Chukyo University Institute of Economics

Discussion Paper Series

January 2021

No. 2005

Urbanization and population contraction

Madoka Muroishi and Akira Yakita

Urbanization and population contraction

Madoka Muroishi and Akira Yakita
Graduate School of Social Sciences, Nanzan University, Nagoya 466-8673, Japan

Abstract
We present a closed overlapping generations model with two regions: urban and rural. Concentration of population to urban areas involves population inflows from rural areas, worsening congestion. Inverse agglomeration in rural areas exerts negative effects on rural income, thereby increasing the attractiveness of urban areas. Because of lower urban fertility rates, urbanization involves population decreases. Therefore, population dynamics might be explained as simultaneous urbanization and population contraction.

Keywords: fertility, migration, population contraction, urbanization

JEL Classification: J11, J13, R13, R23

Correspondence: A Yakita, Faculty of Economics, Nanzan University, 18 Yamasato-cho, Showa-ku, Nagoya 466-8673, Japan
Email: yakita@nanzan-u.ac.jp

1. Introduction

The UN World Urbanization Prospects (2018) reports that urbanization is proceeding steadily worldwide. In fact, urbanization is higher in more economically developed regions than in less developed regions (Fig. 1). According to the UN World Population Prospects (2019), total fertility rates have been lower than the replacement rate in most economically developed countries for decades, although the rates are still higher in less developed economies. Figure 2 portrays changes in the population aged 15– 64 since 1990. Reflecting the fertility differences, the population sizes in more economically developed regions have recently declined, although those in least developed regions has rather shown a plateau during the period. Figure 3 displays net immigrants worldwide since 1990. More economically developed countries received a sizable immigrant flow from less developed regions (UN world Population Prospects 2018). That immigration might slow population and fertility declines in economically developed regions.

Fig. 1(a) Urban population: G7 countries (per cent).

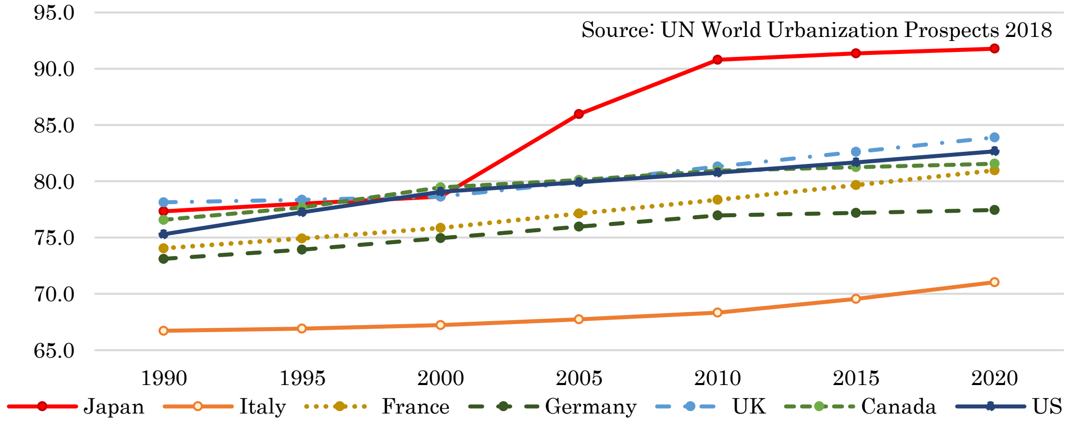
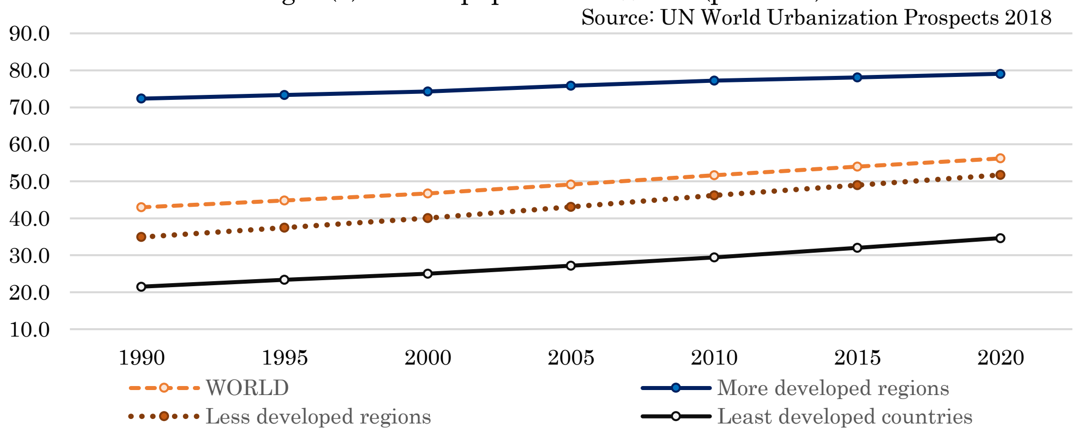


