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# Old-age support policy and fertility with strategic bequest motives

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Abstract

This paper presents an analysis of the effects of public old-age support on individuals' fertility decisions and the long-term equilibrium by assuming strategic bequest motives of individuals in a discrete-time overlapping generations model. An increased wage tax for financing public old-age support reduces the opportunity cost of child-rearing time, although it decreases after-tax income. The increased public support tends to decrease the family old-age support time. Thereby, the net effect of public support on fertility is positive when the rationality constraint is binding. Nevertheless, the tax increase lowers the per-worker capital. Therefore, the net effect of public support on the long-term lifetime utility of an individual is ambiguous. It is also shown that intergenerational exchange based on strategic bequest motives might lead to excessively high fertility rate compared to social optimum.

JEL Classification: D13, J13, J14, J22

Keywords: fertility, old-age support, rationality constraint, strategic bequest motives

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## 1. Introduction

Most advanced economies have experienced extended longevity and declining fertility in past decades. Longer life expectancy increases the possibility of long-term care needs of individuals and entire countries (Mayhew, 2011). Because the health conditions of individuals during the old age period vary among individuals, some countries have public long-term care support systems to supplement or substitute informal supports, e.g., Japan, Germany, Korea, Luxembourg, Austria, Canada, the UK, and the US.[1]

This paper presents an analysis of effects of public old-age support on fertility rates by incorporating endogenous fertility decisions of individuals into the dynamic general equilibrium model based on strategic bequest motives. With strategic bequest motives, a parent obtains additional (marginal) utility and also extra old-age support from an extra child at the cost of rearing time and strategic bequests given to the child. Few studies in the literature describe works on fertility analyses using such a dynamic model based on strategic bequest motives.

As population aging progresses in countries, many theoretical studies of effects of elderly long-term care on family labor supply decisions have been presented (Pestieau and Sato, 2008; Cremer and Roeder, 2013; Ponthiere, 2014; Cremer et al., 2017; Yakita, 2020a). Many empirical analyses of family caregiving effects on the market labor supply of family members have also been presented (van Houtven et al., 2013; Skira, 2015). A vast literature related to Japan has presented analyses of the Japanese Long-Term Care Insurance system on the market labor supply of families (Tamiya et al., 2011; Sugawara and Nakamura, 2014; Yamada and Shimizutani, 2015; Oshio and Usui, 2017; Niimi, 2017; Fu et al., 2017). Most such studies specifically examine the female labor supply because long-term care is often provided by women. The results are mixed. Nevertheless, these theoretical and empirical studies do not consider fertility decisions of families simultaneously.

Since works by Becker, e.g., Becker and Barro (1988), the analysis of fertility decisions have been described in many reports (e.g., Galor and Weil, 1996; de la Croix and Doepke, 2003; Apps and Rees, 2004). Nevertheless, few reports describe the effects of long-term care on the fertility decisions made by families. Recently, Yakita (2023a) presents an analysis of the long-term effects on fertility rate in an overlapping generations model in which children are altruistic toward their parents. He demonstrates that increases in public long-term care provision might not raise the fertility rate, depending on the efficiency of

[1] Canada, the UK, and the US have means-tested public programs.

public care provision.[2] By contrast, this study considers such an issue in a dynamic general equilibrium model including many generations, assuming strategic bequest motives of individuals.

As described herein, the author also considers broader old-age support services, such as inclusion of chore assistances, rather than a narrowly defined long-term care services such as ADL and IADL services.[3] In Japan, for example, the ratio of individuals in need of long-term care among people aged 75 and older is 32.1%; the ratio of those aged 85 and older is 60.6% (Japanese Ministry of Health, Labour and Welfare, 2020).[4] The life expectancy at birth was 81.47 years for men and 87.57 years for women in Japan in 2021 (Japanese Ministry of Health, Labour and Welfare, 2021).[5] Therefore, one might infer that most elderly people are supported by family members or by public programs, even if they do not even need narrowly-defined long-term services such as IADL services.[6] To simplify the arguments, for these analyses we assume that all elderly people receive old-age support both informally from family members and formally from governments. Herein, we consider only old-age support in kind.[7]

The main result is that public old-age support financed through wage taxes raises the fertility rate when the rationality constraint is binding. Under the rationality condition, the cost of family old-age support for young adult children is exactly compensated by bequests from elderly parents. In addition, the positive effect of lower opportunity costs of child rearing outweighs the negative income effects of tax increases. Moreover, that

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[2] Yakita (2022b, 2023b) examines the fertility decisions of parents in a model assuming intergenerational exchange game between children and parents and demonstrates that an increase in the probability of becoming dependent or in the bargaining power of children might lower the fertility rate. Nevertheless, the models are essentially a two-generation model.

[3] Van Houtven et al. (2013 p.244) define chore care as cares including household chores, errands, and transportation.

[4] Ministry of Health, Labour and Welfare of Japan (2020) https://www.mhlw.go.jp/toukei/saikin/hw/life/life21/dl/life21-02.pdf.

[5] Ministry of Health, Labour and Welfare of Japan (2022) https://www.mhlw.go.jp/toukei/saikin/hw/life/tdfk20/dl/tdfk20-05.pdf.

[6] Most municipalities pay for medical service costs of elderly persons in a universal health insurance system in Japan, although elderly patients cover some costs.

[7] Miyazawa (2010), incorporating human capital accumulation as a growth engine, compares the growth effects of public old-age support in cash and in kind. He demonstrates that transfers in kind promote growth to a great degree. Pensions as intergenerational cash transfers have been studied widely, as reported in the literature (e.g., Cipriani and Fioroni, 2023; Tamai, 2023). Nishimura and Zhang (1992) and Yakita (2001), for example, examines the fertility effect of social security.

study found that the policy lowers the capital stock per worker in non-health consumption goods production. Consequently, public old-age support would not necessarily improve the steady-state lifetime utility of individuals.

