of income levels. Financed by taxes on the working generation, agencies of the government provide formal old-age support $z^G\ (<\overline{z})$. Children are altruistic toward their parents in the sense that they are forced to provide informal old-age support for parent (Cremer and Roeder, 2017), making up for the difference $\overline{z}-z^G$. We assume that children are responsible for their parents' health during old age by family norm.[7] In this paper, we assume that old-age support

[6] The old-age support under consideration herein is conceptually wider than either ADL (e.g., eating, bathing, and changing clothes) or IADL (e.g., shopping and handling phone calls). Every elderly individual described herein is assumed to need the support in a broad sense. In Japan, only 33 percent of care-need certificated persons (as support-required (Yo-Shien) levels and as severe-care (Yo-Kaigo) levels in the Japanese Long-Term Care Insurance system) were cared by long-term care facilities in 2020 (Ministry of Health, Labour and Welfare of Japan, 2020).

[7] Barigozzi et al. (2020) describe the responsibility of children as a fact. Because of homogeneous individuals, assuming that individuals provide old-age support by employing labor privately, the results might not be altered fundamentally. Chakrabarti et al. (1993) report that even though parents are uncertain about their children's altruism, the optimal investment in their children's human capital is the efficient one. We do not consider exchange motives for old-age support in this paper.

provision is time intensive both in formal old-age support and informal old-age support.[8] We also assume that time productivity in both provision sectors are equal to μ for the sake of emphasis of management costs in formal old-age support provision.[9] Old-age support production functions are $z^G N_{t-1} = \mu L_t^G$ for formal old-age support and $(\bar{z} - z^G) N_{t-1} = \mu \tilde{L}_t$ for informal old-age support. Herein, $z^G N_{t-1}$ and $(\bar{z} - z^G) N_{t-1}$ respectively represent the total old-age support provided by the government and by families: L_t^G and $\tilde{L}_t$ are the total support time employed in the formal old-age support sector and the total informal support time of working generation, respectively, during period t. We also assume that formal old-age support provision involves management costs in addition to labor cost.

2.1. Individuals

Letting the time endowment of each in a period be unity, the time constraint of a representative worker in period t is then given as

$$1 - (l_t^G + \tilde{l}_t) = \phi n_t + l_t. \tag{1}$$

Herein, ϕ denotes the per-child rearing time, which is assumed to be constant. In addition, l_t represents the labor time employed in the consumption goods production sector, l_t^G denotes labor time employed by government, and $\tilde{l}_t$ stands for the informal old-age support time for the worker's parent at home.[10] Therefore, the market labor time of the worker in period t is $l_t^G + l_t$, whereas $l_t^G + \tilde{l}_t$ stands for total old-age support time, both formal and informal.

We assume that labor in the public sector is compensated using the same wage rate w_t

[8] Miyazawa (2010) also uses such an assumption. The opportunity cost of informal old-age support time at home for each is the wage rate per labor time $w_t h_t$. Therefore, the opportunity cost of old-age support might change along with economic growth paths although the minimum old-age support level remains constant.

[9] Yakita (2023) considers a case in which they are different.

[10] According to OECD (2005), informal long-term care provided by children differs by nation: 24% in Australia, 28% in Germany, 55% in Korea, 60% in Japan, 43% in the UK, and 41% in the USA.

as in the consumption goods production sector because a cost-minimizing government must pay the same wage rate to employ labor in the public support provision sector. Therefore, each chooses the total market labor time supply $l_t^G + l_t$ without distinguishing between being employed by the government and by the consumption goods production sector.

The budget constraint of a worker in period t is expressed as

$$w_t h_t (l_t + l_t^G) - T_t = c_{1t} + n_t e_t + s_t. \quad (2)$$

In that equation, c_{1t} stands for working-period consumption, including consumption by the individual's children, h_t stands for the individual's human capital stock, e_t denotes educational expenditure per child, and s_t expresses his lifecycle savings for retirement period in period t. Additionally, w_t stands for the wage rate per efficient labor in period t, and T_t is the tax payment in period t. The budget constraint of the individual's retirement is given as

$$c_{2t+1} = (1 + r_{t+1}) s_t, \quad (3)$$

where c_{2t+1} represents retirement-period consumption, apart from old-age support services, and where r_{t+1} denotes the interest rate in period $t+1$.

