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Optimal Output and Abatement Technology for Non-point Source Pollution under Ambient Charge: Simultaneous Decisions*

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Abstract

In the case of non-point source pollutions, the regulator is unable to observe individual emission levels of the firms, so based on the emission concentration, it charges uniform environmental tax or gives uniform reward to the firms. The regulator decides on the environmental standard and the tax rate. Assuming these quantities given, in optimizing their profits the firms select optimal abatement technologies and output levels simultaneously. A Cournot duopoly is examined in the paper where each firm faces a two-dimensional decision variable and selects Nash equilibrium strategies. The equilibrium is determined and a comparative analysis is performed between maximum prices, abatement technologies, equilibrium output levels and prices. If the firms are homogeneous, then ambient charges can control the emission concentration, and in the case of heterogeneous firms conditions are given for the effectiveness of the ambient charges.

Keywords: Environmental policy, Ambient charge, Cournot competition, Two-stage game, Non-point cource pollution

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1 Introduction

Oligopoly models are among the most frequently discussed topics in the mathematical economics literature. The classical single-product model without product differentiation has been extended in many different directions including product differentiation, multiproduct models, labor-managed oligopolies, rent-seeking models among many others (Okuguchi, 1976, Okuguchi and Szidarovszky, 1999). Oligopoly models including environmental issues became a very important line of research because of its practical importance and theoretical challenges. There are several lines of research in this broad field. In the case of point-source pollutants, the regulator knows the individual levels of pollution for each firm, so it is able to punish or reward the firms individually. However, in the case of nonpoint source pollution, the regulator cannot monitor the individual emissions with low cost and sufficient accuracy. Therefore standard instruments of environmental policy are not possible. The effects of different environmental regulation policies were examined by several researchers including Downing and White (1986), Jung et al. (1996) and Montero (2002). Segerson (1988) suggested monitoring ambient concentration of pollutants and Xepapadeas (2011) summarized the different control methods. The regulator first selects an environmental standard, and imposes uniform tax on the pollutants if the concentration is above this standard, and gives uniform reward if it is below. The regulator and the firms have two decision variables. The regulator decides about the environmental standard and the environmental tax rate. The firms can select their abatement technologies and the output volumes. One of the important questions is to determine how the environmental standard and the tax rate affect the emission concentration. Ganguli and Raju (2012) examined Bertrand duopolies and showed that increase in the ambient charges might increase the emission concentration, which is called the "perverse" effect. Raju and Ganguli (2013) numerically showed the effectiveness of the ambient charge in Cournot duopolies. This result was shown analytically by Sato (2017). The n-firm generalization of this model was investigated by Matsumoto et al. (2018a) in a dynamic framework, and the corresponding Bertrand model was examined by Ishikawa et al. (2019) showing that the sign of the effect depends on the number of firms, the degree of substitutability, and the heterogeneity of the abatement technologies of the firms. Matsumoto et al. (2018b) considered a one-stage and a two-stage Bertrand duopoly and a three-stage Cournot oligopoly was introduced and examined by Matsumoto et al. (2020), where in the first stage the regulator determines the tax rate of the ambient charge to maximize social welfare, in the second stage each firm selects optimal abatement technology and in the third stage the firms decide on their optimal output levels. In this paper a Cournot oligopoly is considered. The ambient charge rate and the environmental standard are considered given and each firm maximizes its profit as a bivariable function with decision variables being the ambient technology and production level. The profit of each firm includes the revenue, the production cost, the ambient charge (or reward) and the technology installment cost. The equilibrium will be determined and the effectiveness of the ambient charge is shown in the symmetric case, and in the non-symmetric case conditions are given for the effectiveness.

The paper is developed as follows. In Section 2, the equilibrium is determined and shown how the production cost affects the optimal ambient technology and output level, as well as the price. Section 3 considers the symmetric case, when the firms are homogeneous and the effectiveness of the ambient charges on the emission concentration is verified. The non-symmetric case is analyzed in Section 4 with conditions for the effectiveness of the ambient charges. Section 5 offers concluding remarks and offers further research directions.

2 Basic Model

Let us consider a Cournot duopoly market in which firm i produces a differentiated output q_i with a linear price function,

$$p_i = \alpha_i - q_i - \gamma q_j \tag{1}$$

where $\alpha_i > 0$ is the maximum price and $0 \leq \gamma \leq 1$ denotes the degree of product differentiation; two goods are substitutes if $0 < \gamma < 1$, homogeneous if $\gamma = 1$ and independent if $\gamma = 0$. Each firm produces output as well as emits pollution. It is assumed that one unit of production emits one unit of pollution. Let ϕ_i denote the pollution abatement technology of firm i and $0 \leq \phi_i \leq 1$ with a pollution-free technology if $\phi_i = 0$ and a fully-discharged technology if $\phi_i = 1$. If firm i has the marginal cost c_i and the belief that the competitor output will remain unchanged, then its profit is

$$\pi_i(q_i, \phi_i) = (\alpha_i - q_i - \gamma q_j)\, q_i - c_i q_i - (1-\phi_i)^2 - \theta(\phi_i q_i + \phi_j q_j - \bar{E}) \tag{2}$$

where $\bar{E}$ is the ambient standard by a regulator, θ is the ambient tax rate and $(1-\phi_i)^2$ is the installation cost of technology. To have positive profit in case of no pollutions, $\alpha_i > c_i$ is assumed. The rate θ is measured in some monetary unit per emission. It is positive and can be larger than unity (e.g., dollar/ton, yen/kg, etc). However, we assume that $\theta < 1$ under some normalization. We state these assumptions more formally:

Assumption 1: (1) $0 < \gamma < 1,\ 0 < \theta < 1$; (2) $\alpha_k > c_k$ for $k = i, j$.