The remainder of this paper is the following. An overlapping generations model with strategic bequest motives of individuals is introduced in the next section.[8] Section 3 describes an examination of the model dynamics. Section 4 presents comparative static results with respect to old-age support provision. Section 5 compares the results with the social optimum. The objective of a social planner is the individual lifetime utility in steady states. The last section concludes the paper.

## 2. Model

We consider a discrete-time overlapping generations model. For our purposes, we assume unisex individuals. Individuals are homogeneous and live for three periods: childhood, a working period and old-age retirement period. The lifetime is certain, and the length of each period is normalized to unity. An individual is fed by a parent during his childhood. The individual then supplies labor to the labor market, consumes some of the wage income, and rears children in the second working period. The working individual also provides family old-age support for the parent in exchange for bequests, i.e., with strategic bequest motives.[9] A government provides public old-age support, produced using labor, and finances it through wage taxes. Non-health consumption goods are produced under Cobb Douglas technology using capital and labor.

### 2.1 Individuals

While consuming part of the sum of personal wage income and a bequest from a parent, the individual saves the remainder for his old-age retirement in the second period. He allocates the fruits of savings between non-health consumption and makes bequests to his children in exchange for old-age support from them in the third period. The lifetime optimization problem of an individual can be split into two stages: the first is the maximization of utility during the old-age period, the second is the maximization of the lifetime utility in the second working period. We first consider the choice of a strategic bequest in the third period of individual's life and then consider the choice of

[8] The strategic bequest motives are called the exchange motives for old-age caregiving.

[9] This setting is similar to the one which Miyazawa (2010) presents as a case of old-age support in kind. Bernheim et al. (1985) provide evidence that strongly suggests that bequests are used as compensation for services rendered by beneficiaries.

consumption-savings plan in the second period in turn.

An old-age individual in period $t+1$ allocates the fruit of his lifecycle savings between old-age non-health consumption and bequests to maximize old-age utility. The old-age utility function is assumed as $u_{2t+1} = c_{2t+1}^{1-\gamma}[n_t(z_{t+1} + \varepsilon z_{t+1}^G)]^{\gamma}$, where $c_{2t+1}$ is non-health consumption during the old-age period, $n_t$ stands for the number of his children, $z_{t+1}$ represents old-age support per child for a parent, and $z_{t+1}^G$ denotes public old-age support provided by government in per-child terms. Parameter $\varepsilon > 0$ represents the degree of cost-efficiency of public old-age support time relative to family support.[10] The total elderly support per child is given as $z_{t+1} + \varepsilon z_{t+1}^G$ in terms of family support. Letting $b_{t+1}$ be the bequest per child, the rationality constraint can be written as

$$b_{t+1} \geq (1-\tau) w_{t+1} z_{t+1}, \qquad (1)$$

where $\tau \in (0,1)$ is the wage tax rate and where $w_{t+1}$ denotes the wage rate for period $t+1$. No child provides family old-age support unless that child receives a bequest greater than or equal to the opportunity cost of support provision.[11] Letting $I_{t+1}$ be the fruit of the child's lifecycle savings from the working period, the budget constraint in the old-age retirement period is given as[12]

$$I_{t+1} = c_{2t+1} + n_t b_{t+1}. \qquad (2)$$

The optimization problem of the old-age individual is to choose non-health consumption $c_{2t+1}$ and old-age support (demand) $z_{t+1}$ to maximize the old-age utility. Because of maximization by the individual, it is natural to consider equality for the rationality constraint (1). Therefore, the choice of support demand $z_{t+1}$ can be regarded as that of bequests to children $b_{t+1}$. Inserting the equal constraint into the budget constraint, we have

$$I_{t+1} = c_{2t+1} + n_t (1-\tau) w_{t+1} z_{t+1}. \qquad (3)$$

The first-order conditions for old-age utility maximization are

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[10] Although public support is expected to be more efficient than family support, i.e., $\varepsilon \geq 1$, one can have $\varepsilon < 1$ when the public support provision needs goods inputs other than labor or when X-inefficiency exists.

[11] Coefficient of $z_{t+1}$ in constraint (1), $(1-\tau)w_{t+1}$, corresponds to the equilibrium price of the private market of old-age support in a model of exchange motives for caring, which is considered by Klimaviciute et al. (2017). In equilibrium $b_{t+1}$ is the amount paid for $z_{t+1}$.

[12] We assume that parents regard their children as identical, giving them bequests equally.

$$(1-\gamma)c_{2t+1}^{-\gamma}n_t^{\gamma}(z_{t+1}+\varepsilon z_{t+1}^G)^{\gamma}-\mu=0\text{, and} \tag{4a}$$

$$\gamma c_{2t+1}^{1-\gamma}n_t^{\gamma}(z_{t+1}+\varepsilon z_{t+1}^G)^{\gamma-1}-\mu n_t(1-\tau)w_{t+1}=0\,. \tag{4b}$$

Variable $\mu$ is the Lagrange multiplier attached to constraint (3). From (4) we obtain

$$c_{2t+1}=\frac{1-\gamma}{\gamma}n_t(1-\tau)w_{t+1}(z_{t+1}+\varepsilon z_{t+1}^G)\,. \tag{5}$$

By inserting $c_{2t+1}$ from (5) into budget constraint (3), one obtains

$$z_{t+1}+\varepsilon z_{t+1}^G=\gamma[I_{t+1}+n_t(1-\tau)w_{t+1}\varepsilon z_{t+1}^G]/[n_t(1-\tau)w_{t+1}]\,. \tag{6}$$

From (5) and (6) we obtain the indirect old-age utility of

$$u_{2t+1}=(\frac{1-\gamma}{\gamma})^{1-\gamma}[n_t\,(1-\tau)w_{t+1}]^{1-\gamma}(z_{t+1}+\varepsilon z_{t+1}^G)^{1-\gamma}\,n_t^{\gamma}\,(z_{t+1}+\varepsilon z_{t+1}^G)^{\gamma}$$

$$=Q_{t+1}[I_{t+1}+n_t(1-\tau)w_{t+1}\varepsilon z_{t+1}^G]\equiv u_{2t+1}^*\,, \tag{7}$$

where $Q_{t+1}=(1-\gamma)^{1-\gamma}\gamma^{\gamma}[(1-\tau)w_{t+1}]^{-\gamma}$. Because the rationality constraint is satisfied with equality, each child provides old-age support $z_{t+1}$ to his parent in exchange for bequest $b_{t+1}$. Income $I_{t+1}$ and the number of children $n_t$ are already determined in the prior period; public old-age support $z_{t+1}^G$ and tax rate $\tau$ are determined by the government; market prices (i.e., wage rate $w_{t+1}$ and interest rate $r_{t+1}$) are given for individuals.