The human capital of a child h_{t+1} is assumed to be produced under the nested Cobb–Douglas production technology as[11]

$$h_{t+1} = \sigma e_t^{\delta} (h_t^{\beta} \bar{h}_t^{1-\beta})^{1-\delta}. \quad (\sigma > 0,\ \delta, \beta \in (0,1)) \quad (4)$$

Herein, $\bar{h}_t$ denotes the average human capital stock of his parent's generation. Assuming homogeneous individuals, we have $h_t = \bar{h}_t$.

The lifetime utility of an individual is assumed as the following log linear function:

$$U_t = \ln c_{1t} + \varepsilon \ln n_t + \lambda \ln h_{t+1} + \rho \ln c_{2t+1}. \quad (5)$$

The working generation is altruistic toward both children and parents. For expositional ease, we omit the utility from the minimum old-age support $\bar{z}$, which is provided by government and his children. Parameter $\rho \in (0,1)$ is the discount factor for utility from consumption during old age, $\varepsilon > 0$ is the utility weight on the number of children, and $\lambda > 0$ is the utility weight on the human capital per child.[12]

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[11] As described by de la Croix and Michel (2002), other functions with constant returns to scale can be considered for sustainable endogenous growth.

[12] Because the utility weight on working-period consumption is set as equal to unity, ε

We obtain the following optimal plans from the first-order conditions for lifetime utility maximization as[13]

$$c_{1t} = \frac{1}{1+\varepsilon+\rho}[w_t h_t (1-\tilde{l}_t) - T_t], \tag{6}$$

$$n_t = \frac{\varepsilon - \lambda\delta}{\phi(1+\varepsilon+\rho)}[(1-\tilde{l}_t) - \frac{T_t}{w_t h_t}], \tag{7}$$

$$e_t = \frac{\lambda\delta}{\varepsilon - \lambda\delta} w_t h_t, \text{ and} \tag{8}$$

$$s_t = \frac{\rho}{1+\varepsilon+\rho}[w_t h_t (1-\tilde{l}_t) - T_t]. \tag{9}$$

Herein, the individual devotes time to old-age support for parents to a level such that the parent receives total old-age support of the minimum level by the family norm. For expositional simplicity, we do not explicitly include the utility of the parent from the total old-age support in defining individual lifetime utility (5). As might be apparent from (7) and (8), both the number of children and education expenditure per child increase with the wage rate per labor-timer for a given tax payment. However, it is noteworthy that parental education expenditures are unaffected by the tax burden.

2.2. Formal old-age support

The costs of providing formal old-age support are assumed to be $w_t h_t (1+M) L_t^G$, where M denotes a management cost factor. A government pays its employees a wage rate equal to the market wage rate to employ them in the labor market. The management costs include training and re-training costs of formal old-age carers in response to increases in the number of care workers, maintenance of the quality of old-age support, and also the costs of nursing facilities.[14] The plausibility of the existence of such a management cost is suggested in Yakita (2023). The sum of these costs, except the wage payments, is designated as the management cost for these analyses. For simplicity, we measure the management cost in terms of labor costs. The provision cost is assumed to be financed by

and λ might be assumed as less than unity.

[13] We assume that each worker has more than one child, i.e., the marginal benefit of having a child is more valuable than the child's life performance. In other words, we assume that condition $\varepsilon - \lambda\delta > 0$ holds true.

[14] The costs might include the net costs of care-system digitalization (e.g., remote monitoring systems). Such systems are expected to reduce labor-time inputs or to improve time efficiency, reducing the cost of formal old-age support provision. Nevertheless, we do not consider investment decisions on such systems in this paper.

lump-sum taxes on the working generation. This assumption might clarify the effects of management costs of formal old-age support provision. The budget constraint of the government in period t can be written as

$$T_t = w_t h_t (z^G / \mu n_{t-1})(1+M) \tag{10}$$

in per-worker terms.