Fig. 1(b) Urban population: World (per cent).



It has been recognized as a stylized fact that concentrations of population increase income through agglomeration economies, whereas congestion diseconomies lower fertility rates in urban areas. However, concentrations of population in urban areas lead to depopulation in rural areas. Population declines in rural areas decrease agglomeration economies in those regions, thereby lowering regional wages. The lowered income might induce more population outflows.

Fig. 2(a) Population growth rate, 15--64: G7 countries.

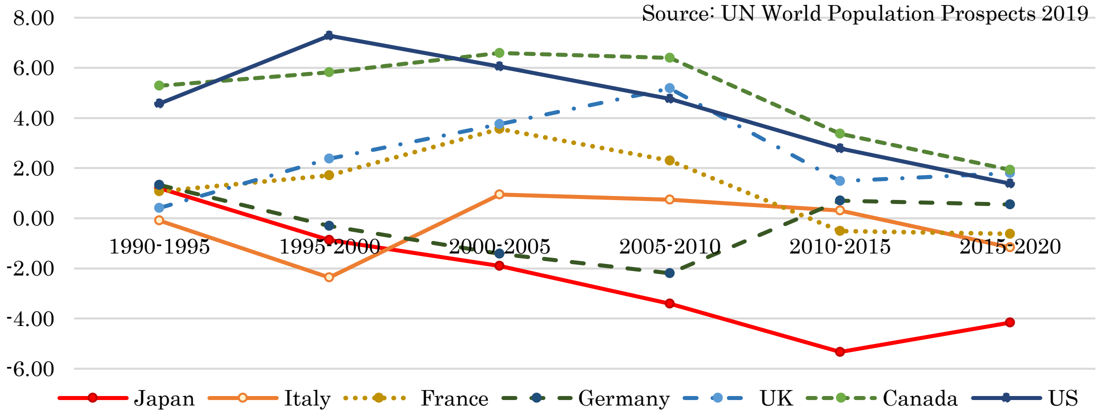
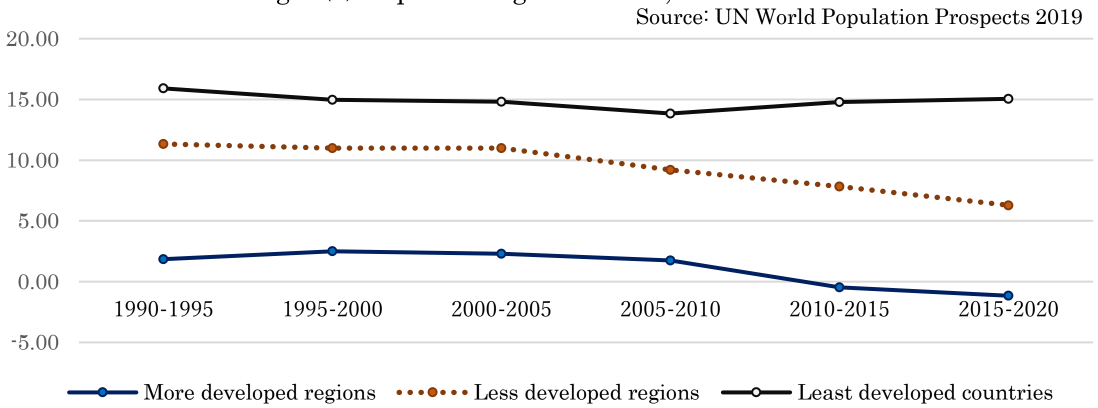


Fig. 2(b) Population growth rate, 15--64: World.