Next, we consider the optimization of working individuals. Each individual chooses the time allocation among market labor supply, child rearing time, and family old-age support time. Each individual also allocates the sum of wage income and bequest received between non-health consumption during the working period and lifecycle savings. Market labor consists of that employed in non-health consumption goods production and that employed in the public old-age support provision sector. For convenience, we denote the former by $l_t$ and the latter by $z_t^G$. We assume that labor in public sector is compensated using the same wage rate $w_t$ as in the non-health consumption goods production sector during period $t$ because a cost-minimizing government must pay the same wage rate to employ labor in the public support provision sector. Assuming that the child-rearing time per child is constant at $\phi>0$, then the time constraint of a working individual is written as

$$l_t + z_t^G + z_t + \phi n_t = 1, \tag{8}$$

where $l_t + z_t^G$ is the market labor supply and where $z_t + \phi n_t$ is the labor supply for family production of family old-age support and childcare.

The budget constraint of a working individual in period $t$ is given as

$$b_t + (1-\tau) w_t (l_t + z_t^G) = c_{1t} + \frac{I_{t+1}}{1+r_{t+1}}. \tag{9}$$

Therein, $c_{1t}$ denotes non-health consumption during the working period. Because the rationality constraint is satisfied with equality for each generation, we have $b_t = (1-\tau) w_t z_t$. Therefore, budget constraint (10) can be rewritten as

$$(1-\tau) w_t (1-\phi n_t) = c_{1t} + \frac{I_{t+1}}{1+r_{t+1}}. \tag{10}$$

The lifetime utility function is assumed as $\tilde{U} = \ln c_{1t} + \sigma \ln n_t + \beta \ln u_{2t+1}^*$. From (7) and because $Q_{t+1}$ is given for the individual, the objective function of the utility maximizing individual can be written as

$$U = \ln c_{1t} + \sigma \ln n_t + \beta \ln[I_{t+1} + n_t (1-\tau) w_{t+1} \varepsilon z_{t+1}^G]. \tag{11}$$

The optimization problem of the individual is to choose non-health consumption $c_{1t}$, the number of children $n_t$, and lifecycle savings $I_{t+1}/(1+r_{t+1}) \equiv s_t$ to maximize lifetime utility (11). The first-order conditions for lifetime utility maximization are

$$\frac{1}{c_{1t}} - \lambda_t = 0, \tag{12}$$

$$\frac{\beta}{I_{t+1} + n_t (1-\tau) w_{t+1} \varepsilon z_{t+1}^G} - \frac{\lambda_t}{1+r_{t+1}} = 0, \text{ and} \tag{13}$$

$$\frac{\sigma}{n_t} + \frac{\beta (1-\tau) w_{t+1} \varepsilon z_{t+1}^G}{I_{t+1} + n_t (1-\tau) w_{t+1} \varepsilon z_{t+1}^G} - \lambda_t (1-\tau) w_t \phi = 0. \tag{14}$$

Variable $\lambda_t$ is the Lagrange multiplier attached to constraint (10). Conditions (12) and (13) give the optimal intertemporal allocation between the working-period consumption and the retired-period consumption, including old-age support services. The first term on the right-hand side of (14) represents the marginal utility of having an extra child. The second term is the marginal utility of old-age support from the child net of bequest (costs) given to the individual. The third term stands for the rearing costs of the child in terms of utility.

From (12) and (13) we obtain

$$c_{1t} = \frac{I_{t+1} + n_t(1-\tau)w_{t+1}\varepsilon z_{t+1}^G}{\beta(1+r_{t+1})}. \tag{15}$$

Using (10) and (15) and defining $G_{t+1} = \frac{n_t(1-\tau)w_{t+1}\varepsilon z_{t+1}^G}{1+r_{t+1}}$, we have

$$c_{1t} = \frac{1}{1+\beta}[(1-\tau)w_t(1-\phi n_t) + G_{t+1}]. \tag{16}$$

Herein, $G_{t+1}$ is the discounted present value of public old-age support in terms of family old-age support. Using the definition of lifecycle savings, we obtain

$$s_t = \frac{1}{1+\beta}[\beta(1-\tau)w_t(1-\phi n_t) - G_{t+1}]. \tag{17}$$

We also have

$$n_t = \frac{1}{\phi}[\frac{\sigma}{1+\beta+\sigma} + \frac{G_{t+1}}{(1-\tau)w_t}]. \tag{18}$$

A greater discounted present value of public old-age support engenders a greater number of children. It is noteworthy that equation (15)–(18) give implicit solutions for hon-health consumption, savings, and the number of children. They will be obtained in the next section.

## 2.2 Government

Government provides public old-age support to old-age individuals, employing labor from the labor market at the market wage rate. The government old-age support provision is financed by a wage income tax, balancing the budget in each period. We assume that the government expends tax revenues only for old-age support. By maintaining a balanced budget, the government budget constraint is given as

$$\tau w_t(l_t + z_t^G)N_t = w_t z_t^G N_t. \tag{19}$$

Variable $N_t$ stands for the number of workers in period $t$. The left-hand side represents the tax revenues. The right-hand side is the labor cost of public old-age support provision.[13] From (19) it follows that $\tau w_t l_t = (1-\tau)w_t z_t^G$ in per-worker terms.