2.3. Consumption goods production

The production function of consumption goods is assumed as

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \quad (\alpha \in (0,1)) \tag{11}$$

where Y_t, K_t, and $L_t = h_t l_t N_t$ respectively represent the aggregate output, aggregate physical capital, and aggregate effective labor in period t. Assuming perfectly competitive markets, the following conditions hold as

$$w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha} = (1-\alpha) k_t^{\alpha} (h_t l_t)^{-\alpha}, \text{ and} \tag{12}$$

$$1 + r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha} = \alpha k_t^{\alpha-1} (h_t l_t)^{1-\alpha}, \tag{13}$$

where $k_t = K_t / N_t$ stands for physical capital per worker. The factor prices are, respectively, equal to the marginal productivity of the factor.

3. Dynamic System and the Long-Term Equilibrium

As described in this section, we first analyze the short-term market equilibrium in the next subsection and then the long-term equilibrium in Subsection 3.2.

3.1. Market equilibrium in the short term

The market-clearing condition in the labor market in period t is given as

$$(1 - \tilde{l}_t - \phi n_t) N_t = (N_t l_t) + L_t^G, \tag{14}$$

where the left-hand side is the labor time supply of the working generation, and where the right-hand side represents the sum of the labor time demand of the consumption goods production sector and that of the formal old-age support provision sector, i.e., $L_t^G = N_t l_t^G$.

The total informal old-age support time in period t is $\tilde{L}_t = N_t \tilde{l}_t$.

The difference between the minimum old-age support and the formal old-age support is met by family old-age support provision, i.e., $\overline{z}N_{t-1} - \mu L_t^G = \mu \tilde{L}_t$, where the second term on the left-hand side represents formal old-age support provision by government $z^G N_{t-1}$.

The equilibrium condition in the capital market can be expressed as

$$s_t N_t = K_{t+1}\text{, or, in per-worker terms, } s_t = n_t k_{t+1}. \tag{15}$$

The left-hand side of (15) denotes the savings of the period-t working generation. The right-hand side is capital stock in the consumption goods production sector in period $t+1$. We assume here that the capital stock depreciates completely after one-period use in production.

3.2. Dynamic system

The state variables in this dynamic model are the fertility rate n_t, human capital per worker h_t, and physical capital per worker k_t. The dynamic system can be described by the three equations of these variables.

By inserting the government budget (10) and informal old-age support production function $(\overline{z} - z^G)N_{t-1} = \mu \tilde{l}_t N_t$ into (7), we obtain the following rule of motion in the fertility rate as

$$n_t = \frac{\varepsilon - \lambda\delta}{\phi(1+\varepsilon+\rho)}\left(1 - \frac{\overline{z} + Mz^G}{\mu n_{t-1}}\right). \tag{16}$$

From the human capital production function (4) and by inserting (12) into (8), we obtain

$$h_{t+1} = \sigma\left[\frac{\phi\lambda\delta}{\varepsilon - \lambda\delta}\right]^{\delta}\left[\left(\frac{k_t}{h_t}\right)/\,l_t\right]^{\alpha\delta} h_t. \tag{17}$$

Substituting (10) and (12) into (9) gives the following equation:

$$s_t = \frac{\rho(1-\alpha)}{1+\varepsilon+\rho}\left(1 - \frac{\overline{z} + Mz^G}{\mu n_{t-1}}\right)\left(\frac{k_t}{h_t}\right)^{\alpha} l_t^{-\alpha} h_t. \tag{18}$$

Inserting (16) and (18) into (15), we have the dynamic equation of per worker physical capital as

$$k_{t+1} = \frac{\phi\rho(1-\alpha)}{\varepsilon - \lambda\delta}\left(\frac{k_t}{h_t}\right)^{\alpha} l_t^{-\alpha} h_t. \tag{19}$$

The dynamic system of this model consists of the three equations (16), (17), and (19). In the system, the labor employed in the consumption goods production sector is described as

$$l_t = \frac{1+\rho+\lambda\delta}{1+\varepsilon+\rho} - \frac{\bar{z}}{\mu n_{t-1}(1+\varepsilon+\rho)}[(1+\rho+\lambda\delta)-(\varepsilon-\lambda\delta)M\frac{z^G}{\bar{z}}], \tag{20}$$

where we use the time constraint (1) and old-age support production functions. The labor supply of the working generation implicitly reflects the old-age support time, informal and formal, and child-rearing time. The old-age support time in turn depends on the ratio of the population size of working generation to the population size of the retired generation or the fertility rate of the retired generation.