The relevant literature reports determination of the dynamics of overall population in an economy by the interaction between the regional distribution and the population size (Sato and Yamamoto 2005; Sato 2007; Yakita 2011; and Muroishi and Yakita 2020). Actually, Sato and Yamamoto (2005) and Yakita (2011) assume that concentration of population causes agglomeration only in urban, but not in rural areas. Sato (2007) and Muroishi and Yakita (2020) infer that the degrees of congestion diseconomies are identical among regions. By contrast, we analyze the population dynamics of a two-region economy by assuming that the degree of agglomeration is higher in urban than in rural areas and that there are congestion diseconomies only in rural areas. A salient feature is that the rural wage rate increases with the number of workers because of agglomeration in rural areas, in contrast to the assumption in Sato and Yamamoto (2005).

The main result is that, if the initial population size is sufficiently low, then the economy will undergo both urbanization and population contraction simultaneously. However, as time passes, the populations are fully urbanized; they converge to a stationary size because population contraction mitigates congestion in urban areas.

Fig. 3(a) Net number of immigrants: G7 countries (thousands).
Source: UN World Population Prospects 2019

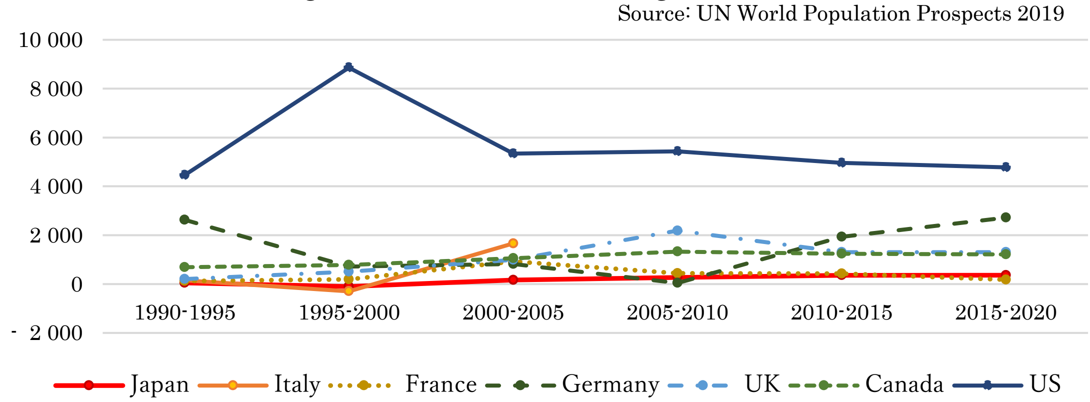
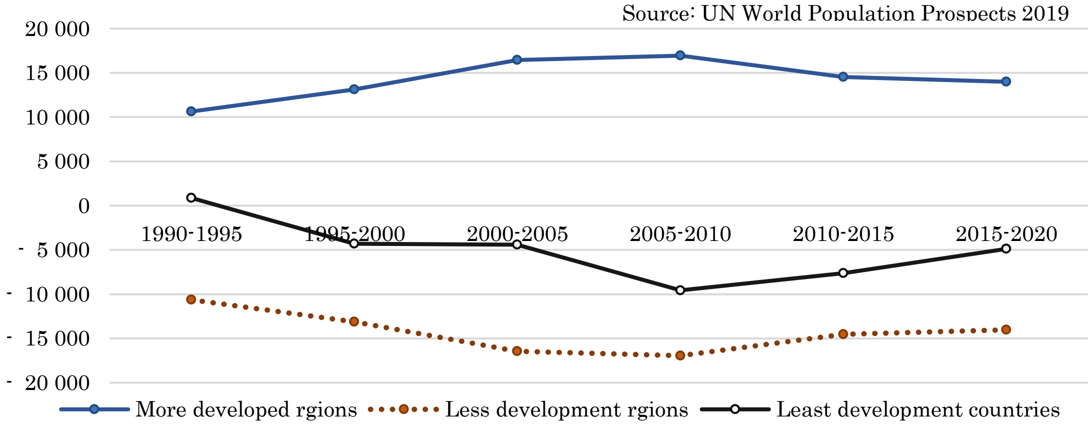


Fig. 3(b) Net number of immigrants: World (thousands).
Source: UN World Population Prospects 2019



The next section introduces a closed overlapping-generations model of a two-region economy of urban and rural regions. The population dynamics is analyzed in Section 3. Section 4 concludes the paper.