For firm i, both q_i and ϕ_i are strategic variables and the first-order conditions for an interior maximum are obtained by differentiating (2) with respect to q_i and ϕ_i,

$$\frac{\partial \pi_i}{\partial q_i} = \alpha_i - 2q_i - \gamma q_j - c_i - \theta\phi_i = 0 \tag{3}$$

and

$$\frac{\partial \pi_i}{\partial \phi_i} = -\theta q_i + 2(1-\phi_i) = 0. \tag{4}$$

The second-order conditions are certainly satisfied since $\pi_i(q_i, \phi_i)$ is strictly concave.[1] From (4), the optimum output is determined as

$$q_i = \frac{2(1-\phi_i)}{\theta} \tag{5}$$

which is positive if $\phi_i < 1$ and zero if $\phi_i = 1$. From (1) and (3) we see that the price is set to sell its optimum output,

$$p_i = q_i + c_i + \theta\phi_i \tag{6}$$

which is also positive if $\phi_i \leq 1$. Once the optimal technology of firm i, ϕ_i^*, is determined, then its optimal output and price are determined through equations (5) and (6).

The optimal ϕ_i is next determined. Substituting (5) into (3) yields the following form of the first-order condition for firm i's optimal technology,

$$\alpha_i - \frac{4(1-\phi_i)}{\theta} - \frac{2\gamma(1-\phi_j)}{\theta} - c_i - \theta\phi_i = 0$$

or

$$\phi_i\left(4-\theta^2\right) + 2\gamma\phi_j = 4 + 2\gamma + (c_i - \alpha_i)\,\theta. \tag{7}$$

In the same way, the corresponding first-order condition for firm j is obtained by interchanging i and j in equation (7),

$$\phi_j\left(4-\theta^2\right) + 2\gamma\phi_i = 4 + 2\gamma + (c_j - \alpha_j)\,\theta. \tag{8}$$

Let $\beta_i = \alpha_i - c_i$ and $\beta_j = \alpha_j - c_j$, both of which are positive by Assumption 1(2). Solving equations (7) and (8) simultaneously yields the optimal abatement technologies

$$\phi_i^* = \frac{(4-\theta^2-2\gamma)(4+2\gamma) + \left[2\gamma\beta_j - (4-\theta^2)\beta_i\right]\theta}{(4-\theta^2)^2 - 4\gamma^2} \tag{9}$$

and

$$\phi_j^* = \frac{(4-\theta^2-2\gamma)(4+2\gamma) + \left[2\gamma\beta_i - (4-\theta^2)\beta_j\right]\theta}{(4-\theta^2)^2 - 4\gamma^2} \tag{10}$$

where the denominators of (9) and (10) are positive due to $0 < \theta < 1$ and $0 < \gamma < 1$,

$$(4-\theta^2)^2 - 4\gamma^2 \geq 9 - 4 > 0.$$

Hence, the feasible conditions $0 \leq \phi_i^* \leq 1$ are spelled out as follows by arranging the numerator of (9),

$$f_1(\beta_i) \leq \beta_j \leq f_2(\beta_i) \tag{11}$$

[1] The Jacobian of $\pi_i(q_i, \phi_i)$ with respect to q_i and ϕ_i is

$$\boldsymbol{J} = \begin{pmatrix} -2 & -\theta \\ -\theta & -2 \end{pmatrix} \text{ with minors } -2 < 0 \text{ and } 4-\theta^2 > 0,$$

so $\boldsymbol{J}$ is negative definite, therefore $\pi_i(q_i, \phi_i)$ is strictly concave as a bivariable function.

where

$$f_1(\beta_i) = \frac{4-\theta^2}{2\gamma}\beta_i - \frac{(4-\theta^2-2\gamma)(2+\gamma)}{\gamma\theta} \tag{12}$$

and

$$f_2(\beta_i) = \frac{4-\theta^2}{2\gamma}\beta_i - \frac{(4-\theta^2-2\gamma)\theta}{2\gamma}. \tag{13}$$

Equations (12) and (13) indicate that the upper and lower bounds of the feasible region for $0 \le \phi_i^* \le 1$ are parallel lines in the (β_i, β_j) plane, having a positive slope

$$\frac{4-\theta^2}{2\gamma} \ge \frac{3}{2}$$

and negative intercepts,

$$0 > -\frac{(4-\theta^2-2\gamma)\theta}{2\gamma} > -\frac{(4-\theta^2-2\gamma)(4+2\gamma)}{2\gamma\theta}.$$

Due to the symmetry between ϕ_i^* and ϕ_j^*, the feasible conditions $0 \le \phi_j^* \le 1$ are obtained by interchanging i and j of relation (11),

$$f_1(\beta_j) \le \beta_i \le f_2(\beta_j)$$

or

$$f_2^{-1}(\beta_i) \le \beta_j \le f_1^{-1}(\beta_i) \tag{14}$$

where

$$f_1^{-1}(\beta_i) = \frac{2\gamma}{4-\theta^2}\beta_i + \frac{(4-\theta^2-2\gamma)(4+2\gamma)}{\left(4-\theta^2\right)\theta} \tag{15}$$

and

$$f_2^{-1}(\beta_i) = \frac{2\gamma}{4-\theta^2}\beta_i + \frac{(4-\theta^2-2\gamma)\theta}{4-\theta^2}. \tag{16}$$

Similarly, (15) and (16) indicate that the upper and lower bounds of the feasible region for $0 \le \phi_j^* \le 1$ are parallel lines with a positive slope,

$$\frac{2\gamma}{4-\theta^2} \le \frac{2}{3}$$

and positive intercepts

$$0 < \frac{(4-\theta^2-2\gamma)\theta}{4-\theta^2} < \frac{(4-\theta^2-2\gamma)(4+2\gamma)}{(4-\theta^2)\theta}.$$

These results are summarized as follows:

Theorem 1 *The optimal technologies ϕ_i^* and ϕ_j^* are nonnegative and not greater than unity for β_i and β_j if*

$$f_1(\beta_i) \le \beta_j \le f_2(\beta_i) \text{ and } f_2^{-1}(\beta_i) \le \beta_j \le f_1^{-1}(\beta_i).$$