By defining a new variable $\upsilon_t = k_t / h_t$ and thereby simplifying the analyses of the long-term equilibrium of the system, equations (17) and (19) can be united into the following dynamic equation:

$$\upsilon_{t+1} = \frac{\phi^{1-\delta}\rho(1-\alpha)}{\sigma(\lambda\delta)^{\delta}(\varepsilon-\lambda\delta)^{1-\delta}}(\upsilon_t / l_t)^{\alpha(1-\delta)}. \tag{21}$$

Therefore, the dynamic system becomes a set of two equations of the fertility rate n_t and the physical capital–human capital ratio υ_t, (16) and (21).

As might be readily apparent, equation (16) is a difference equation only of the fertility rate n_t. Assuming the existence of stable long-term fertility rate n for analytical purposes, the fertility rate satisfies the following condition.[15]

$$n = \frac{\varepsilon-\lambda\delta}{\phi(1+\varepsilon+\rho)}(1-\frac{\bar{z}+Mz^G}{\mu n}) \tag{16'}$$

From (20), when n remains constant, l_t also becomes constant, i.e., l. In this case, equation (21) becomes a difference equation of υ_t. Because $0 < \alpha(1-\delta) < 1$, a stable steady-state solution υ exists as

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[15] Equation (16) is a quadratic function in n. If the discriminant condition $\mu(\varepsilon-\lambda\delta)-4\phi(1+\varepsilon+\rho)(\bar{z}+Mz^G) > 0$ holds, then a stable solution is obtained. This condition gives the maximum level of management factor $M^{\max}$ for n to be a real value. The stability condition is given as $\frac{dn_t}{dn_{t-1}} = \frac{\varepsilon-\lambda\delta}{\phi(1+\varepsilon+\rho)}\frac{\bar{z}+Mz^G}{\mu n^2} \in (0,1)$.

$$\upsilon = [\frac{\phi^{1-\delta}\rho(1-\alpha)}{\sigma(\lambda\delta)^{\delta}(\varepsilon-\lambda\delta)^{1-\delta}} l^{-\alpha(1-\delta)}]^{\frac{1}{1-\alpha(1-\delta)}} . \quad (22)$$

The growth rates of human capital and of physical capital become equal in the long term, where the growth rate is given from (17) as

$$\frac{h_{t+1}}{h_t} = \sigma[\frac{\phi\lambda\delta}{\varepsilon-\lambda\delta}]^{\delta}(\upsilon / l)^{\alpha\delta} , \quad (23)$$

where from (22), we have

$$\frac{\upsilon}{l} = [\frac{(\frac{\phi(1-\alpha)}{\varepsilon-\lambda\delta})^{1-\delta}}{\sigma(\lambda\delta)^{\delta} l}]^{\frac{1}{1-\alpha(1-\delta)}} . \quad (24)$$

Therefore, the human capital growth rate gives the balanced growth rate, which can be described as $\gamma \ (= h_{t+1} / h_t - 1 = k_{t+1} / k_t - 1)$.

From the above findings, we might have two comparative statics results. First, the balanced growth rate γ is higher when the per-child rearing time ϕ is greater. From (16’), it is apparent that an increase of the per-child rearing time decreases the stable fertility rate. The lower fertility rate in turn decreases the employment of labor time in the consumption goods production (see (20)). Therefore, together with (22) and (24), the increase in the per-child rearing time increases the physical capital–human capital ratio. Consequently, the increased per-child rearing time raises the wage rate and thereby education expenditure through the income effect (see (8)). This result reflects a quantity–quality trade off of children.