2. Model

The regions are respectively designated as 'urban (u)' and 'rural (r).' Each generation consists of individuals who live for two periods: childhood and adulthood. Individuals have identical preferences and live for two periods. In adulthood, individuals choose a region in which to reside and work. They rear children during adulthood. The adult population in the economy in period t is denoted as $N_t = N_{ut} + N_{rt}$, where N_i stands for the number of adults in region i. The fertility rate of the economy in period t is $n_t = (n_{ut}N_{ut} + n_{rt}N_{rt}) / N_t$, where n_{it} is the regional fertility rate ($i = r, u$). The movement of the population size can be written as $N_{t+1} = n_t N_t$.

Assuming, as did Sato and Yamamoto (2005), that the congestion cost in urban is measured in time costs, the net time endowment available for urban adults can be written as

$$l_{ut} = \exp[-GN_{ut}] < 1. \tag{1}$$

Parameter G measures the degree of congestion diseconomies. The gross time endowment is normalized to one. Without congestion, the time endowment for adults in rural regions is therefore equal to one, i.e., $l_{rt} = 1$. Denoting the rearing time per child as z, which is assumed to be constant, the budget constraint of adults in region i at period t can be written as

$$(l_{it} - zn_{it})w_{it} = c_{it}, \tag{2}$$

where w_{it} and c_{it} respectively stand for the wage rate and consumption in region i at period t. Assuming the lifetime utility of an individual in region i at period t to be represented as $U_{it} = c_{it}^{\alpha} n_{it}^{1-\alpha}$, where $0 < \alpha < 1$ ($i = r, u$), we obtain the optimal plans as

$$c_{it} = \alpha l_{it} w_{it}, \tag{3}$$

$$n_{it} = \frac{1-\alpha}{b} l_{it}. \tag{4}$$

We assume here that the degree of agglomeration effect and therefore the wage rate is higher in the urban region than in the rural region, i.e., the urban wage rate

$$w_{ut} = B\exp(\beta N_{ut}) \tag{5}$$

is higher than the rural wage rate

$$w_{rt} = R\exp(\gamma N_{rt}), \tag{6}$$

where $B > R(>0)$ and $\beta \geq \gamma(>0)$ hold. The former inequality guarantees the possibility that adults choose to reside in an urban area, although they face congestion costs. The last inequality means that the degree of agglomeration economy is greater in the urban area.

By inserting the optimal plans into the utility function, we obtain the indirect function as $V_{it} = \alpha^{\alpha}(\frac{1-\alpha}{z})^{1-\alpha} l_{it}(w_{it})^{\alpha}$. Because of free residence choices of adults between regions, the utility levels are equalized in the migration equilibrium, i.e., $V_{ut} = V_{rt}$. From (1), (5), and (6), the arbitrary condition can be rewritten as

$$\exp(-GN_{ut})[B\exp(\beta N_{ut})]^{\alpha} = [R\exp(\gamma N_{rt})]^{\alpha} . \tag{7}$$

From (7), the equilibrium distribution of adults between regions in period t can be obtained for total population N_t. From (7), the urban population is given as

$$N_{ut} = \frac{\lambda + \gamma N_t}{\beta - \frac{G}{\alpha} + \gamma} , \tag{8}$$

where $\lambda = \ln R - \ln B < 0$. The migration-stability condition is $\frac{d}{dN_{ut}}(V_{ut} - V_{rt}) < 0$, i.e.,

$$\alpha\{B\exp[(\beta - \frac{G}{\alpha})N_{ut}]\}^{\alpha}(\beta - \frac{G}{\alpha} + \gamma) < 0 . \tag{9}$$

An adult is worse off if the adult migrates to the other region. We assume that condition $\beta - (G/\alpha) + \gamma < 0$ is satisfied. The rural agglomeration is smaller than the urban congestion net of agglomeration, i.e., $\gamma < (G/\alpha) - \beta$. Nevertheless, because we must have $(\lambda <)\lambda + \gamma N_t < 0$ for N_{ut} to be positive in (8), B is sufficiently greater than R, i.e., the urban wage rate is sufficiently high to attract workers.