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[13] Public old-age support for an elderly person in period $t$, $z_t^G$, is expressed in per adult-child terms. Therefore, each elderly person receives public support

2.3 Non-health consumption goods production

The production function of non-health consumption goods is assumed as

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \quad (0 < \alpha < 1). \tag{20}$$

In that equation, $Y_t$ is the aggregate output; $K_t$ and $L_t$ represent the aggregate capital and labor in period $t$. The production function can be rewritten as $y_t = k_t^{\alpha} l_t^{1-\alpha}$ in per-worker terms, where $y_t = Y_t / N_t$, $k_t = K_t / N_t$ and $l_t = L_t / N_t$. Assuming perfectly competitive factor markets, we have

$$w_t = (1-\alpha) k_t^{\alpha} l_t^{-\alpha} = (1-\alpha) y_t / l_t, \text{ and} \tag{21}$$

$$1 + r_t = \alpha k_t^{\alpha-1} l_t^{1-\alpha} = \alpha y_t / k_t. \tag{22}$$

The marginal productivity of each factor is equal to the factor price.

2.4 Capital market

The clearing condition of capital market is given as

$$K_{t+1} = s_t N_t \text{ or, in per worker terms, } n_t k_{t+1} = s_t. \tag{23}$$

3. Dynamics and Long-Term Equilibrium

In this section we study the dynamics of the economy. First, we consider temporary equilibrium; and then we examine the dynamics of the system.

3.1 Temporary equilibrium

We first analyze a temporary equilibrium in a period, period $t$. From the government budget constraint and from the definition of $G_{t+1}$, we have $G_{t+1} = \varepsilon \theta n_t k_{t+1}$, where $\theta = [(1-\alpha)/\alpha]\tau$ is the wage tax burden relative to capital income (hereinafter, it is called the tax parameter) . Using (17) and (23), we obtain $n_t k_{t+1} = \varepsilon \theta \beta (1-\tau) w_t (1-\phi n_t) / (1+\beta+\varepsilon\theta)$. Therefore, we have

$$G_{t+1} = \frac{\varepsilon \theta \beta (1-\tau) w_t (1-\phi n_t)}{1+\beta+\varepsilon\theta}. \tag{24}$$

From (12), (16), and (24), we obtain

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$n_{t-1} z_t^G = N_t z_t^G / N_{t-1}$ from the next generation of their children.

$$\frac{1}{\lambda_t} = \frac{1+\varepsilon\theta}{1+\beta+\varepsilon\theta}(1-\tau)w_t(1-\phi n_t). \tag{25}$$

Inserting $G_{t+1}$ from (24) and $1/\lambda_t$ from (25) into condition $\lambda_t G_{t+1} + \sigma = \lambda_t(1-\tau)w_t\phi n_t$, which is obtained from (13) and (14), one can obtain

$$(1-\phi n_t)[\varepsilon\theta\beta + \sigma(1+\varepsilon\theta)] = (1+\beta+\varepsilon\theta)\phi n_t. \tag{26}$$

Therefore, the fertility rate in period $t$ is given as

$$n_t = \frac{\sigma(1+\varepsilon\theta)+\varepsilon\theta\beta}{\phi(1+\varepsilon\theta)(1+\beta+\sigma)]}. \tag{27}$$

The right-hand side of (27) does not depend on the period. The fertility rate in each period is constant, i.e., $n_t = n$ for all $t$, when the wage tax rate $\tau$ and $\theta$ are kept constant. It is noteworthy that even when parents are non-altruistic toward their children, i.e., $\sigma = 0$, parents want to have children merely for exchange motives.[14]

From (6), (23) and definition $s_t \equiv I_{t+1}/(1+r_{t+1})$, we have

$$(1-\tau)w_{t+1}[z_{t+1} + \varepsilon z_{t+1}^G(1-\gamma)] = \gamma(1+r_{t+1})k_{t+1}. \tag{28a}$$

From (21), (22), and from the government budget constraint (19), it follows that

$$z_{t+1} = \{[\gamma-(1-\gamma)\varepsilon\theta]/\theta\}z_{t+1}^G. \tag{28b}$$

The government budget constraint (19) is rewritten as

$$l_{t+1} = \frac{1-\alpha(1+\theta)}{\alpha\theta}z_{t+1}^G. \tag{29}$$

where (21) and (22) are used. Inserting these into the time constraint, one obtains

$$[\frac{1-\alpha(1+\theta)}{\alpha\theta} + \frac{\gamma-(1-\gamma)\varepsilon\theta+\theta}{\theta}]z_{t+1}^G = 1-\phi n,$$

from which one has the following:

$$z_{t+1}^G = \frac{\alpha\theta}{1-\alpha(1-\gamma)(1+\varepsilon\theta)}\frac{1+\beta+\varepsilon\theta}{(1+\varepsilon\theta)(1+\beta+\sigma)}. \tag{30}$$

From the time constraint (8), we obtain

[14] This case corresponds to a model of children as investment goods, described by Cochrane (1975). Adult children provide 'future consumption' of old-age support for parents. Even in this case, young adults care for both their children and elderly parents, i.e., sandwich caregivers. Nevertheless, a case of $\alpha = 0$ is apparently unrealistic because one has $n_t = 0$ without public old-age support. Although public long-term care systems vary among countries, countries such as Germany, Japan, France, Korea and the Netherlands introduced or reformed public long-term care system around 2000.

$$z_t = \frac{\alpha[\gamma - (1-\gamma)\varepsilon\theta]}{1-\alpha(1-\gamma)(1+\varepsilon\theta)} \frac{1+\beta+\varepsilon\theta}{(1+\varepsilon\theta)(1+\beta+\sigma)}. \quad (31)$$

The right-hand sides of (29), (30), and (31) do not depend on the period. Therefore, for a given tax rate, not only public old-age support, but also the labor employment in non-health consumption goods sector, and family old-age support time are constant through all periods, i.e., $z_t^G = z^G$, $l_t = l$ and $z_t = z$ for all $t$. For family and public old-age support to be positive, we must have $1-\alpha(1+\theta) > 0$ and $\gamma - \varepsilon\theta(1-\gamma) > 0$. We assume that these two conditions are satisfied in this paper.[15]