The pairs (β_i, β_j) satisfying both conditions (11) and (14) are illustrated in Figure 1 with $\gamma = 0.6$ and $\theta = 0.8$.[2] The two dotted red lines describe $\beta_j = f_1(\beta_i)$ and $\beta_j = f_2(\beta_i)$ while the two dotted blue lines describe $\beta_j = f_1^{-1}(\beta_i)$ and $\beta_j = f_2^{-1}(\beta_i)$. Conditions (11) and (14) are simultaneously satisfied in the yellow parallelogram that will be called the *feasible region* and is given as

$$R = \{(\beta_i, \beta_j) \mid f_2(\beta_i) \geq \beta_j \geq f_1(\beta_i) \text{ and } f_2(\beta_j) \geq \beta_i \geq f_1(\beta_j)\}.$$

It is to be noticed that the second condition can be written as $f_1^{-1}(\beta_i) \geq \beta_j \geq f_2^{-1}(\beta_i)$. $\beta^M(\theta)$ and $\beta^m(\theta)$ in Figure 1 denote the maximum and minimum values of β and are given as

$$\beta^M(\theta) = \frac{4 + 2\gamma}{\theta} \text{ and } \beta^m(\theta) = \theta.$$

As will be seen later, the diagonal between two points $\left(\beta^M(\theta), \beta^M(\theta)\right)$ and $(\beta^m(\theta), \beta^m(\theta))$ is the feasible interval when the firms are homogeneous in the sense that $\beta_i = \beta_j$.

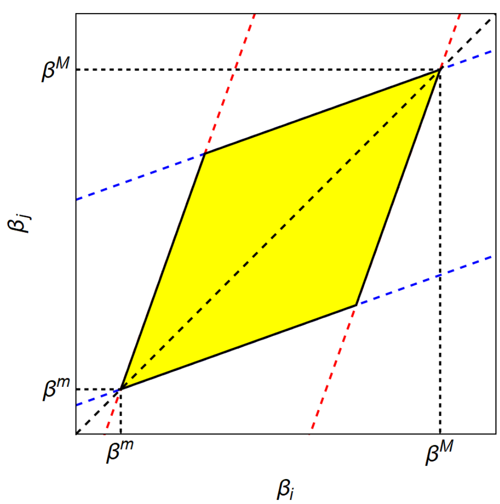


Figure 1. Feasible region for $0 \leq \phi_i^*,\ \phi_j^* \leq 1$

If the maximum prices are the same (i.e., $\alpha_i = \alpha_j$), then subtracting (10) from (9) gives

$$\phi_i^* - \phi_j^* = \frac{\theta}{4 - \theta^2 - 2\gamma}(c_i - c_j) \tag{17}$$

that implies

$$c_i \lesseqgtr c_j \Longrightarrow \phi_i^* \lesseqgtr \phi_j^*.$$

[2] Any other combinations for γ and θ generate essentially the same shape.

Through equations (5) and (6), we have

$$\phi_i^* \lesseqgtr \phi_j^* \Longrightarrow q_i^* \gtreqless q_j^* \Longrightarrow p_i^* \lesseqgtr p_j^*.$$

We summarize magnitude relations among the optimal decisions as follows:

Theorem 2 *If $\alpha_i = \alpha_j$ holds in addition to Assumption 1, then a firm with a lower production cost chooses a more efficient abatement technology, produces more output and charges less price.*

3 Homogenous Firms

To examine effects caused by a change in the ambient charge on output and the total pollution, we start with the simplified case in which the following assumption is imposed:

Assumption 2. $\alpha_i = \alpha_j = \alpha$ and $c_i = c_j = c$.

Under Assumption 2, the firms become homogenous in a sense that the price and the cost functions are the same. Their optimal decisions are identical. In particular, Assumption 2 implies $\beta_i = \beta_j = \beta$ for which (9) and (10) give identical optimal abetment technology,

$$\phi_i^* = \phi_j^* = \phi^* = \frac{\theta\left(\beta^M(\theta) - \beta\right)}{4 - \theta^2 + 2\gamma} \tag{18}$$

from which

$$\phi^* - 1 = \frac{\theta(\beta^m(\theta) - \beta)}{4 - \theta^2 + 2\gamma}.$$

Since the denominators are positive, we have the following as Corollary of Theorem 1:

Corollary 1 *Under Assumptions 1 and 2, the optimal technology satisfies* $0 < \phi^* < 1$ *if an only if* $\beta^m(\theta) < \beta < \beta^M(\theta)$.

Optimal technology ϕ^* is then substituted into (5) to have the optimal output,

$$q_i^* = q_j^* = q^* = \frac{2(1-\phi^*)}{\theta} = \frac{2\left(\beta - \beta^m(\theta)\right)}{4 - \theta^2 + 2\gamma} > 0 \text{ if } \phi^* < 1. \tag{19}$$

The optimal price is then determined via (6) as

$$p_i^* = p_j^* = p^* = q^* + c + \theta\phi^* > 0.$$

Notice that Corollary 1 is visualized in Figure 2 in which $0 \leq \phi^* \leq 1$ in the yellow region and the loci of $\beta = \beta^M(\theta)$ and $\beta = \beta^m(\theta)$ are the upper and lower boundaries.[3]

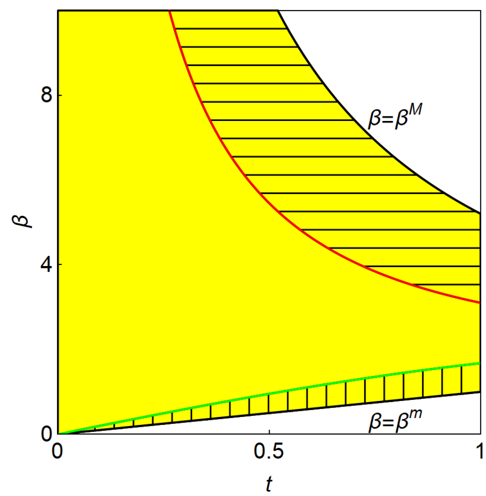


Figure 2. Region of (θ, β) for $0 \leq \phi^* \leq 1$
when $\gamma = 0.6$ and $\theta = 0.8$