Second, higher efficiency in human capital production σ also engenders a higher growth rate for the following reason. From (4), the increased production efficiency boosts human capital accumulation for a given parental education expenditure. Greater amounts of human capital per worker lower the physical capital–human capital ratio (see (22)), lowering the wage rate (see (12)). Although the low wage rate works to decrease parental education expenditure (see (8)), the positive effect of the higher human capital production efficiency dominates the negative effects of the low wage rate, consequently increasing the growth rate (see (23)). The steady-state effects of other parameters are indeterminate *a priori*.

4. Formal Old-Age Support Program

This section presents an analysis of the effects of an increase in formal old-age support provision by the government. The policy change engenders two immediate effects on the economy: one is decreased informal old-age support of families because old-age support provision remains minimal; the other is an increased tax burden to finance a greater formal support. We are concerned with the long-term effects on the economy in the following.

The long-term effect on the fertility rate is obtained from (16') as

$$\frac{dn}{dz^G} = -\frac{M}{\mu n[\frac{\phi(1+\varepsilon+\rho)}{\varepsilon-\lambda\delta} - \frac{\bar{z}+Mz^G}{\mu n^2}]} < 0 . \tag{25}$$

The denominator on the right-hand side of (25) is positive from the stability condition of the long-term fertility rate. The increased formal old-age support lowers the long-term fertility rate. We have the following lemma:

Lemma 1 *If formal old-age support provision involves management costs, then an increase in formal old-age support provision lowers the long-term fertility rate.*

In this model setting, formal old-age support is less cost-efficient than informal old-age support because of the existence of management costs. The increased tax burden brought about by an increase in formal old-age support lowers the number of children of the working generation through a negative income effect. This result is consistent with that obtained by Yakita (2023).[16]

Next, the effect of an increase in formal old-age support on the labor of the consumption goods production sector is obtained from (20) as

$$\frac{dl}{dz^G} = \frac{(\varepsilon-\lambda\delta)(1-\frac{z^G}{n}\frac{dn}{dz^G})\bar{z}}{\mu n(1+\varepsilon+\rho)}[M - \frac{(1+\rho+\lambda\delta)(-\frac{z^G}{n}\frac{dn}{dz^G})}{(\varepsilon-\lambda\delta)(1-\frac{z^G}{n}\frac{dn}{dz^G})}\frac{\bar{z}}{z^G}] . \tag{26}$$

From (25), both the coefficient of the bracketed terms and the second term in the bracket on the right-hand side of (26) are positive. Therefore, we obtain the following results.

$$\frac{dl}{dz^G} \gtreqless 0 \quad \text{as} \quad M \gtreqless \frac{(1+\rho+\lambda\delta)(-\frac{z^G}{n}\frac{dn}{dz^G})}{(\varepsilon-\lambda\delta)(1-\frac{z^G}{n}\frac{dn}{dz^G})}\frac{\bar{z}}{z^G} [\equiv H] . \tag{27}$$

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[16] As demonstrated by Yakita (2023), when the time productivity related to formal support and informal support differ, increases in old-age support might not lower the fertility rate, depending on the magnitude of management costs.

In those equations, $H > 0$.[17] If the management cost factor M in formal old-age support provision is sufficiently small, i.e., if $M < H$, then increases in formal old-age support decrease labor employment in the consumption goods production sector, i.e., $dl / dz^G < 0$. In this case, the market per-worker labor time employment, which is the sum of employments of the government and the consumption goods production sector, $l^G + l$, does not necessarily increase, although government employment increases. Also, if $M = H$, then changes in formal old-age support do not alter the labor employment in the consumption goods production sector, i.e., $dl / dz^G = 0$. If the management cost factor is great, i.e., if $M > H$, then the increased formal old-age support expands employment in the consumption goods production sector, i.e., $dl / dz^G > 0$. In this case, the market labor supply of the working generation increases.[18]