### 3.2 Dynamics of the economy

From (17), (23), and $G_{t+1} = \varepsilon\theta n_t k_{t+1}$, we obtain the rule of motion of capital per worker as

$$k_{t+1} = \frac{\beta(1-\tau)(1-\phi n)(1-\alpha)l^{-\alpha}}{(1+\beta+\varepsilon\theta)n} k_t^{\alpha}. \quad (32)$$

Because the coefficient of $k_t^{\alpha}$ is positively constant and because $0 < \alpha < 1$, the capital per worker has stable dynamics. In addition, the long-term equilibrium steady state is unique. From (32) the long-term steady state capital per worker is given as

$$k = [\frac{\beta(1-\tau)(1-\phi n)(1-\alpha)l^{-\alpha}}{(1+\beta+\varepsilon\theta)n}]^{1/(1-\alpha)}, \quad (33)$$

where $\tau = [\alpha/(1-\alpha)]\theta$.

## 4. Effects of Public Old-Age Support Policy

In this section, we analyze the effects of public old-age support policy changes on the steady state. The next subsection presents analyses of the effects. Subsection 4.2 provides a numerical example.

### 4.1 Policy effects

The effect on the fertility rate is ascertained by differentiating (27) with respect to the tax parameter $\theta$ as

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[15] The latter condition is the same as the condition for family old-age support to be positive in Miyazawa (2010).

$$\frac{dn}{d\theta} = \frac{1}{\phi(1+\beta+\sigma)} \frac{\varepsilon\beta}{(1+\varepsilon\theta)^2} > 0\,. \tag{34}$$

Therefore, we obtain the following result.

**Proposition 1** *We assume that individuals have strategic bequest motives and that the rationality constrain is binding. Then, increases in the tax for public old-age support always raise the fertility rate.*

The result can be interpreted as follows. When public and family old-age support are substitutes, an increase in public old-age support will reduce family support. It might increase the number of children and the market labor supply by freeing adult-children's time from providing family old-age support. Nevertheless, with strategic exchanges between elderly parents and young adult children, children's old-age support is provided correspondingly to the bequest received (see the rationality constraint with equality). That is, the intergenerational transfers in families, i.e., bequests, are neutralized by family old-age support in this model.[16] Therefore, the after-tax wage rate does not affect the decisions related to family old-age support provision. However, the fertility decision is affected by the after-tax wage rate, i.e., the opportunity cost of rearing children. Decreases in the after-tax wage rate lower the opportunity cost of child rearing and therefore raises the fertility rate more than offsetting the negative income effect of the tax increases.[17] Therefore, public old-age support financed by wage taxes increases the fertility rate. The lowered wage rate might also reduce the market labor supply.

It is noteworthy that the result does not depend on whether public old-age support is cost-efficient relative to family support. That is in contrast to the result reported by Yakita (2023a), which assumes altruistic elderly care supply for parents. Yakita (2023a) demonstrated that an expansion of public long-term care lowers fertility when public long-term care provision is less cost-efficient than family care. In fact, that can be the case even when public long-term care is more cost-efficient. His result is based on the assumption that public long-term care provision needs goods inputs other than labor. As described in this paper, by contrast, public old-age support negatively affects fertility regardless of whether public old-age support is more cost-efficient, even when produced

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[16] It holds true al far as bequests or family old-age supports are not constrained to corner solutions.

[17] The dominance of price effect is supported by the empirical studies of Jones and Tertilt (2008) and Jones et al. (2010), who find that fertility and wage rates are negatively related in most countries.

using labor alone.

Next, we present the analyses of policy effects on the time allocation of working individuals. The effect on public old-age support is demonstrated by differentiating (29) with respect to $\theta$ as

$$\frac{dz^G}{d\theta} = \frac{1}{(1+\varepsilon\theta)(1+\beta+\sigma)}[\frac{\alpha[1-\alpha(1-\gamma)](1+\beta+\varepsilon\theta)}{[1-\alpha(1-\gamma)(1+\varepsilon\theta)]^2} - \frac{\varepsilon\beta}{1+\varepsilon\theta}]. \tag{35}$$

Although the first term in brackets on the right-hand side is positive, the second term is negative from (34). Therefore, the sign of the effect is not determined *a priori*. However, one can reasonably consider that an increase in the tax rate will increase public old-age support. Otherwise, no government have an incentive to introduce the tax for old-age support provision. Therefore, we assume that the sign of (35) is positive in the following.

The policy effect on family old-age support is derived from (31) as

$$\frac{dz}{d\theta} = \frac{1}{[1-\alpha(1-\gamma)(1+\varepsilon\theta)](1+\varepsilon\theta)(1+\beta+\sigma)}[\frac{\alpha(\alpha-1)\varepsilon(1-\gamma)(1+\beta+\varepsilon\theta)}{1-\alpha(1-\gamma)(1+\varepsilon\theta)}$$

$$-\frac{\varepsilon\beta\alpha[\gamma-(1-\gamma)\varepsilon\theta]\phi}{1+\varepsilon\theta}] < 0. \tag{36}$$

Increases in the tax rate decrease family old-age support. This result can be interpreted as follows. Tax increases reduce the disposable income of workers. The negative income effect leads to decreases in family support even if bequests from the parent are fixed. If public old-age support increases, then this increase enforces the effect. In fact, improvements in the cost-efficiency of public old-age support provision reduce family support, i.e., $dz/d\varepsilon < 0$. That freed-up time might be allocated to rearing children and market labor supply, i.e., $dn/d\varepsilon > 0$ from (27).[18] However, summing up above comparative static analyses, the net policy effects on the market labor supply $l$ and $l + z^G$ are indeterminate. Therefore, we have the following proposition.