The total amount of production pollution is the sum of the individual pollutiona and given by

$$E^* \left(= \phi_i^* q_i^* + \phi_j^* q_j^* \right) = 2\phi^* q^*.$$

Differentiating $\phi^* q^*$ with respect to θ yields, after arranging the terms,

$$\frac{\partial \left(\phi^* q^* \right)}{\partial \theta} = -\frac{2\left[(4 + 2\gamma + 3\theta^2)\beta^2 - 2\theta(12 + 6\gamma + \theta^2)\beta + 2(2+\gamma)(2 + \gamma + 3\theta^2) \right]}{(4 + 2\gamma - \theta^2)^3}. \tag{20}$$

The denominator is positive. Since the numerator is quadratic in β, equating it to zero and solving for β yield two solutions

$$\beta_{\pm} = \frac{\theta(\theta^2 + 6\gamma + 12) \pm \sqrt{-\left(4 - \theta^2 + 2\gamma\right)^3}}{4 + 2\gamma + 3\theta^2}.$$

Since the discriminant is negative, the numerator of (20) is positive. Therefore,

$$\frac{\partial E^*}{\partial \theta} = 2\frac{\partial \left(\phi^* q^* \right)}{\partial \theta} < 0. \tag{21}$$

The direction of inequality implies that increasing the ambient charge rate decreases the total amount of pollution. This result is summarized as follows:

[3] We will refer to the red negative- and green positive-sloping curves inside the yellow region below.

Theorem 3 *If the firms are homogenous and $\beta^m(\theta) \leq \beta \leq \beta^M(\theta)$, then changing the ambient charge tax rate can control the concentration of the NPS pollution,*

$$\frac{\partial E^*}{\partial \theta} < 0.$$

We consider why the ambient charge can be effective in controlling pollution. Differentiating (18) and (19) with respect to θ gives

$$\frac{\partial \phi^*}{\partial \theta} = \frac{4+2\gamma+\theta^2}{4+2\gamma-\theta^2}\left[\frac{2\theta(4+2\gamma)}{4+2\gamma+\theta^2} - \beta\right] \tag{22}$$

and

$$\frac{\partial q^*}{\partial \theta} = \frac{4\theta}{\left(4+2\gamma-\theta^2\right)^2}\left[\beta - \frac{4+2\gamma+\theta^2}{2\theta}\right]. \tag{23}$$

Accordingly, two new functions are introduced,

$$\beta_0(\theta) = \frac{2\theta(4+2\gamma)}{4+2\gamma+\theta^2} \text{ and } \beta_1(\theta) = \frac{4+2\gamma+\theta^2}{2\theta}.$$

The $\beta = \beta_0(\theta)$ curve corresponds to the positive-sloping green curve located just above the $\beta = \beta^m(t)$ line in Figure 2 and the following relations hold,

$$\frac{\partial \phi^*}{\partial \theta} \gtreqless 0 \text{ according to } \beta \lesseqgtr \beta_0(\theta).$$

Thus in the vertically-striped yellow region surrounded by the two curves, $\beta = \beta^m(\theta)$ and $\beta = \beta_0(\theta)$ in Figure 2, the following inequality holds,

$$\frac{\partial \phi^*}{\partial \theta} > 0.$$

Similarly the $\beta = \beta_1(\theta)$ curve corresponds to the negative-sloping red curve located just below the $\beta = \beta^M(\theta)$ curve and the following relations hold,

$$\frac{\partial q^*}{\partial \theta} \gtreqless 0 \text{ according to } \beta \gtreqless \beta_1(\theta).$$

In the horizontally-striped yellow region surrounded by these two curves, $\beta = \beta^M(\theta)$ and $\beta = \beta_1(\theta)$, we have

$$\frac{\partial q^*}{\partial \theta} > 0.$$

Summarizing the results, we can divide the yellow region into three subregions in which

$$\begin{aligned}
&(i)\ \beta^m(\theta) \leq \beta \leq \beta_0(\theta),\ \frac{\partial q^*}{\partial \theta} < 0 \text{ and } \frac{\partial \phi^*}{\partial \theta} \geq 0,\\
&(ii)\ \beta_0(\theta) < \beta < \beta_1(\theta),\ \frac{\partial q^*}{\partial \theta} < 0 \text{ and } \frac{\partial \phi^*}{\partial \theta} < 0,\\
&(iii)\ \beta_1(\theta) \leq \beta \leq \beta^M(\theta),\ \frac{\partial q^*}{\partial \theta} \geq 0 \text{ and } \frac{\partial \phi^*}{\partial \theta} < 0.
\end{aligned} \tag{24}$$

If the change in the ambient tax rate has an unfavorable effect on the variables, we call it a *perverse* effect. Although the regulator cannot observe the individual firm's reactions to a change in θ, their optimal responses are summarized as follows:

Theorem 4 *The ambient tax rate has a perverse effect on the optimal technology decision if β is small enough in the sense that $\beta^m(\theta) \leq \beta \leq \beta_0(\theta)$ and on the optimal output decision if β is large enough in the sense of $\beta_1(\theta) \leq \beta \leq \beta^M(\theta)$ whereas it has a normal effect on both variables if β takes a normal value in the sense of $\beta_0(\theta) < \beta < \beta_1(\theta)$.*

We now consider the magnitude of these normal and perverse effects. Differentiating $E^* = 2\phi^* q^*$ with respect to θ and arranging the terms present

$$\frac{\partial E^*}{\partial \theta} = 2\frac{E^*}{\theta}\left(\varepsilon_\phi + \varepsilon_q\right)$$

where ε_ϕ and ε_q denote the elasticity of technology and the elasticity of output for the ambient charge tax and are defined, respectively, by

$$\varepsilon_\phi = \frac{\theta}{\phi^*}\frac{\partial \phi^*}{\partial \theta} \text{ and } \varepsilon_q = \frac{\theta}{q^*}\frac{\partial q^*}{\partial \theta}.$$

