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The Optimality of Inflation Targeting: The Scope of Hyperbolic Discounting

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Abstract

I incorporate hyperbolic discounting preference into a monetary search model as in Lagos and Wright (2005). Through this extension, I show that inflation targeting is not an optimal policy rule when the agents are naive.

Keywords: Hyperbolic discounting; Monetary search model; Monetary policy; Inflation targeting

JEL *classification*: E52; E70

1 Introduction

Central banks often adopt inflation targeting. Its purposes are to avoid the dynamic-inconsistent monetary policy and to stabilize agents' expectations of future inflation and economy. Thus, the effectiveness of inflation targeting depends on the agents' expectations of future inflation. Literature shows that imperfect information impacts the effectiveness of inflation targeting and the other policy rules of the central banks when the agents cannot correctly predict the states of the economy. Ball, Mankiw, and Reis (2005) develop a model where all the firms could not immediately obtain the required information to set their prices as in Mankiw and Reis (2002). They indicate that price-level targeting is an optimal policy rather than inflation targeting. Agliari, Massaro, Pecora, and Spelta (2017) study the New Keynesian model where there are costs to obtain the information of economic states as in Sims (2003). They also prove that the achievement of the inflation rate target depend on the costs. Angeletos and La'O (2020) study

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a model where the signals of the information of the economic states owned firms are different between the firms as in Morris and Shin (1998). They show that the nominal GDP targeting is more desirable than price-level targeting. In addition, studies analyze another method of forming expectations. Grauwe (2011), Ho, Lin, and Yeh (2021), and Molnar (2020) incorporate heuristics into the New Keynesian model. In these studies, the agents predict the future economic variables through a simple expectation rule, which is irrational. They indicate that inflation targeting can be suboptimal. Another factor that affects the expectation is a time-inconsistent preference, i.e., it changes over time. According to O'Donoghue and Rabin (1999), the agents with hyperbolic discounting, a discount method casing time-inconsistent preference, fail to correctly predict their future behavior as they are naive agents who do not understand that preference change over time. However, there is no previous research that discusses this point. Therefore, this study investigates the effect of time-inconsistent preference on the optimality of inflation targeting.

This study provides a theoretical analysis. I incorporate hyperbolic discounting into a monetary search model provided by Lagos and Wright (2005), hereafter, called the LW model. Through this extension, I show that inflation targeting is not an optimal policy rule when the agents are naive because the agents cannot correctly predict their and other agents' future behavior. It is to be noted that my model does not include macroeconomic shocks as in other literature. In other words, the central bank in this model does not need to care about macroeconomic stability. However, I emphasize that inflation targeting is not optimal even if there is no macroeconomic shock, which is this study's contribution to existing literature.

I review related literature to bring a perspective. The standard models discuss monetary policy rules by defining or deriving a loss function. This increases if the inflation rate or the output gap increases. Since there are several literature using the loss function, I study a recent paper. Billi (2017) discusses the optimality of GDP targeting when the nominal interest rate hits the zero lower bound by using the New Keynesian model. Another literature that discusses the central bank's targeting rules is reviewed by Walsh (2010).

Some literatures study policy rules or stabilization monetary policy by using the monetary search model. Berentsen and Waller (2011) show that price-level targeting is the optimal policy rule for the short-term, i.e., if the current price level deviates from the long-term price path due to some shocks, the central bank controls the amount of money to be injected to return the price level to the target level. Berentsen and Waller (2015) discuss that when the probability of a successful transaction is determined by the number of the agents entering the market, an optimal stabilization monetary policy could be derived. Boel and Waller (2019) discuss the optimal stabilization policy at the zero lower bound in the scenario when the agents hit idiosyncratic preference shocks. All the studies focus on the liquidity constraint of the buyer where the central bank injects money to relax the liquidity constraint. In contrast, my study focusses on the agent's forecast error due to time-inconsistent preference.

The following literatures study monetary policy with a behavioral economics model. Graham and Snower (2008, 2013) and Maeda (2018a, 2018b) study the case of hyperbolic discounting. Maeda (2020), Hori, Futagami, and Morimoto (2021) and Futagami and Maeda (2022) study non-unitary discounting in which discount rates differ between goods. Hiraguchi (2018) studies the model where the agents have the temptation to spend all their money. However, these papers do not discuss the monetary policy rules that is presented in my study.