Next, we consider the effect on the balanced growth path. First, from (22), the physical capital–human capital ratio decreases with labor employment in the consumption goods production sector, i.e., $\frac{d\upsilon}{dl} = [\frac{-\alpha(1-\delta)}{1-\alpha(1-\delta)}]\frac{\upsilon}{l} < 0$. Second, we obtain $\frac{d(h_{t+1}/h_t)}{d(\upsilon/l)} = \alpha\delta\frac{h_{t+1}/h_t}{\upsilon/l} > 0$ from (23) and $\frac{d(\upsilon/l)}{dl} = [\frac{-1}{1-\alpha(1-\delta)}](\upsilon/l^2) < 0$ from (24). Therefore, the balanced growth rate decreases along with the labor employed in the consumption goods production sector, i.e., $d\gamma / dl < 0$. Together with (26), if the management cost factor M in formal old-age support provision is sufficiently small (great), then increases in formal old-age support raise (lower) the balanced growth rate. Therefore, if the management cost factor M in formal old-age support provision is sufficiently small (great), then increases in formal old-age support raise (lower) the physical capital–human capital ratio. Summing up, we can obtain the following result:

Proposition 1 *When the management costs in formal old-age support provision are sufficiently small (great), an increase in formal old-age support provision increases (decreases) the balanced growth rate and also raises (lowers) the physical capital–human*

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[17] It is noteworthy that the right-hand side of the condition in (27) includes the management cost factor, i.e., $H(M)$. For analytical purposes, we assume the existence of a unique $M \in (0, M^{\max})$ satisfying (27). The possibility that $dl / dz^G < 0$ for all $M \in (0, M^{\max})$ cannot be ruled out *a priori*. An explanation for this assumption is given in Appendix A.

[18] Based on the analysis reported by Aya (2014) and assuming different time efficiency between formal and informal long-term care provision, Yakita (2023) obtains $M = 0.55$ for Japan.

capital ratio.

The explanation underlying the results described above can be presented as described below. Increases in formal old-age support z^G require tax increases for financing them, T. The increased tax burden decreases the number of children n and lifecycle savings s (see (7) and (9)). The smaller number of children increases old-age support time inputs, formal and family, l^G and $\tilde{l}$. Because total support $\bar{z}$ remains constant, the increase of formal support works to decrease family old-age support time.

When the management cost factor of formal old-age support provision M is great, the decreases in the fertility rate and lifecycle savings become large through the large income effects (see (7) and (9)). Therefore, although the time inputs for old-age support $l^G + \tilde{l} = \bar{z} / (\mu n)$ increase, child rearing time is affected by the increased tax burden, i.e., the management cost factor M. When M is great, the increased formal old-age support, together with the lowered fertility rate, largely lowers child rearing time $\phi n = \frac{\varepsilon - \lambda\delta}{1+\varepsilon+\rho}(1 - \frac{\bar{z} + Mz^G}{\mu n})$. The great decrease in the fertility rate might engender a higher labor time supply in the consumption goods production $l = 1 - (l^G + \tilde{l}) - \phi n$ (see time constraint (1)), possibly overwhelming the effect of increases of total old-age support time $l^G + \tilde{l}$. This result might be interpreted as that individuals might increase the market labor supply largely to compensate the largely decreased disposable income. The comparative statics result presented in (27) reflects these effects. Because the effects of the increased formal old-age support on lifecycle savings and the fertility rate are both great, the effect on per-worker physical capital k is indeterminate (see (15)). If the per-worker physical capital changes slightly, then the physical capital–effective labor ratio in the consumption goods production sector $\upsilon / l = k / (hl)$ decreases with an increase of the formal old-age support (see (24)). This effect lowers the wage rate (see (12)), thereby decreasing parental education expenditures on children (see (8)). Therefore, when the management cost factor is great, the increased formal old-age support deters the balanced growth rate (see (23)).

By contrast, when the management cost factor M is small, the decrease of the fertility rate caused by increases in formal old-age support is small because of the small increase of the tax burden. The individual's time freed by the increased formal old-age support works to increase the disposable income, thereby working to increase savings and the number of children (see (7) and (9)). Therefore, in this case, the labor time supply to consumption goods production decreases because of the time constraint (1). The comparative statics results in (27) also reflect these effects. The decrease of lifecycle savings is also small because of small tax increase. The decreased labor supply might increase the physical capital–effective labor ratio (see (24)). Thereby, the wage rate increases and therefore parental education expenditure also increases (see (8)). Consequently, when the management cost factor is small, increases in formal old-age support raise the balanced growth rate (see (23)). It is noteworthy that, in this case of a small management cost factor, because of a small burden the management costs, the family old-age support time freed by increased formal old-age support works to increase the disposable income of individuals greatly.