Since θ can have a perverse effect as discussed in Theorem 4, the sign of $\partial E^*/\partial\theta$ depends on the relative magnitudes between ε_ϕ and ε_q in absolute values. In case (*ii*) of (24) or in the non-striped yellow region of Figure 2, both elasticities are negative, implying that $\partial E^*/\partial\theta < 0$. On the other hand, the sign of $\partial E^*/\partial\theta$ in case (*i*) or in case (*iii*) is ambiguous since the two elasticities are of opposite sign. However, Theorem 3 indicates $\partial E^*/\partial\theta < 0$ in both regions. Hence, in case (*i*) or in the vertically-striped yellow region, the negative elasticity of the optimal output dominates the positive elasticity of ϕ^*. In case (*iii*) or in the horizontally-striped yellow region, the negative elasticity of the optimal technology dominates the positive elasticity of q^*. Theorem 3 demonstrates the effectiveness of the ambient charge on the total concentration. We reach the same result through a different route.

Theorem 5 *The ambient charge can control the NPS pollution $\phi^* q^*$ emitted by an individual firm since the normal effect dominates the perverse effect. In consequence,*

$$\frac{\partial E^*}{\partial \theta} = 2\frac{\partial\left(\phi^* q^*\right)}{\partial \theta} < 0.$$

4 Heterogeneous Firms

We now consider the ambient charge effect when the firms are heterogeneous. The total amount of pollution is

$$E^* = E_i^* + E_j^* \text{ where } E_k^* = \phi_k^* q_k^* \text{ for } k = i, j$$

and its θ-derivative is

$$\frac{\partial E^*}{\partial \theta} = \frac{\partial \left(\phi_i^* q_i^*\right)}{\partial \theta} + \frac{\partial \left(\phi_j^* q_j^*\right)}{\partial \theta} \tag{25}$$

where

$$\frac{\partial \left(\phi_k^* q_k^*\right)}{\partial \theta} = \frac{E_k^*}{\theta} \left(\frac{\theta}{\phi_k^*} \frac{\partial \phi_k^*}{\partial \theta} + \frac{\theta}{q_k^*} \frac{\partial q_k^*}{\partial \theta} \right). \tag{26}$$

The following transformation is obtained by applying equation (5) and its θ-derivative,

$$\frac{\theta}{q_k^*} \frac{\partial q_k^*}{\partial \theta} = - \left(\frac{\phi_k^*}{1 - \phi_k^*} \frac{\theta}{\phi_k^*} \frac{\partial \phi_k^*}{\partial \theta} + 1 \right)$$

that is substituted into (26) to obtain

$$\frac{\partial \left(\phi_k^* q_k^*\right)}{\partial \theta} = \frac{E_k^*}{\theta} \left(\frac{1 - 2\phi_k^*}{1 - \phi_k^*} \frac{\theta}{\phi_k^*} \frac{\partial \phi_k^*}{\partial \theta} - 1 \right). \tag{27}$$

For notational convenience, we focus on the pollution amount emitted by firm i for a while. Using the functions $f_1(\beta_i)$ and $f_2(\beta_i)$ defined in (12) and (13), we can rewrite equation (9), the optimal technology selected by firm i, as

$$\phi_i^* = \frac{2\gamma\theta \left[\beta_j - f_1(\beta_i)\right]}{(4 - \theta^2)^2 - 4\gamma^2}$$

with which, we have

$$1 - \phi_i^* = \frac{-2\gamma\theta \left[\beta_j - f_2(\beta_i)\right]}{(4 - \theta^2)^2 - 4\gamma^2}$$

and

$$1 - 2\phi_i^* = \frac{-4\gamma\theta \left[\beta_j - g_1(\beta_i)\right]}{(4 - \theta^2)^2 - 4\gamma^2}$$

where

$$g_1(\beta_i) = \frac{4 - \theta^2}{2\gamma} \beta_i - \frac{\left(4 - \theta^2 - 2\gamma\right)\left(4 + \theta^2 + 2\gamma\right)}{4\gamma\theta}.$$

Differentiating ϕ_i^* with respect to θ and arranging the terms present

$$\frac{\partial \phi_i^*}{\partial \theta} = \frac{2\gamma \left[(4 - \theta^2)(4 + 3\theta^2) - 4\gamma^2\right] \left[\beta_j - g_2(\beta_i)\right]}{\left[(4 - \theta^2)^2 - 4\gamma^2\right]^2}$$

where

$$g_2(\beta_i) = \frac{(4-\theta^2)^2(4+\theta^2) - 4\gamma^2(4-3\theta^2)}{2\gamma\left[(4-\theta^2)(4+3\theta^2) - 4\gamma^2\right]}\beta_i - \frac{2\theta(2+\gamma)\left(4-\theta^2-2\gamma\right)^2}{\gamma\left[(4-\theta^2)(4+3\theta^2)-4\gamma^2\right]}.$$

Using these terms, we have

$$\frac{1-2\phi_i^*}{1-\phi_i^*}\frac{\theta}{\phi_i^*}\frac{\partial\phi_i^*}{\partial\theta} = \frac{A}{B}\frac{\left[\beta_j - g_1(\beta_i)\right]\left[\beta_j - g_2(\beta_i)\right]}{\left[\beta_j - f_1(\beta_i)\right]\left[\beta_j - f_2(\beta_i)\right]} \tag{28}$$

with

$$A = 2\left[(4-\theta^2)(4+3\theta^2) - 4\gamma^2\right] > 0,$$

and

$$B = (4-\theta^2)^2 - 4\gamma^2 > 0.$$

Hence, the right hand side of equation (27) for firm i can be written as

$$\frac{\partial\left(\phi_i^* q_i^*\right)}{\partial\theta} = \frac{E_k^*}{\theta}\left(\frac{A}{B}\frac{\left[\beta_j - g_1(\beta_i)\right]\left[\beta_j - g_2(\beta_i)\right]}{\left[\beta_j - f_1(\beta_i)\right]\left[\beta_j - f_2(\beta_i)\right]} - 1\right). \tag{29}$$