The rest of the paper follows the mentioned flow. Section 2 explains the setting of the model. Section 3 solves the agent's optimization problem. Section 4 derives the equilibrium and discusses the optimality of inflation targeting. Section 5 concludes the study.

2 The model

There is a continuum of agents with a unit measure. The agents infinitely live. Time is discrete and goes from $t = 0$ to ∞. Each period has two subperiods. We call the first subperiod "day" and the second subperiod "night." Each subperiod has a goods market and the goods traded in them cannot be carried over after subperiods.

Before the agents enter the day, they are divided into "seller" and "buyer." The agents become sellers with the probability $n \in (0, 1)$, and buyers with the probability $1 - n$. During the day, the sellers produce and sell goods and the buyers buy and consume them. To focus on the agent's time-inconsistency, I adopt a simple assumption; i.e., the market is perfectly competitive[1] [2]. This assumption seems to be the same as a centralized market. However, we call this market a decentralized market (DM) following the original setting of the LW model. We assume that there are no methods of trade record-keeping. Therefore, the buyers have to use the money to buy the goods.

[1] Berentsen, Camera, and Waller (2007) also adopt this assumption.

[2] We can incorporate random matching and Nash bargaining into the decentralized market to determine the quantity and the price of goods as in the original LW model. However, the difference between the original setting and the setting of the perfectly competitive, decentralized market in my model, with time-inconsistency, is qualitatively the same as the difference in the model without time-inconsistency because the Nash bargaining process is not for intertemporal decision making.

At night, all the agents simultaneously decide the amount of consumption, labor supply, and money holdings to be carried over to the next period. These markets are the same as a standard model; a centralized, perfectly competitive market. We call the goods market at the night a centralized market (CM). In this market, a unit of goods is produced by a unit of labor. Since we assume that the labor market is perfectly competitive, the real wage is equal to one.

Money is divisible and storable however, intrinsically useless. M_t is the amount of money issued before period t. The central bank issues and transfers money to the agents equally at night. I define T_t as the transfer of money to agents. Then, $T_t = M_{t+1} - M_t$.

Next, I explain the agent's preference. The agent obtains utility from consumption and disutility from production and labor supply in each subperiod. We assume that the utility function is additively separable and quasilinear;

$$\mathcal{U}(q_t^b, q_t^s, x_t, h_t) = u(q_t^b) - c(q_t^s) + U(x_t) - h_t, \tag{1}$$

where $u(q)$ is the utility from consuming q units of goods and $c(q)$ is the cost of producing q units of goods in the day. $U(x)$ is the utility from consuming x units of goods and h is the disutility of supplying h units of labor in the night. The functions u, c, and U are twice continuously differentiable and satisfy $u(0) = c(0) = 0$, $u' > 0$, $c' > 0$, $U' > 0$, $u'' < 0$ $c'' \geq 0$, $U'' < 0$, $u'(0) = U'(0) = \infty$, and $u'(\infty) = U'(\infty) = 0$. h is positive and has upper bound $\bar{h}$. As mentioned in the introduction, the future utility is hyperbolically discounted. Therefore, the lifetime utility in the period, t, is denoted by

$$Z_t = u(q_t^b) - c(q_t^s) + U(x_t) - h_t + \beta \sum_{i=1}^{\infty} \delta^i \left[u(q_{t+i}^b) - c(q_{t+i}^s) + U(x_{t+i}) - h_{t+i} \right], \tag{2}$$

where $\delta \in (0, 1)$ is a discount factor for the long-term. Since we assume that $\beta \in (0, 1]$, the agents evaluate current utility rather than future utility. In other words, β is a parameter denoting present bias.

Previous studies in behavioral economics consider two types of agents. The first is a sophisticated agent. They understand that their preference change over time. In this model, they know that the future themselves will also have the present bias parameter β and resolve the optimization problem. In other words, they know that the future themselves will maximize (2). The second is a naive agent. They do not understand that their preference changes over time. In this model, they understand that the future themselves will not have the present bias and their current decision is not changed by the future themselves. In other words, they understand that the future themselves will maximize $\sum_{i=1}^{\infty} \delta^i \mathcal{U}(q^b_{t+i}, q^s_{t+i}, x_{t+i}, h_{t+i})$. I summarize the agents' types as follows.

Definition 1. *In this model, the sophisticated agents and the naive agents predict their future behavior as follows*

1. *Sophisticated agents predict that the future themselves will maximize* (2).
2. *Naive agents predict that the future themselves will maximize* $\sum_{i=1}^{\infty} \delta^j \mathcal{U}(q^b_{t+i}, q^s_{t+i}, x_{t+i}, h_{t+i})$.