Given the migration-stability condition, we analyze the population dynamics of the economy for a given population size. First, from (8), we have $N_{ut} > 0$ only for $N_t < -\lambda/\gamma$. When $N_t \geq -\lambda/\gamma$, we have a corner solution $N_{ut} = 0$ and $N_{t+1} = n_r N_t$, where $n_r = (1-\alpha)/z$ is the rural fertility rate. All adults reside in the rural region. We assume here that $n_r > 1$.[1]

Second, from (8), we can obtain $N_t = N_{ut}$ when $N_t = \lambda/[\beta - (G/\alpha)]$. If $\lambda/(\beta - G/\alpha) > N_t$, then $N_{ut} > (\lambda + \gamma\frac{\lambda}{\beta - G/\alpha})/(\beta - G/\alpha + \gamma) = \lambda/(\beta - G/\alpha) > N_t$, which is impossible. Therefore, we also have a corner solution $N_t = N_{ut}$ for $\lambda/[\beta - (G/\alpha)] > N_t$. In this case, all adults choose to reside and work in the urban region. The evolution of the overall population is given as

[1] If $n_r \leq 1$, then the overall population does never increase.

$N_{t+1} = n_{ut}N_t = n_r \exp(-GN_t)N_t$.

Finally, because of the migration-stability condition (9), we have $0 < N_{ut} < N_t$ for $-\lambda/\gamma > N_t > \lambda/[\beta-(G/\alpha)]$. Adults are distributed both in urban and in rural regions. In this case, the evolution of population is given as

$$N_{t+1} = n_{ut}N_{ut} + n_r N_{rt}$$
$$= n_r\{-\frac{\lambda}{\Omega} + (1-\frac{\gamma}{\Omega})N_t + \frac{\lambda+\gamma N_t}{\Omega}\exp(-G\frac{\lambda+\gamma N_t}{\Omega})\}, \qquad (10)$$

where $\Omega = \beta-(G/\alpha)+\gamma < 0$. From (8) we obtain

$$\frac{d}{dN_t}(\frac{N_{ut}}{N_t}) = \frac{-\lambda}{(N_t)^2\Omega} < 0. \qquad (11)$$

The share of the urban population deceases with the overall population size. Increases (decreases) in the population size lower (raise) the urban share of population. Therefore, urbanization is associated with population contraction. It is noteworthy that this result runs against those presented in earlier reports of the conventional literature.

3. Dynamics

For analyses of the population dynamics, two possibilities can be considered at $N_t = \lambda/(\beta-G/\alpha)$: (i) $n_{ut} = n_r \exp[-G\lambda/(\beta-G/\alpha)] \geq 1$ and (ii) $n_{ut} = n_r \exp[-G\lambda/(\beta-G/\alpha)] < 1$.

In case (i), we have $N_{t+1} = n_{ut}N_t \geq N_t$ for $N_t \leq \lambda/(\beta-G/\alpha)$. When N_t becomes greater than $\lambda/(\beta-G/\alpha)$, the share of urban population decreases from (11). The dynamics runs against the facts reported in the Introduction. Therefore, this case is apparently implausible.

Next, in case (ii), a stationary equilibrium prevails because $N_{t+1} = n_r N_t > N_t$ at $N_t = -\lambda/\gamma$. In other words, a stationary equilibrium population size exists if condition $1 < n_r < \exp[\lambda G/(\beta-G/\alpha)]$ is satisfied. Curve $N_{t+1} = n_t N_t \equiv \Lambda(N_t)$ crosses the 45-degree line from below, as shown in Figure 4.[2] The stationary population equilibrium is unstable.[3] Urbanization therefore involves population contraction. After the population size shrinks to become smaller than $\lambda/(\beta-G/\alpha)$, the population is fully urbanized. It

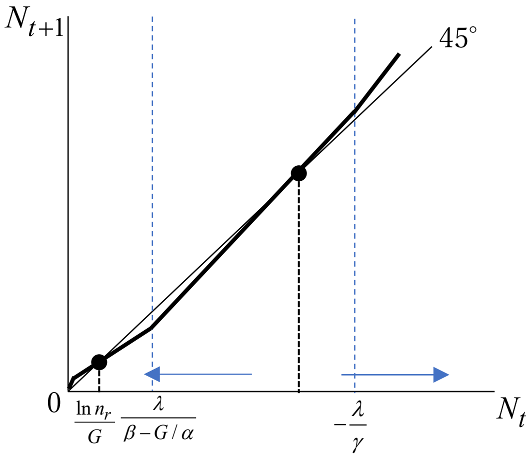
---

[2] Because all adults reside in rural areas when $N_t = -\lambda/\gamma$, we have $N_{t+1} = n_r N_t > N_t$.