Figure 3(A) presents the division of the feasible region given in Figure 1. The loci of $\beta_j = f_2(\beta_i)$ and $\beta_j = f_1(\beta_i)$ are the left hand and right hand boundaries and the loci of $\beta_i = f_1(\beta_j)$ and $\beta_i = f_2(\beta_j)$ are the lower and upper boundaries. Two loci of $\beta_j = g_1(\beta_i)$ and $\beta_j = g_2(\beta_i)$ divide the feasible region for firm i into three regions as shown in Figure 3(A) where $0 < dg_2(\beta_i)/d\beta_i < dg_1(\beta_i)/d\beta_i$. The following inequalities hold in the divided subregions:

$$\begin{aligned} R_{1i} &= \{(\beta_i, \beta_j) \in R \mid \beta_j > g_2(\beta_i) \text{ and } \beta_j > g_1(\beta_i)\}, \\ R_{2i} &= \{(\beta_i, \beta_j) \in R \mid \beta_j < g_2(\beta_i) \text{ and } \beta_j > g_1(\beta_i)\}, \\ R_{3i} &= \{(\beta_i, \beta_j) \in R \mid \beta_j < g_2(\beta_i) \text{ and } \beta_j < g_1(\beta_i)\}. \end{aligned}$$

It is confirmed that $f_1(\beta_i) \leq \beta_j \leq f_2(\beta_i)$. Consequently, we have

$$\frac{\left[\beta_j - g_1(\beta_i)\right]\left[\beta_j - g_2(\beta_i)\right]}{\left[\beta_j - f_1(\beta_i)\right]\left[\beta_j - f_2(\beta_i)\right]}\begin{cases} < 0 \text{ for } (\beta_i, \beta_j) \in R_{1i}, \\ > 0 \text{ for } (\beta_i, \beta_j) \in R_{2i}, \\ < 0 \text{ for } (\beta_i, \beta_j) \in R_{3i}, \end{cases} \tag{30}$$

Therefore, equation (29) and relation (30) imply the following:

Lemma 1 *For the pair of (β_i, β_j) in region $R_{1i} \cup R_{3i}$, the ambient charge tax rate is effective in reducing firm i's pollution,*

$$\frac{\partial\left(\phi_i^* q_i^*\right)}{\partial\theta} < 0.$$

In region R_{2i}, since $f_2(\beta_i) > g_2(\beta_i) > g_1(\beta_i) > f_1(\beta_i)$, multiplying by -1 and adding β_j to each term gives the following order,

$$\beta_j - f_2(\beta_i) < \beta_j - g_2(\beta_i) < 0 < \beta_j - g_1(\beta_i) < \beta_j - f_1(\beta_i)$$

which then implies that

$$\frac{\left[\beta_j - g_1(\beta_i)\right]\left[\beta_j - g_2(\beta_i)\right]}{\left[\beta_j - f_1(\beta_i)\right]\left[\beta_j - f_2(\beta_i)\right]} < 1$$

as

$$0 < \frac{\beta_j - g_2(\beta_i)}{\beta_j - f_2(\beta_i)} < 1 \text{ and } 0 < \frac{\beta_j - g_1(\beta_i)}{\beta_j - f_1(\beta_i)} < 1.$$

On the other hand,

$$A - B = 4(4-\gamma^2) + \theta^2(24 - 7\theta^2) \geq 12 \text{ for } 0 \leq \gamma \leq 1 \text{ and } 0 \leq \theta \leq 1.$$

Hence $A/B > 1$ implying that the sign of $\partial\left(\phi_i^* q_i^*\right)/\partial\theta$ is ambiguous in region R_{2i}. However, it is possible to check the sign in a different way. Let us denote the parenthesized factor of (29) by

$$\varphi_i\left(\beta_i, \beta_j\right) = \frac{A}{B}\frac{\left[\beta_j - g_1(\beta_i)\right]\left[\beta_j - g_2(\beta_i)\right]}{\left[\beta_j - f_1(\beta_i)\right]\left[\beta_j - f_2(\beta_i)\right]} - 1. \tag{31}$$

As can be seen in Figure 3(B) in which $\gamma = 0.6$ and $\theta = 0.8$, the maximum value of $\varphi_i\left(\beta_i, \beta_j\right)$ is denoted as the red point and realized by maximizers, β_i^m and β_j^m, satisfying $\beta_j^m = f_2^{-1}(\beta_i^m)$. Point $(\beta_i^m, \beta_j^m, -1)$ is denoted as the black point. We numerically determine those critical values. To this end, we first substitute $\beta_j = f_2^{-1}(\beta_i)$ into $\varphi_i\left(\beta_i, \beta_j\right)$,[4]

$$\Phi_i(\beta_i) = \varphi_i\left(\beta_i, f_2^{-1}(\beta_i)\right) = \frac{15608 - 15426\beta_i + 4805\beta_i^2}{513\left(5-\beta_i\right)\left(4-5\beta_i\right)}.$$

Then differentiating $\Phi_i(\beta_i)$ with respect to β_i, equating it to zero and solving for β_i yield

$$\beta_i^m \simeq 1.84,\ \beta_j^m \left(= f_2^{-1}(\beta_i^m)\right) \simeq 1.17 \text{ and } \varphi_i\left(\beta_i^m, \beta_j^m\right) \simeq -0.416.$$

Here $\varphi_i\left(\beta_i^m, \beta_j^m\right)$ is the maximum value of $\varphi_i\left(\beta_i, \beta_j\right)$ in region R_{2i} in which point $\left(\beta_i^m, \beta_j^m\right)$ is denoted by the black point on the $\beta_j = f_2^{-1}(\beta_i)$ locus. Repeating the same procedure under the conditions of $0 < \gamma < 1$ and $0 < \theta < 1$, we have the following result:

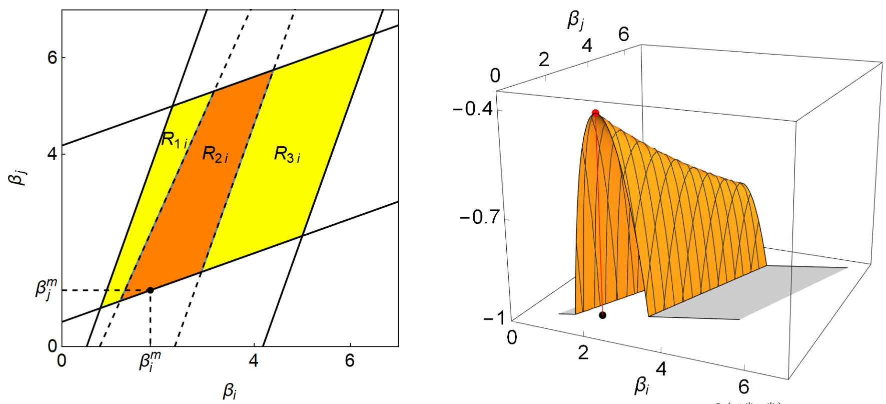
Lemma 2 *Given Assumption 1, the maximum value of* $\partial\left(\phi_i^* q_i^*\right)/\partial\theta$ *is negative for any pair of* (β_i, β_j) *in region* R_{2i}.

[4] The calculations are done with Mathematica, version 12.1.

Proof. See the Appendix. ■

Lemmas 1 and 2 imply the following;

$$\frac{\partial\left(\phi_i^* q_i^*\right)}{\partial\theta} < 0 \text{ for } (\beta_i, \beta_j) \in R. \tag{32}$$



(A) The feasible region

(B) Graph of $\frac{\partial(\phi_i^* q_i^*)}{\partial\theta}$

Figure 3. $\dfrac{\partial\left(\phi_i^* q_i^*\right)}{\partial\theta} < 0$ for firm i under $\gamma = 0.6$ and $\theta = 0.8$

For firm j, we repeat the same procedure and can show that firm j's pollution amount is negatively related with the ambient tax rate. Indeed, following the definition of $\varphi_i\left(\beta_i, \beta_j\right)$ in equation (31), we can define the corresponding function for firm j by interchanging β_i with β_j,

$$\varphi_j\left(\beta_i, \beta_j\right) = \frac{1 - 2\phi_j^*}{1 - \phi_j^*}\frac{\theta}{\phi_j^*}\frac{\partial\phi_j^*}{\partial\theta} - 1 = \frac{A}{B}\frac{\left[\beta_i - g_1(\beta_j)\right]\left[\beta_i - g_2(\beta_j)\right]}{\left[\beta_i - f_1(\beta_j)\right]\left[\beta_i - f_2(\beta_j)\right]} - 1$$

and substituting $\beta_i = f_2(\beta_j)$ into $\varphi_j\left(\beta_i, \beta_j\right)$ gives

$$\Phi_j(\beta_j) = \varphi_j\left(f_2(\beta_j), \beta_j\right)$$

The feasible region is the same and is differently divided into three subregions by the two loci of $\beta_i = g_1(\beta_j)$ and $\beta_i = g_2(\beta_j)$ as

$$\begin{aligned} R_{1j} &= \{(\beta_i, \beta_j) \in R \mid \beta_i > g_2(\beta_j) \text{ and } \beta_i > g_1(\beta_j)\}, \\ R_{2j} &= \{(\beta_i, \beta_j) \in R \mid \beta_i < g_2(\beta_j) \text{ and } \beta_i > g_1(\beta_j)\}, \\ R_{3j} &= \{(\beta_i, \beta_j) \in R \mid \beta_i < g_2(\beta_j) \text{ and } \beta_i < g_1(\beta_j)\}. \end{aligned}$$

Due to its definition, $f_2(\beta_j) \geq \beta_i \geq f_1(\beta_j)$ holds in region R. Hence, applying Lemmas 1 and 2 leads to the following for firm j,

$$\frac{\partial\left(\phi_j^* q_j^*\right)}{\partial\theta} < 0 \text{ for } (\beta_i, \beta_j) \in R. \tag{33}$$

Figure 4 describes the yellow half-cylinder of $\varphi_i\left(\beta_i,\beta_j\right)$ and the blue half-cylinder of $\varphi_j\left(\beta_i,\beta_j\right)$ for $(\beta_i,\beta_j)\in R$. Both are restricted to the gray colored region that is the feasible region of (β_i,β_j) and corresponds to the yellow parallelogram in Figure 1 and Figure 3(A). Notice that the highest value is negative.

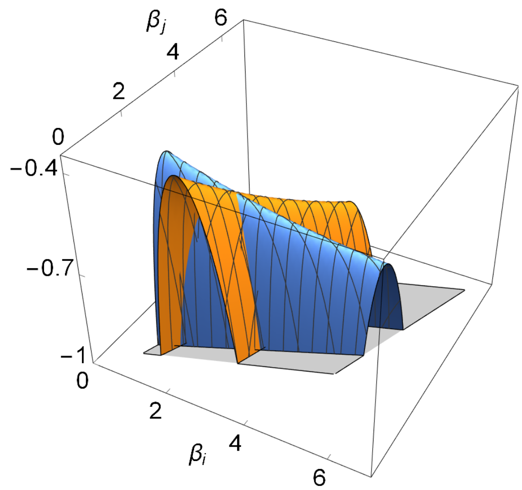


Figure 4. The sign of $\partial E^*/\partial\theta$ when $\gamma=0.6$ and $\theta=0.8$

From (32) and (33), we arrive at the following result for the heterogeneous firms:

Theorem 6 *Given Assumption 1, the ambient charge can control the total amount of NPS pollutions when the firms are heterogeneous,*

$$\frac{\partial E^*}{\partial\theta}=\frac{\partial\left(\phi_i^* q_i^*\right)}{\partial\theta}+\frac{\partial\left(\phi_j^* q_j^*\right)}{\partial\theta}<0.$$