In this study, we consider two cases. The first where all the agents are sophisticated and the second where all the agents are naive.

3 The agent's optimization problem

3.1 The current problem

Before we discuss the agent's optimization problem, please note that I mention notations hereafter. I omit the variables' subscript, t and add +1 to the variables in the next period.

We start by solving the seller's problem. An individual feels the disutility, $c(q^s)$, by producing the goods during the day (hereafter, the DM goods), and experiences net utility, $U(x)-h$, during the night. We solve this by using the Bellman equation. We can define the seller's problem as follows:

$$V_0^s(m) = \max_{q^s, x, h, m_{+1}} [-c(q^s) + U(x) - h + \beta\delta V(m_{+1})], \tag{3}$$

where m denotes money holdings. $V(m)$ denotes the expected value that is obtained from future consumption and production, i.e., the sum of the expected discounted future utility. The definition of $V(m)$ is different for the sophisticated and the naive agent (explained after this subsection). The agent obtains the nominal income, pq^s, where p is the nominal price of the DM goods from the production during the day. When ϕ denotes the real price of money (the inverse of the price of the goods in the CM), the real income is given by ϕpq^s. As mentioned before, the real wage at night is one. Thus, the seller's budget constraint is denoted by

$$h = x + \phi m_{+1} - \phi p q^s - \phi(m + T). \tag{4}$$

Substituting (4) into (3), we obtain

$$V_0^s(m) = \max_{q^s}[\phi p q^s - c(q^s)] + \max_{x}[U(x) - x] + \phi(m + T) + \max_{m_{+1}}[-\phi m_{+1} + \beta\delta V(m_{+1})]. \tag{5}$$

Since the DM is perfectly competitive, the seller behaves with price ϕp, as given. Then, the first-order condition of q^s is

$$\phi p = c'(q^s). \tag{6}$$

From the second term of (5), the first order condition of x is given by

$$U'(x) = 1. \tag{7}$$

Let x^* denote the value of satisfying (7). Moreover, we assume that $U(x^*) > x^*$ is satisfied for x^* to have a positive value. From the last term of (5), we find that m_{+1} does not depend on m or on whether the individual is a seller or a buyer during the day. We will seek the first-order condition of m_{+1} in the next subsection.

Next, we solve the buyer's problem. Since the buyers are anonymous, they have to hold the money before they consume it. Therefore, the buyer faces the following constraint:

$$pq^b \leq m. \tag{8}$$

The buyer's budget constraint is given by

$$h = x + \phi m_{+1} + \phi p q^b - \phi(m + T). \tag{9}$$

Calculating like the seller's problem, the value function of the buyer is denoted by

$$V_0^b(m) = \max_{pq^b \leq m}[u(q^b) - \phi p q^b] + \max_{x}[U(x) - x] + \phi(m + T) + \max_{m_{+1}}[-\phi m_{+1} + \beta\delta V(m_{+1})]. \tag{10}$$

From (10), the buyer's optimal conditions of x and m_{+1} are the same as those of the seller. If

(8) binds, $q^b = m/p$. If it does not bind, $q^b = q^*$, which satisfies $u'(q^*) = \phi p$. Hereafter, we consider the case where (8) binds.

3.2 The sophisticated agent's expectation and money holdings

To determine money holdings, we have to get the future value, $V(m)$. Since it is the sum of the expected discounted future utility, we define it as follows:

$$V(m) = E \sum_{i=0}^{\infty} \delta^i \left[u(q_{t+i}^b) - c(q_{t+i}^s) + U(x_{t+i}) - h_{t+i} \right], \tag{11}$$

where E is an expectation operator. Since the probabilities of the individuals becoming sellers and buyers are n and $1-n$, respectively, we can rewrite (11) as follows:

$$V(m) = (1-n)u(q_{so}^b) - nc(q_{so}^s) + [(1-n)U(x_{so}^b) + nU(x_{so}^s)] + [(1-n)h_{so}^b + nh_{so}^s] + \delta V(m_{so,+1}), \tag{12}$$

where the superscripts b and s, respectively, denote the values when the individual will become a buyer and a seller in the future, and the subscript so denotes a sophisticated agent's expected value. From definition 1, the current individuals predict that the future themselves maximize (2); that is, the predicted future objective function is the same as the current one. This means that the future themselves' behavior is the same to the current individuals'. Since I have shown that q^s, x and m_{+1} do not depend on m in subsection 3.1, we can rewrite the last term of (5) and (10) as follows:

$$\max_{m_{+1}} [-\phi m_{+1} + \beta\delta\{(1-n)v(m_{+1}) + \phi_{+1} m_{+1}\}], \tag{13}$$

where $v(m) \equiv u(q^b) - \phi p q^b$, and I omit the variables which do not depend on m_{+1}. The optimal condition of money holdings is denoted by

$$\frac{\phi}{\phi_{+1}} = \beta\delta \left[(1-n) \frac{u'(q_{+1}^b)}{\phi_{+1} p_{+1}} + n \right]. \tag{14}$$

We have already obtained the real price of the DM goods by (6). Using it, we can rewrite (14) as follows:

$$\frac{\phi}{\phi_{+1}} = \beta\delta\left[(1-n)\frac{u'(q^b_{+1})}{c'(q^s_{+1})} + n\right]. \tag{15}$$

3.3 The naive agent's expectation and money holdings

From definition 1, the naive agents predict, in the future, they would maximize $\sum_{i=0}^{\infty}\delta^j\mathcal{U}(q^b_{t+j+i}, q^s_{t+j+i}, x_{t+j}, h_{t+j+i})$. This is the same as maximizing $V(m)$. Therefore, we can rewrite (11) as follows:

$$V(m) = \max_{q^b_{na}, q^s_{na}, x_{na}, h_{na}} [E\sum_{i=0}^{\infty}\delta^i\left[u(q^b_{na,t+i}) - c(q^s_{na,t+i}) + U(x_{na,t+i}) - h_{na,t+i}\right], \tag{16}$$

where the variables added to the subscript, na, denote the naive agent's expected value. As in subsection 3.1, first, we solve the future seller's problem. The future seller's value is denoted by

$$V^s(m_{na}) = \max_{q^s_{na}, x_{na}, h_{na}, m_{na,+1}} [-c(q^s_{na}) + U(x_{na}) - h_{na} + \delta V(m_{na,+1})]. \tag{17}$$

Substituting the seller's budget constraint, (4), into (17), we get

$$\begin{aligned} V^s(m_{na}) =& \max_{q^s_{na}}[\phi_{na}p_{na}q^s_{na} - c(q^s_{na})] + \max_{x_{na}}[U(x_{na}) - x_{na}] + \phi_{na}(m_{na} + T_{na}) \\ &+ \max_{m_{na,+1}}[-\phi_{na}m_{na,,+1} + \delta V(m_{na,+1})]. \end{aligned} \tag{18}$$

From this equation, we obtain the following optimal conditions:

$$\phi_{na}p_{na} = c'(q^s_{na}), \tag{19}$$

$$U'(x_{na}) = 1. \tag{20}$$

(20) is the same as (7). Therefore, $x_{na} = x^*$.

The future buyer's problem is denoted by

$$V^b_{na}(m_{na}) = \max_{p_{na}q^b_{na} \leq m_{na}} [u(q^b_{na}) - \phi_{na}p_{na}q^b_{na}] + \max_{x_{na}}[U(x_{na}) - x_{na}] + \phi(m_{na} + T_{na})$$
$$+ \max_{m_{na,+1}} [-\phi m_{na,+1} + \delta V(m_{na,+1})], \tag{21}$$
$$s.t. \quad (8) \; and \; (9).$$

Therefore, since we assume that (8) binds, $q^b_{na} = m_{na}/p_{na}$. The optimal condition of x_{na} is denoted by (20).

Next, we seek the expected money holdings, $m_{na,+1}$. We can rewrite (16) as follows:

$$V(m_{na}) = (1-n)u(q^b_{na}) - nc(q^s_{na}) + [(1-n)U(x^b_{na}) + nU(x^s_{na})] + [(1-n)h^b_{na} + nh^s_{na}] + \delta V(m_{na,+1}). \tag{22}$$

As in the current problem, we can rewrite the last term of (18) and (21) as follows:

$$\max_{m_{na,+1}} [-\phi_{na}m_{na,+1} + \delta\{(1-n)v(m_{na,+1}) + \phi_{na,+1}m_{na,+1}\}]. \tag{23}$$