The result presented above contrasts to the result obtained by Miyazawa (2010), who assumes strategic bequest motives. He shows, assuming exchange motives for bequests and a constant population scale, that even if formal old-age support provision is less cost-efficient than family support, the increased formal old-age support boosts economic growth: The positive valuation effect of formal old-age support dominates the disincentive effect in the strategic behavior. Canta et al. (2016) also demonstrate that if family long-term care (LTC) insurance is sizably cost-efficient, then a payroll tax increase for formal LTC financing might increase the capital stock because individuals compensate the crowded-out family help by increasing savings, in a model of constant population with a family norm. By contrast, we assume endogenous fertility decisions of individuals in addition to parental educational expenditure, engendering endogenous economic growth. By contrast, the present analyses illustrate that even cost-inefficient public old-age support might accelerate economic growth because of endogenous fertility decisions of individuals.

5. Concluding Remarks

We have demonstrated that the effects of formal old-age support on economic growth depend crucially on the magnitude of management costs in formal old-age support provision. If formal old-age support is provided cost-inefficiently with lower management costs, then increases in formal old-age support raise the balanced economic growth rate and the physical capital–human capital ratio, but that increased formal old-age support lowers the long-term fertility rate. By contrast, if formal old-age support provision is sufficiently cost-inefficient, then the increased formal old-age support deters economic growth and lowers the physical capital–human capital ratio in the long term. The present analyses have policy implications for old-age support program design in population-aging countries. For population-aging economies, making formal old-age support production cost-efficient engenders high economic growth along with lowering of the fertility rate.[19]

The main message from our analyses is that the negative effect of management costs on the economy might be offset by labor time efficiency of formal old-age support provision.[20] With more labor-time efficient formal old-age support, an increase in formal old-age support needs a smaller increase of labor time input of the government sector, freeing larger time of individuals from family old-age support and thereby suppressing decline in fertility. The freed time might increase the wage income of individuals and therefore human capital accumulation, consequently accelerating economic growth, if the tax burden increase caused is depressed by higher labor time efficiency of formal old-age support provision.

Three remarks can also be made. First, we have not considered child policies because this study emphasizes investigation of the effects of formal old-age support. Government pursues a better tradeoff between elderly care and child care in policy making in the real world (e.g., Yakita, 2018; Connelly, 1992). Second, we have not considered the probability of becoming dependent and the uncertain lifetime of elderly people. Such a consideration is presented rather commonly in the literature (e.g., Cremer and Roeder, 2017; Yakita, 2023). This paper especially emphasizes the effects of formal old-age support on economic growth. Third, because of extended longevity, grandparents might bear the burden of child rearing care, at least partly. Not only might grandparent's child care enable parents to supply more labor time (Miyazawa, 2016), but also grandchildren's care for grandparents might reduce the necessity for formal elderly care by stimulating them cognitively (Arpino et al., 2014; Rapp et al., 2023).

Appendix A: Management cost factor M satisfying condition $M = H$ of (27)

We have the following from (25):

$$-\frac{z^G}{n}\frac{dn}{dz^G} = \frac{z^G M}{\mu n^2 \dfrac{\phi(1+\varepsilon+\rho)}{\varepsilon-\lambda\delta} - (\bar{z} + M z^G)} [\equiv \Lambda] . \tag{A1}$$

The right-hand side of the condition of (27) can be rewritten as

[19] Effects of time efficiency of old-age support are briefly discussed in Appendix B.