5 Concluding Remarks

In this paper Cournot duopolies with product differentiation are considered, when the profit of each firm includes revenue, production cost, ambient environmental tax (or reward) and technology investment cost. The action of the regulator is assumed given, and the firms select their ambient technologies and output levels. A two-person game is therefore defined, when the players have two-dimensional strategies. First the Nash equilibrium is determined and comparative study is performed between maximal prices, abatement technologies, output levels and prices at the equilibrium levels. In the case of homogeneous firms the effectiveness of the ambient charges is proved on the emission concentration. If the firms are heterogeneous, then conditions are derived for effectiveness. The research reported in this paper can be continued and extended into several directions. In the paper we considered linear price and cost functions as well as a simple quadratic technology investment cost. More complex, mainly nonlinear function types might make the results more interesting. Oligopolies with n firms can be modeled as an $(n+1)$-player game, when the firms and the regulator are the players. Its equilibrium analysis might offer some interesting results. We can also introduce the dynamic extensions of these models with and without time delays.

Appendix

In this Appendix, Lemma 2 is proved under the conditions of $0 < \gamma < 1$ and $0 < \theta < 1$ (that is, Assumption 1(1)). We start with restating equation (31),

$$\varphi_i\left(\beta_i, \beta_j\right) = \frac{A}{B}\frac{\left[\beta_j - g_1(\beta_i)\right]\left[\beta_j - g_2(\beta_i)\right]}{\left[\beta_j - f_1(\beta_i)\right]\left[\beta_j - f_2(\beta_i)\right]} - 1.$$

Substituting $\beta_j = f_2^{-1}(\beta_i)$ into $\varphi_i\left(\beta_i, \beta_j\right)$ gives

$$\Phi_i(\beta_i) = \varphi_i\left(\beta_i, f_2^{-1}(\beta_i)\right) = -\frac{a\beta_i^2 + b\beta_i + c}{\left(\theta - \beta_i\right)\left(4/\theta - \beta_i\right)\left[\left(4-\theta\right)^2 - 4\gamma^2\right]} \tag{A-1}$$

where

$$\begin{aligned} a &= \left(4 - \theta^2\right)\left(4 + 3\theta\right) - 4\gamma^2 > 0, \\ b &= -\theta\left[(4 - \gamma^2)\left(12 - 6\gamma + \theta^2\right) - 8\gamma_2\right] < 0, \\ c &= 2\left(4 - \theta^2\right)\left[4(2-\gamma) + \theta^2(6-\gamma)\right] - 4\gamma^2\theta^2 > 0. \end{aligned}$$

The discriminant of the quadratic polynomial $a\beta_i^2 + b\beta_i + c$ is negative,

$$D = 4(4-\theta^2)\left[8(2-\gamma)\gamma^2 - (4-\theta^2)^2(4 - 2\gamma - \theta^2)\right] < 0,$$

which means that the numerator of (A-1) is positive. Since $\left(4-\theta\right)^2 - 4\gamma^2 > 0$ for $0 < \gamma < 1$ and $0 < \theta < 1$, we have

$$\Phi_i(\beta_i) < 0 \text{ if } \theta < \beta_i < \frac{4}{\theta}.$$

To find maximizers for $\Phi_i(\beta_i)$, we solve $\partial\Phi_i/\partial\beta_i = 0$ for β_i and obtain two solutions,

$$\beta_i^{\pm} = \frac{2\gamma\theta\left(4 + \theta^2 - 2\gamma\right) \pm \sqrt{2}(4-\theta^2)\sqrt{(2-\gamma)\left[(4-\theta^2)^2 - 4\gamma^2\right]}}{\left[(4-\theta^2)^2 - 4\gamma^2\right] + 4\gamma\theta^2}$$

where the denominator is positive. Each term in the numerator of β_i^- is positive. Factorizing the difference of these squares gives

$$-2\left[4(2-\gamma)\left(2 - \theta^2 + \gamma\right) + \theta^4\right]\left[\left(32 - 16(\gamma + \theta^2) + \left(2\gamma^2 + (2-\gamma)\theta^2\right)\theta^2\right)\right] < 0$$

as the first and second bracketed terms are positive. Hence $\beta_i^- < 0$ and is eliminated from further considerations. On the other hand, β_i^+ is definitely positive and can be confirmed to satisfy the negativity conditions,

$$\theta < \beta_i^+ < \frac{4}{\theta}. \tag{A-2}$$

In particular,

$$\beta_i^+ - \theta = \frac{(4-\theta^2)\sqrt{4-\theta^2-2\gamma}\left[\sqrt{2}\sqrt{(2-\gamma)(4-\theta^2+2\gamma)} - \theta\sqrt{4-\theta^2-2\gamma}\right]}{\left[(4-\theta^2)^2 - 4\gamma^2\right] + 4\gamma\theta^2}$$

Squaring each term in the brackets of the numerator and making subtraction present

$$2(2-\gamma)(4-\theta^2+2\gamma) - \theta^2\left(4-\theta^2-2\gamma\right) = 4(2-\gamma)(2-\theta^2+\gamma) > 0$$

that implies the first inequality of (A-2). Next,

$$\beta_i^+ - \frac{4}{\theta} = \frac{(4-\theta^2)\sqrt{(2-\gamma)\left(4-\theta^2+2\gamma\right)}\left[\sqrt{2}\theta\sqrt{(4-\theta^2-2\gamma)} - \sqrt{(2-\gamma)\left(4-\theta^2+2\gamma\right)}\right]}{\theta[(4-\theta^2)^2 - 4\gamma^2 + 4\gamma\theta]}$$

where the denominator is positive and each term in the numerator is also positive. The difference of the squares of the two terms inside the brackets yields

$$-2\left[\theta^4 + 4(2-\gamma)(2-\theta^2+\gamma)\right] < 0$$

which implies the second inequality of (A-2). Hence, the larger solution β_i^+ satisfies the negativity conditions and makes the maximum value of $\Phi_i(\beta_i)$ negative. Let us denote β_i^+ by β_i^m, then the proof of Lemma 2 is completed.

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