If $q^b_{na} = m_{na}/p_{na}$, the optimal condition of the expected money holdings is denoted by

$$\frac{\phi_{na}}{\phi_{na,+1}} = \delta \left[(1-n)\frac{u'(q^b_{na,+1})}{\phi_{na,+1}p_{na,+1}} + n\right]. \tag{24}$$

From the conditions (19), (20), and (24), we have obtained the expected values of the naive agents. Using it, we seek the current money holdings. The current problem is the same for the sophisticated and the naive agents. Therefore, we solve the problem (13). The problem corresponding to (13) is denoted by

$$\max_{m_{+1}}[-\phi m_{+1} + \beta\delta\{(1-n)v(m_{+1}) + \phi_{na,+1}m_{+1}\}]. \tag{25}$$

Note that we do not have to replace $m_{na,+1}$ with m_{+1} because the current agents can decide the current nominal money holdings. Solving (25), we obtain

$$\frac{\phi}{\phi_{na,+1}} = \beta\delta \left[(1-n)\frac{u'(q^b_{na,+1})}{\phi_{na,+1}p_{na,+1}} + n\right]. \tag{26}$$

To completely determine the agents' expectations, we need their price expectations. The prices are determined in each market. Therefore, we also need an agent's expectation of the other agents' behavior. We assume that the agent predicts as follows

Assumption 1. *The agent predicts that the other agents also behave as naive agents.*

Due to the above assumption and the agents' utility functions being the same, the DM goods' price is given by (19). Substituting it into (24) and (26), we obtain[3].

$$\frac{\phi_{na}}{\phi_{na,+1}} = \delta \left[(1-n) \frac{u'(q^b_{na,+1})}{c'(q^s_{na,+1})} + n \right], \tag{27}$$

$$\frac{\phi}{\phi_{na,+1}} = \beta\delta \left[(1-n) \frac{u'(q^b_{na,+1})}{c'(q^s_{na,+1})} + n \right]. \tag{28}$$

4 Equilibrium and Policy

4.1 The market clearing conditions

Since we have assumed that the probability of becoming a seller and a buyer is n and $1-n$, respectively, and there is a continuum of agents with a unit measure, n sellers and $1-n$ buyers exist. Therefore, the clearing condition of the DM is denoted by

$$(1-n)q^b = nq^s. \tag{29}$$

Hereafter, I will define q as the buyer's consumption amount. Therefore, $q^s = [(1-n)q]/n$. Since the productivity of labor is one and the consumption of the CM goods is denoted by (6), the clearing condition of the CM is denoted by

$$x^* = (1-n)h^b + nh^s. \tag{30}$$

Since the agents' money holdings must be equal to the money supply, we obtain

$$m = M. \tag{31}$$

[3]The equilibrium concept is also used by Gabrieli and Ghosal (2013), Ojima (2017), and Futagami and Maeda (2022).

4.2 The equilibrium path and monetary policy in the sophisticated agent economy

First, we seek the equilibrium path in the sophisticated agent economy. If the constraint (8) binds, we can rewrite the left hand side of (15) in the equilibria as follows:

$$\frac{\phi pq}{\phi_{+1}p_{+1}q_{+1}}\frac{m_{+1}}{m} = \frac{c'\left(\frac{1-n}{n}q\right)q}{c'\left(\frac{1-n}{n}q_{+1}\right)q_{+1}}\frac{M_{+1}}{M}, \tag{32}$$

where (29) and (31) are substituted. Substituting this into (15) and calculating it, we obtain the equilibrium path of the DM goods as follows:

$$c'\left(\frac{1-n}{n}q\right)q = c'\left(\frac{1-n}{n}q_{+1}\right)\frac{q_{+1}\beta\delta}{\gamma}\left[(1-n)\frac{u'(q_{+1})}{c'\left(\frac{1-n}{n}q_{+1}\right)} + n\right], \tag{33}$$

where $\gamma \equiv M_{+1}/M$. From (33), we find that we need the dynamics of money to characterize the equilibrium path of the DM goods. Therefore, we consider a monetary policy rule. We assume that the growth rate of money is constant, i.e., $\gamma = \bar{\gamma} > 0$. Therefore, the dynamics in our model are the same as the other LW models, except the existing present bias parameter, β. There are two types of equilibria in the LW models. The first is the monetary equilibrium, in which the economy immediately jumps to the unstable steady state and money has a real value that is positive and constant. The second is the inflationary equilibrium, in which the economy converges to a stable steady state and the real value of money becomes zero. In this study, we focus on the monetary equilibrium. In the steady-state, (33) can be written as follows