[3] Curve $N_{t+1} = n_t N_t$ is presented as a line for expositional simplicity. However, it is generally nonlinear. Therefore, although the stationary equilibrium might be multiple, the equilibrium with the smallest population size between $\lambda/(\beta-G/\alpha)$ and $-\lambda/\gamma$ is unstable. At stable stationary equilibrium between $\lambda/(\beta-G/\alpha)$ and $-\lambda/\gamma$ if exists, there are regions of two types, rural and urban, as presented in Fig. A1.

declines further as time passes. However, we have $N_{t+1} = N_t$ at $N_t = \ln n_r / G$, where $n_r \exp(-GN_t) = 1$. Therefore, fully urbanized economies might have a stable stationary population.[4]

Fig. 4 Population dynamics.



Therefore, we have the following propositions.

Proposition 1. *Insofar as* $1 < n_r < \exp[\lambda G / (\beta - G / \alpha)]$ *is satisfied, a stationary equilibrium of population size exists. However, the stationary equilibrium with the smallest population size between* $\lambda / (\beta - G / \alpha)$ *and* $-\lambda / \gamma$ *is unstable.*

Proposition 2. *If the initial population size is greater than but close to* $\lambda / (\beta - G / \alpha) > 0$*, then there are two regions: urban and rural. The population eventually urbanizes fully; simultaneously, its size contracts over time. However, when* $N_t = \ln n_r / G$*, the fully urbanized population becomes stationary.*

The greater the degree of congestion diseconomy (G) exists, the wider the range of instability ($\ln n_r / G < N_t < \lambda / (\beta - G / \alpha)$) becomes, as depicted in Figure 4. The stationary population size is greater if the degree of congestion diseconomy is smaller and if the rural fertility rate without congestion is higher.

The intuition behind these results is the following. The initial population is smaller than the unstable stationary level: the population size decreases and converges toward a fully urbanized stable population. In fact, concentration of the population in urban

---

[4] Condition $\Lambda(N_t) = N_t$ is also satisfied at the origin in Fig. 4.

regions enhances agglomeration economies in the urban region, consequently raising the urban wage rate, but it also decreases agglomeration economies in rural areas, thereby lowering the rural wage rate. Higher wages attract workers to urban areas. Increases in the number of urban workers accompany a lower overall fertility rate because the urban fertility rate is lower.[5]

4. Concluding remarks

Results of this study have shown that population urbanization and contraction occur simultaneously in a simple model. However, after full urbanization, decreased population size mitigates congestion diseconomies, exerting positive effects on fertility, and eventually bringing the economy to a stable fully urbanized stationary population.

The simplicity of the model entails some important deficits. If the initial population size is sufficiently great, then the population might continue to increase. However, there must be a limit on the population size imposed by an implicit resource constraint, e.g., the land scale. Second, immigration might affect the population size and population dynamics in European countries. These are interesting subjects which shall be reserved for future research.

Acknowledgments

The second author acknowledges financial support from the Japan Society for the Promotion of Science KAKENHI [Grant No. 19H01503].

Declaration of interest

None.

References

Muroishi, M., Yakita, A. (2020) Agglomeration economies, congestion diseconomies, and fertility in a two-region economy. Lett. Spat. Resour. Sci., in press. https://doi.org/10.1007/s12076-020-00264z

Sato, Y. (2007) Economic geography, fertility and migration. J. Urban Econ. 61(2), 372-

[5] If the initial population size is smaller than $\ln n_r / G$, then the size increases to the stationary one with both agglomeration and congestion.