**Proposition 2** *The effects of old-age support tax changes on the total old-age support, family and public, and the market labor supply are indeterminate a priori.*

Finally, the policy effect on capital per worker in the non-health consumption production is derived from (33) as

[18] Assuming altruistic family long-term care, Yakita (2023a) demonstrates that improvements in the cost efficiency of public long-term care can increase the fertility rate.

$$\frac{dk}{d\theta} = -\frac{\varepsilon k}{1+\beta+\varepsilon\theta} < 0. \quad (37)$$

Per-worker capital decreases with the tax rate. Because the policy effects on market labor employment are ambiguous, the effects on the steady-state wage rate and interest rate are also ambiguous. Therefore, the effects on non-health consumption during working and old-age periods and on lifetime utility are indeterminate *a priori*. The following proposition holds.

**Proposition 3** *Increases in the tax rate lowers the steady-state level of per worker capital stock.*

4.2 Numerical example

In this subsection, we present numerical results obtained by assuming model parameters. According to de la Croix and Doepke (2003), we set the utility weight of children as $\sigma = 0.271$ and the discount factor as $\beta = 0.99^{120} = 0.299$.[19] We further assume here that one period lasts 30 years. The income share of capital is assumed to be $\alpha = 0.3$, as usual in the macroeconomics literature. The scale parameter for non-health consumption goods is set to unity for this example. Although the burden rate for old-age support in kind varies from country to country, we set $\tau = 0.10$ as a benchmark rate because, for example, the social security benefits-GDP ratio was 23.6%, about half of which was of pension benefits in Japan in 2019[20]. In this case, we have $\theta = 0.23$. The contribution ratio of old-age support in the old-age utility is assumed to be $\gamma = 0.864$. Liu et al. (2023) show that an increase in Long-term Care Insurance (LTCI) increases non-health consumption by 0.157%. The elasticity of substitution between non-health consumption and old-age support might be nearly zero (e.g., ADL), whereas the demand for old-age support depends on its price (Sano et al., 2022).[21] We consider here that the magnitude of the marginal effect approximates an elasticity. The cost-efficiency of public

[19] De la Croix and Doepke (2003) also assume the same utility weight on human capital of children.

[20] Ministry of Health, Labour and Welfare of Japan (https://www.mhlw.go.jp/stf/newpage_21509.html: cited on 4 July 2023). Japanese Long-Term Care Insurance does not provide cash transfers.

[21] Tax change effects are qualitatively unaltered for a smaller $\gamma$. It is noteworthy here that changes in $\gamma$ do not affect the steady state fertility rate because the right-hand side of (27) does not include $\gamma$.

old-age support is set as $\varepsilon = 0.5$. Yakita (2023) calculates it from empirical works for Japanese long-term care industry such as Aya (2014).[22] Per-child rearing time is assumed variously in the literature. We set the variable to generate the fertility rate for unity, i.e., $\phi = 0.19$. This value is approximately equal to 0.15, which is assumed by de la Croix and Doepke (2003). The parameter set engenders the equilibrium fertility rate of about unity. With these parameters, we calculate the steady state values of endogenous variables of the model. For comparative statics, we consider variations of $\tau$ by 0.01 from the benchmark case. The results are demonstrated in Table 1.

As shown in Proposition 1, a tax rise for old-age support increases the fertility rate. The capital stock per worker and family support decrease. With the assumed parameters, the tax rise increases public old-age support, as expected by the policymaker. The market labor $l + z^G$ decrease with the tax rise. Although old-age non-health consumption decreases, the total old-age support $z + \varepsilon z^G$ increases. Consequently, the net effect on the old-age utility $u_2^*$ is positive. Nevertheless, the negative effect on the working-period non-health consumption through negative income effect overwhelms the positive effect on fertility, consequently lowering the lifetime utility $\tilde{U}$.[23,24]

Table 1 Simulation results

| $\tau$ | $z^G$ | $n$ | $z$ | $l$ | $k$ | $u_2$ * | $\tilde{U}$ |
|---|---|---|---|---|---|---|---|
| 0.09 | 0.053 | 1.004 | 0.2160 | 0.540 | 0.0553 | 0.162 | –2.129 |
| 0.10 | 0.059 | 1.013 | 0.2153 | 0.533 | 0.0532 | 0.165 | –2.141 |
| 0.11 | 0.065 | 1.022 | 0.2145 | 0.526 | 0.0512 | 0.168 | –2.153 |

## 5. Social optimum

As described earlier, it was demonstrated that tax rises for public old-age support always increase the fertility rate, although they lower the per-worker capital stock. In this

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[22] The relative cost efficiency of public long-term care is calculated as $1/1.55$ by Yakita (2023) when the labor productivities of public and family elderly care are equal. Because elderly people might prefer broader old-age support by family members to public support, we assume a lower value.

[23] Effects on non-health consumption during the working and retired periods are not presented in Table 1.

[24] Sensitivity analyses ensure that these qualitative results hold for wider ranges of parameters.

section, we present evaluation of the policy of public old-age support by comparing the decentralized long-term equilibrium under public old-age support policy with the social optimum. In doing so, the role of strategic bequest motives in the dynamic allocation might be clarified.

The long-term equilibrium is achieved as a steady state. Therefore, we consider the social optimum as the steady state which maximizes the lifetime utility of an individual by controlling the resource allocation centrally. For our purposes, we designate the total old-age support per worker as $Z$.

The social optimization problem of the social planner can be formalized as

$$\underset{c_1,c_2,n,Z,k,l}{Max} \quad \tilde{U} = \ln c_1 + \sigma \ln n + \beta(1-\gamma)\ln c_2 + \beta\gamma \ln(nZ)$$

subject to $k^{\alpha} l^{1-\alpha} - c_1 - \frac{c_2}{n} - nk = 0$, and (38)

$$1 - l - Z - \phi n = 0 . \quad (39)$$

Constraint (37) is the resource constraint, also, (38) is the time constraint per individual. Derivation of the solution is set aside in the Appendix. From the first-order conditions for maximization, we obtain the optimal resource allocation as shown below.

$$\frac{c_1}{y} = \frac{1-\alpha}{1+\beta(1-\gamma)} , \quad (40a)$$

$$\frac{c_2 / n}{y} = \frac{(1-\alpha)\beta(1-\gamma)}{1+\beta(1-\gamma)} , \text{ and} \quad (40b)$$

$$\frac{nk}{y} = \alpha . \quad (40c)$$

From (40c) we can immediately obtain $\alpha y / k [= \alpha k^{\alpha-1} l^{1-\alpha}] = n$. This is the Golden Rule condition for capital accumulation.