$$\bar{\gamma} = \beta\delta\left[(1-n)\frac{u'(\bar{q})}{c'\left(\frac{1-n}{n}\bar{q}\right)} + n\right] \tag{34}$$

or

$$\frac{\bar{\gamma}-\beta\delta}{\beta\delta} = (1-n)\left[\frac{u'(\bar{q})}{c'\left(\frac{1-n}{n}\bar{q}\right)} - 1\right], \tag{35}$$

where $\bar{q}$ denotes the consumption of the DM goods in the steady-state. To obtain the optimal monetary policy, we have to define a welfare function. We adopt the welfare function that is the sum of the buyers' and sellers' welfare. Therefore, the welfare in each period is denoted by

$$\mathcal{W} = (1-n)u(q^b) - nc(q^s), \tag{36}$$

where the term of the utility and the disutility from consuming and producing the CM goods are omitted because they do not depend on money holdings. Although measuring welfare is difficult in models with time inconsistency because the agents' preferences change over time, nevertheless, the following lemma is obtained.

Lemma 1. *The welfare is maximized if $q = q^*$ for all t.*

Proof. Using (29), we can rewrite (36) as follows:

$$\mathcal{W} = (1-n)u(q) - nc\left(\frac{1-n}{n}q\right). \tag{37}$$

The maximizing condition of this equation is denoted by

$$(1-n)u'(q) - (1-n)c'\left(\frac{1-n}{n}q\right) = 0. \tag{38}$$

From (6), $\phi p = c'(q^s)$. Therefore, (38) is rewritten

$$u'(q^*) = c'\left(\frac{1-n}{n}q^*\right) = \phi p. \tag{39}$$

If the above condition is satisfied for all the periods, then the welfare is maximized because $\mathcal{W}$ is the level of welfare at each period. □

From this lemma, we obtain the following proposition.

Proposition 1. *The optimal monetary policy in the sophisticated agent economy is $\bar{\gamma} = \beta\delta$.*

Proof. Since $u'(q) = c'(q^s)$ when $q = q^*$, the right hand side of (35) approaches zero. The growth rate of money must equal $\beta\delta$ to achieve $q = q^*$ in the equilibrium □

We had better focus on the optimal allocation achieved when the growth rate of money is constant. I will provide the interpretation of this proposition after we discuss the naive agent scenario in the next subsection.

4.3 The equilibrium path and monetary policy in the naive agent economy

From assumption 1, all agents' behavior is the same. Therefore, the expected market clearing conditions are denoted by

$$(1-n)q^b_{na} = nq^s_{na}, \tag{40}$$

$$x^* = (1-n)h^b_{na} + nh^s_{na}, \tag{41}$$

$$m_{na} = M_{na}. \tag{42}$$

Since the above conditions and $q^b_{na} = m_{na}/p_{na}$ are satisfied, we can rewrite (27), and obtain the equilibrium path of the expected DM goods in the naive agent economy, as follows:

$$c'\left(\frac{1-n}{n}q_{na}\right)q_{na} = c'\left(\frac{1-n}{n}q_{na,+1}\right)\frac{q_{na,+1}\delta}{\gamma_{na}}\left[(1-n)\frac{u'(q_{na,+1})}{c'\left(\frac{1-n}{n}q_{na,+1}\right)} + n\right], \tag{43}$$

where q_{na} is the amount of expected consumption of the DM goods and γ_{na} is the expected growth rate of money. We also assume that the monetary equilibrium is chosen and the growth rate of money is constant; $\gamma_{na} = \bar{\gamma}$. Thus, we obtain

$$\bar{\gamma} = \delta\left[(1-n)\frac{u'(\bar{q}_{na})}{c'\left(\frac{1-n}{n}\bar{q}_{na}\right)} + n\right], \tag{44}$$

where $\bar{q}_{na}$ denotes the expected consumption of the DM goods in the steady-state. We can also rewrite (28) like (43), i.e.,

$$c'\left(\frac{1-n}{n}q\right)q = c'\left(\frac{1-n}{n}q_{na,+1}\right)\frac{q_{na,+1}\beta\delta}{\gamma}\left[(1-n)\frac{u'(q_{na,+1})}{c'\left(\frac{1-n}{n}q_{na,+1}\right)} + n\right]. \tag{45}$$

When $\gamma = \bar{\gamma}$ and we substitute it into (45), we obtain

$$c'\left(\frac{1-n}{n}q\right)q = \beta c'\left(\frac{1-n}{n}\bar{q}_{na}\right)\bar{q}_{na} \tag{46}$$

Since $c''(q^s) \geq 0$ and $\beta \in (0,1]$, we obtain the following proposition.