[20] When time efficiency in formal old-age provision is higher, the tax burden becomes smaller (see (10)). Yakita (2023) considers a case in which time efficiency differs between public and family old-age support sectors.

$$H = \frac{(1+\rho+\lambda\delta)\Lambda}{(\varepsilon-\lambda\delta)(1+\Lambda)}, \tag{A2}$$

from which we obtain

$$\frac{dH}{dM} = \frac{1+\rho+\lambda\delta}{\varepsilon-\lambda\delta}\frac{1}{(1+\Lambda)^2}\frac{d\Lambda}{dM}. \tag{A3}$$

From (A1) we obtain

$$\frac{d\Lambda}{dM} = \frac{\Lambda}{M}\{1-\frac{\Lambda[2n\mu\frac{\phi(1+\varepsilon+\rho)}{\varepsilon-\lambda\delta}\frac{dn}{dM}-z^G]}{z^G}\} > 0. \tag{A4}$$

where we have from (16'):

$$\frac{dn}{dM} = (-\frac{z^G}{\mu n})/[\frac{\phi(1+\varepsilon+\rho)}{\varepsilon-\lambda\delta}-\frac{\bar{z}+Mz^G}{\mu n^2}] < 0. \tag{A5}$$

The right-hand side of the condition H increases with management cost factor M, i.e., $dH/dM > 0$. Because we have $H = 0$ when $M = 0$, the condition (27) is satisfied with equality.

As M increases, both sides increase. The right-hand side is apparently nonlinear. From (A1) and (A5), we obtain $\Lambda \to 0$ as $M \to 0$. From (A4), we also have $d\Lambda/dM \to \infty$ as $M \to 0$. Therefore, one might have $dH/dM \to \infty$ as $M \to 0$ from (A3).

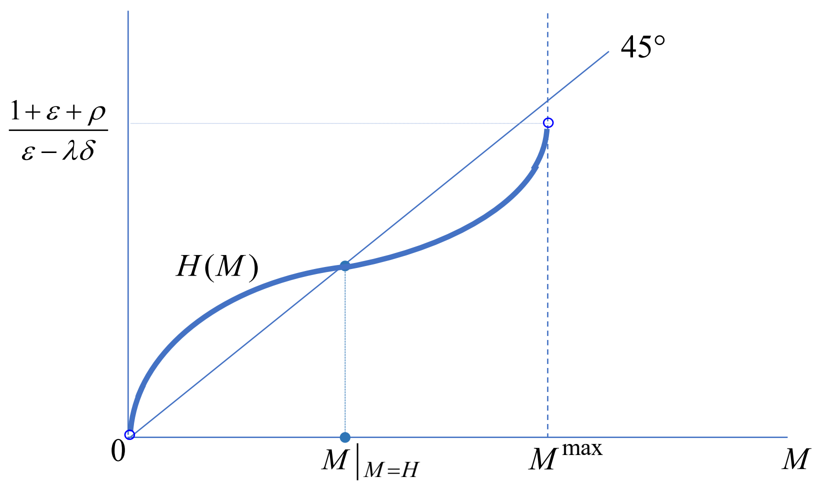

Figure A1 M satisfying condition $M = H$ in (27)

Appendix B: Effects of Increased Support Time Productivity

This appendix presents an analysis of the effects of increases in the support-time productivity of informal old-age support. It is noteworthy that, for these analyses, the support-time productivity is equal in both informal and formal old-age support provision.

From (16), we have

$$\frac{dn}{d\mu} = \frac{\overline{z} + Mz^G}{n\mu^2[\frac{\phi(1+\varepsilon+\rho)}{\varepsilon-\lambda\delta} - \frac{\overline{z}+Mz^G}{\mu n^2}]} > 0. \tag{A6}$$

Because increases in the support-time productivity of old-age support provision reduce old-age support time, the freed time might be allocated to child rearing. From (20) we also obtain

$$\frac{dl}{d\mu} = \frac{(1+\varepsilon+\rho)\overline{z} - (\varepsilon-\lambda\delta)Mz^G}{\mu^2 n(1+\varepsilon+\rho)} > 0. \tag{A7}$$

The increased productivity increases labor employed in the consumption goods production sector. Therefore, together with (22), the physical capital–human capital ratio decreases. From (23) and (24), the balanced growth rate becomes lower. A brief explanation of these results is the following. The increased labor in the consumption goods production sector lowers the wage rate. Therefore, because of the greater number of children, parents reduce their per-child education expenses and also savings. However, the results are not comparable to those reported by Yakita (2023), who assumes different labor-time productivity between informal and formal old-age support.

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