Using (38)–(40), we obtain the following solution:

$$n_{so} = \frac{(1-\alpha)(\sigma+\beta) - \alpha[1+\beta(1-\gamma)]}{\phi\{(1-\alpha)(1+\sigma+2\beta) - \alpha[1+\beta(1-\gamma)]\}} , \quad (41)$$

$$l_{so} = \frac{(1-\alpha)[1+\beta(1-\gamma)]}{(1-\alpha)(1+\sigma+2\beta) - \alpha[1+\beta(1-\gamma)]} , \text{ and} \quad (42)$$

$$Z_{so} = \frac{\beta\gamma(1-\alpha)}{(1-\alpha)(1+\sigma+2\beta) - \alpha[1+\beta(1-\gamma)]} . \quad (43)$$

Subscript $so$ designates the optimal value of variables in the Golden Rule optimum.[25]

[25] With the parameters assumed in the preceding section, the social optimum fertility rate

For these values to be non-negative, the denominators on the right-hand sides of these equations must be positive. We assume that these conditions are satisfied. For expositional purposes, we also assume that the social optimum fertility rate is positive, i.e., $n_{so} > 0$.[26]

Comparing these optimum solutions with those obtained under decentralization with public old-age support policy, we can infer the following: First, the tax rate which achieves the optimum fertility rate is obtainable from (27) and (41), i.e., $\tau = [\alpha / (1-\alpha)]\theta$ satisfying the following condition.

$$\theta\varepsilon = \frac{(1+\beta)\{\beta(1-\alpha) - \alpha[1+\beta(1-\gamma)]\}}{(1-\alpha)\beta(\sigma+\beta) + \alpha[1+\beta(1-\gamma)]}. \quad (44)$$

Two remarks must be made: First, tax rate $\theta$ is inversely proportion to the relative cost-efficiency of public old-age support $\varepsilon$. When the public support efficiency is higher, the tax rate is lower by comparison.[27] Nevertheless, we cannot rule out the possibility that the decentralized fertility rate with zero wage tax, i.e., with no old-age support policy, can be higher than the social optimum fertility rate. To illustrate this point, we set $\tau = 0$ in (27). Then, the decentralized fertility rate without policies is

$$n_{ss|\tau=0} = \frac{\sigma}{\phi(1+\beta+\sigma)}. \quad (27')$$

From (41) and (27'), we can demonstrate that the difference between two rates $n_{so} - n_{ss|\tau=0}$ as

$$n_{so} - n_{ss|\tau=0} = \frac{(1+\beta)\{(1-\alpha)\beta - \alpha[1+\beta(1-\gamma)]\}}{\phi\{(1-\alpha)(1+\sigma+2\beta) - \alpha[1+\beta(1-\gamma)]\}(1+\beta+\sigma)}. \quad (45)$$

The sign of expression (44) cannot be ascertained *a priori*. If it is negative, i.e., if

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is $n_{so} = 0.460$. Although the optimum rate is too low to maintain the population size in the steady state, some strategic bequest motives are observed, e.g., in Horioka et al (2018). Therefore, the equilibrium fertility rate would be higher than the social optimum even without public old-age support. The optimum level of lifetime utility is $\tilde{U}_{so} = -1.085$. This finding implies that if government also aims to maintain, or even increase, the population size, then the optimal policy would be to provide sufficient public old-age support, lowering the lifetime utility. By contrast, if, for example, $\alpha = 0.2$, other things being equal, then the social optimum fertility rate $n_{so}$ would be greater than unity.

[26] Nevertheless, we cannot rule out the possibility of $n_{so} \leq 0$ *a priori*.

[27] The optimal tax rate might be zero because of the non-negative constraint. For the parameters assumed in a numerical example, the right-hand side of (44) is $-0.309$. Therefore, we must have a corner solution $\tau = 0$ as the optimal policy in this case. Nevertheless, when $\alpha = 0.2$ with other parameters unchanged, we have $\theta\varepsilon = 0.1182 > 0$ and hence $\tau = 0.045$.

$n_{so} - n_{ss|\tau=0} < 0$, then the public old-age support provision fails to achieve the optimum fertility rate because increases in $\tau$ always raise the fertility rate, as condition (34) indicates.[28]

Second, even if a wage tax $\tau$ achieves the optimum fertility rate, the tax rate might not achieve the overall resource allocation efficiency.[29] The tax rate which equalizes $l_{ss}$ in (30), with $l_{so}$ in (42) perhaps not equalizing $n_{so}$ with $n_{ss}$.

Summing up the above arguments, we obtain the following proposition.

**Proposition 4** *Suppose that individuals have strategic bequest motives. Then, old-age support policy alone might not lead the decentralized market equilibrium to the social optimum.*

In our model, there are two potential sources of inefficiency, i.e., finite lifetimes and strategic behaviors. Nevertheless, the numerical example implies that strategic bequest motives might keep fertility rate higher. We have only old-age support financed by wage taxes. Dynamic efficiency in the sense of a golden rule might require multiple policy measures.

## 6. Concluding Remarks

For a dynamic general equilibrium model in which old-age support is provided by children to parents based on strategic bequest motives, an analysis of fertility decisions of individuals is presented. The public old-age support provision increases the fertility rate and lowers the capital stock per worker, thereby engendering ambiguous effects on the lifetime utility of individuals. Comparing the decentralized long-term equilibrium with the golden rule social optimum, we can infer that public old-age support provision might not be socially desirable under strategic bequest behaviors of individuals. Even when no public old-age support is provided, strategic bequest motives of individuals might engender a fertility rate higher than the social optimum rate.

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[28] For the parameters in the numerical example, the value of $n_{so} - n_{ss|\tau=0}$ is $-0.858$ when $\tau = 0$. When $\alpha = 0.2$ with other parameters unchanged, the difference becomes $0.183 > 0$.