Proposition 2. *The current consumption of the DM goods is smaller than the expected consumption.*

Proof. See the discussion immediately above for the proof. □

We obtain this proposition although there is a present bias, which is unusual. However, it is not unusual when we consider the following From (28), the low β causes the low price of the current money. It also causes the low real price of the DM goods. We find that this low price declines the incentive of the production of the DM goods from (6). Therefore, the current consumption becomes small.

We also obtain the following proposition about monetary policy.

Proposition 3. *The policy rule, which is the constant growth rate of money, cannot induce the current consumption of the DM goods in the optimal quantity in naive agent economy.*

Proof. If $q^* < q^b$, $u'(q^b) < \phi p$ because $u''(q^b) < 0$. Therefore, the agents have no incentive to consume more DM goods than q^* , and thus, $\bar{q}_{na} \leq q^*$. From proposition 2, the current consumption of the DM goods is smaller than the expected consumption. Therefore, the current consumption of the DM goods is smaller than q^*. □

4.4 Discussion

Adopting the constant growth rate of money means that the central bank keeps the expected inflation rate constant[4], i.e., the central bank adopts inflation targeting. From our analysis, it is evident that the central bank can induce the optimal production in a sophisticated agent economy, but not in a naive agent economy through inflation targeting. The cause of this difference between the sophisticated and the naiveagent economy is the difference in how expectations are formed. The sophisticated agents can correctly predict their future behavior, unlike the naive agents. The naive agents predict that the future themselves hold more money (from (23)) and the future inflation rate is higher (from (24)) than the current ones because they predict that the future themselves do not have present bias. Moreover, the naive agents also predict that the future themselves will work more to hold more money than the current themselves. However, these expectations are incorrect. Therefore, in the naive agent economy, the optimal output cannot be realized.

One solution to this problem is to diverge the expected and the realized inflation rates as follows.

[4] ϕ/ϕ_{+1} is the inflation in this model because ϕ is the real price of money. From (34) or (44), we find that it is constant if the growth rate of money is constant.

Proposition 4. *For all t, q equals to q^* if*

$$\gamma_{na} = \delta, \tag{47}$$

$$\gamma = \beta\delta. \tag{48}$$

Proof. If the future expected growth rate of money, γ_{na}, is constant, the expected future inflation rate is denoted given by $\phi_{na}/\phi_{na,+1} - 1 = \gamma_{na} - 1$. Thus, we can rewrite (27) as follows

$$\frac{\gamma_{na} - \delta}{\delta} = (1-n)\left[\frac{u'(\bar{q}_{na})}{c'\left(\frac{1-n}{n}\bar{q}_{na}\right)} - 1\right]. \tag{49}$$

From this equation, we find that $\bar{q}_{na} = q^*$ if $\gamma_{na} = \delta$. Moreover, from (28), we obtain

$$c'\left(\frac{1-n}{n}q\right)q = c'\left(\frac{1-n}{n}q^*\right)q^*\frac{\beta\delta}{\gamma}. \tag{50}$$

From this equation, if $\gamma = \beta\delta$, $q = q^*$. □

However, it is difficult that the central bank would observe how the agents predict the future inflation rate in a real economy. Therefore, the other solution is that the central bank does not commit to the future inflation rate. If the central bank commits that $q = q^*$, and all the naive agents believe the commitment, we obtain

$$\frac{\phi_{na}}{\phi_{na,+1}} = \delta, \tag{51}$$

$$\frac{\phi}{\phi_{na,+1}} = \beta\delta. \tag{52}$$

These are the same as the inflation rates of proposition 4. Therefore, I conclude that the output control rule is better than inflation targeting in a naive agent economy.

5 Conclusion

I have developed the monetary search model where the agents have present bias given by hyperbolic discounting. Through this model, I have shown that inflation targeting cannot maximize welfare and suggested that output level should be the target of the monetary policy. These are novel results of this paper.

However, my model is the simple LW model without macroeconomic shocks. Inflation targeting also has the goal of preventing destabilization caused by macroeconomic shocks. Therefore, we should analyze the relationship between the stabilization policy and the time-inconsistent preference in the future.

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