387. https://doi.org/10.1016/j.jue.2006.08.002

Sato, Y., Yamamoto, K. (2005) Population concentration, urbanization, and demographic transition. J. Urban Econ. 58(1), 45-61. https://doi.org/10.1016/j.jue.2005.01.0004

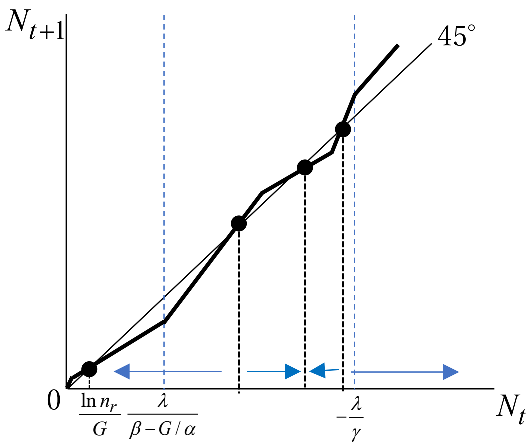
The World Urbanization Prospects (2018) The World Urbanization Prospects 2018. United Nations Department of Economic and Social Affairs, Population Division.

The World Population Prospects (2019) The World Population Prospects: The 2019 Revision. United Nations Department of Economic and Social Affairs, Population Division.

Yakita, S. (2011) Regional public goods, migration, and growth. Lett. Spat. Resour. Sci. 4(2), 129-138. https://di.org/10.1007/s12076-011-0057-9

Appendix figure

Fig. A1 Case of three stationary equilibrium of population size.

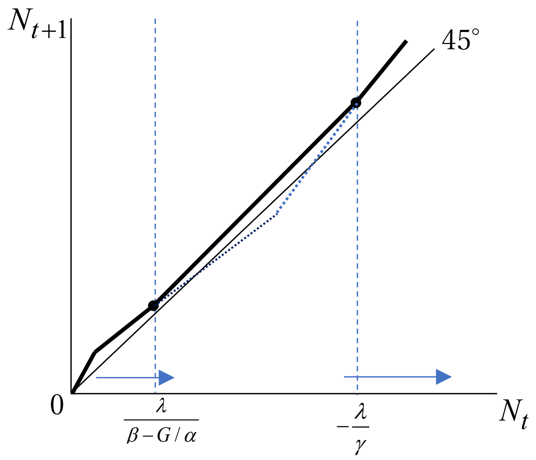


Supplementary note

S1. Dynamics – case (i)

Consider case (i) $n_{ut} = n_r \exp[-G\lambda/(\beta - G/\alpha)] \geq 1$ at $N_t = \lambda/(\beta - G/\alpha)$, which is labeled as "implausible" in the text In this case, we have $N_{t+1} = n_{ut}N_t = n_{ut}N_{ut} > N_t$, as described in the text. Therefore, the (total) population size increases. For $N_t > \lambda/(\beta - G/\alpha)$, some adults in the next generation choose to reside in rural areas. Because $n_{ut} = n_r \exp(-GN_{ut}) < n_r \exp[-G\lambda/(\beta - G/\alpha)]$, the urban fertility rate might decline. However, because $1 \leq n_r \exp[-G\lambda/(\beta - G/\alpha)] < n_r$ and because of (11), the overall fertility rate $n_t = (n_{ut}N_{ut} + n_r N_{rt})/N_t$ plausibly increases. As the population size increases, the urban population *share* decreases and the rural population *size* increases. The changed population shares increase the overall fertility rate and therefore the population size. At $N_t = -\lambda/\gamma$, we have $N_t = N_{rt}$. Therefore, $N_{t+1} = n_r N_t > N_t$ where $N_t = N_{rt}$ for $N_t \geq -\lambda/\gamma$. The population size monotonically increases over time. The dynamics of population size in this case can be presented in Figure S1.

Fig. S1 Population dynamics – case (i).



Because function $N_{t+1} = n_t N_t \equiv \Lambda(N_t)$ is not necessarily monotonic, we might have stationary population sizes between $\lambda/(\beta - G/\alpha)$ and $-\lambda/\gamma$. Such a case is represented by a dotted line in Figure S1. The stationary equilibrium with the smallest population size is stable. Adults reside both in rural and urban areas. In this case, the economy urbanizes with depopulation on the transition from a greater size of population to the stable equilibrium. We cannot exclude this possibility theoretically. However, it is noteworthy that a reverse urbanization occurs on the transition from $N_t = \lambda/(\beta - G/\alpha)$ to the stable equilibrium. The evidence in the Introduction does not necessarily support the case. Also, the high-income-country urban population share of 81 per cent in 2018 is expected to rise *further* to 88 per cent in 2050 in the UN World Urbanization Prospects 2018. (273 words)