[29] There are multiple tax rates which equalize the decentralized steady-state employment of (29) and the social optimum employment of (41) in the non-health consumption goods sector.

Further research directions can be inferred. First, as Horioka et al. (2018) demonstrate, individuals might have both altruistic and strategic bequest motives simultaneously.[30] The analyses can be extended to such a case. Although the effect of public old-age support on fertility does not depend on the cost-efficiency of public support in this study, Yakita (2023a) demonstrates that public long-term care provision lowers fertility if the public provision is inefficient. Second, we have not considered any child policy. If old-age support affects child policy, then that must be considered in a model simultaneously. Third, whether an old individual becomes dependent and the degree to which dependency exists can be expected to vary among individuals. Such uncertainty must be considered. Many works such as those reported by Pestieau and Sato (2008) and by Cremer and Roeder (2013) introduce such uncertainty. Fourth, we have assumed only child-rearing time cost. This assumption simplifies the analyses together with Cobb Douglas utility functions. Goods costs of rearing and educating children might be significant. Consideration of these costs makes the fertility decisions dependent on the relative magnitudes of the wage rate and the goods price (Becker and Barro, 1988).

Finally, and more importantly, we have assumed that steady-state old-age support, family plus public, is determined as an interior solution. Nevertheless, given the level of public support, individuals might want to make the level of family support negative. In other words, the public old-age support is too great relative to old-age non-health consumption. If the level of public support is merely equal to the necessary level for old-age individuals to remain alive, then the chosen fertility rate might be a corner solution to their utility maximization. In this case, the negative income effect of tax increases becomes significant for individuals, consequently affecting the fertility rate negatively. Therefore, in such a situation, together with positive effects of the opportunity cost change, the net effect of a tax increase on the fertility rate might be indeterminate *a priori*. This presents an interesting case for future research in this area.

Declaration

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[30] Klimaviciute et al. (2014) demonstrate, using SHARE data, that long-term caring is driven by altruism or by family norm, whereas Alessie et al. (2014) show that exchange motive is important in intergenerational transfers.

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Appendix: Social optimum

Assuming Lagrange multipliers attached to constraints (38) and (39) respectively as $q_1$ and $q_2$, the first-order conditions for the optimum are given as

$$\frac{1}{c_1} - q_1 = 0, \tag{A1}$$

$$\frac{\beta(1-\gamma)}{c_2} - q_1 \frac{1}{n} = 0, \tag{A2}$$

$$\frac{\sigma + \beta\gamma}{n} - q_1\left(-\frac{c_2}{n^2} + k\right) - q_2\phi = 0, \tag{A3}$$

$$\frac{\beta\gamma}{Z} - q_2 = 0, \tag{A4}$$

$$q_1(\alpha k^{\alpha-1} l^{1-\alpha} - n) = 0, \text{ and} \tag{A5}$$

$$q_1(1-\alpha)k^{\alpha} l^{-\alpha} - q_2 = 0. \tag{A6}$$

Condition (A5) stipulates that the marginal productivity of capital equals the population growth rate, i.e., the Golden Rule for capital accumulation. Inserting (A1), (A5), and (A6) into the resource constraint (38), one obtains

$$\frac{1}{yq_1} = \frac{1-\alpha}{1+\beta(1-\gamma)}. \tag{A7}$$

Together with (A1), we have

$$\frac{c_1}{y} = \frac{1-\alpha}{1+\beta(1-\gamma)}. \tag{A8}$$

From (A2) and (A7), we have

$$\frac{c_2 / n}{y} = \frac{(1-\alpha)\beta(1-\gamma)}{1+\beta(1-\gamma)}. \tag{A9}$$

Conditions (A8) and (A9) are the same with (40a) and (40b) in the text. Inserting (A5) and (A9) into (A3), and using (A7), we obtain

$$\sigma + \beta - \frac{\alpha[1+\beta(1-\gamma)]}{1-\alpha} = q_2 \phi n \,. \tag{A10}$$

We also have, from (A6) and (A7),

$$l = [1+\beta(1-\gamma)] / q_2 \,. \tag{A11}$$

Therefore, inserting (A11) and (A4) into the time constraint (39), we obtain

$$q_2(1-\phi n) = 1+\beta \,. \tag{A12}$$

Eliminating $q_2$ from (A10) and (A12), one can have the following equation.

$$\frac{\phi n}{1-\phi n} = \frac{(\sigma+\beta)(1-\alpha)-\alpha[1+\beta(1-\gamma)]}{(1-\alpha)(1+\beta)} \,. \tag{A13}$$

Therefore, from (A13), we obtain the social optimum fertility rate as

$$n = \frac{(\sigma+\beta)(1-\alpha)-\alpha[1+\beta(1-\gamma)]}{\phi\{(1+\sigma+2\beta)(1-\alpha)-\alpha[1+\beta(1-\gamma)]\}} \,. \tag{A14}$$

From (A11) and (A12), we have

$$l = \frac{(1-\alpha)[1+\beta(1-\gamma)]}{(1+\sigma+2\beta)(1-\alpha)-\alpha[1+\beta(1-\gamma)]} \,. \tag{A15}$$

Inserting (A14) and (A15) into the time constraint, we also have

$$Z = \frac{\beta\gamma(1-\alpha)}{(1+\sigma+2\beta)(1-\alpha)-\alpha[1+\beta(1-\gamma)]} \,. \tag{A16}$$

Solutions in (A14)–(A16) are identical with those in (41)–(43) of the text.

We also obtain, using (A5),

$$k = (n/\alpha)^{1/(\alpha-1)} l \,, \tag{A17}$$

from (A1) and (A7),

$$c_1 = \frac{(1-\alpha k^{\alpha} l^{1-\alpha}}{1+\beta(1-\gamma)} \,, \tag{A18}$$

and, from (A8) and (A9),

$$c_2 = c_1 \beta(1-\gamma) n \,, \tag{A19}$$

where $n$ and $l$ are given by (A14) and (A15).