Chukyo University Institute of Economics

Discussion Paper Series

December 2019

No. 1909

Ambient Charge Effects on Non-point Source Pollution in a Three-stage Game

Akio Matsumoto, Keiko Nakayama, Makoto Okamura
Ferenc Szidarovszky

Ambient Charge Effects on Non-point Source Pollution in a Three-stage Game

Akio Matsumoto* Keiko Nakayama† Makoto Okamura‡

Ferenc Szidarovszky§

Abstract

In the case of non-point source (NPS) pollutions the individual sources cannot be monitored, the regulator can observe only the total emission level. Therefore the standard emission controlling instruments cannot be applied. The regulator defines an emission standard and if the pollution concentration is above this standard, then the firms are uniformly taxed and if it is below the standard then the firms receive uniform subsidies. In the case of a duopoly this is a three person game between the firms and the regulator. The firms are profit maximizers, the regulator wants to maximize social welfare. A leader-follower model is defined where the regulator is the leader with defining the environmental standard, and the firms are the followers in finding their optimal output levels and abatement technologies. The firms use a two-stage process, first finding their optimal output levels and then determining the best abatement technologies with optimal output levels. The three stages are described and optimal choices are determined. It is also shown that the total emission level can be effectively controlled by the ambient taxes.

Keywords: Three-stage game, Leader-follower model, Non-point source Pollutions, Ambient charge, Cournot competition

*Department of Economics, Chuo University, 742-1, Higashi-Nakano, Hachioji, Tokyo, 192-0393, Japan, akiom@tamacc.chuo-u.ac.jp.

†Department of Economics, Chukyo Univesity, 101-2, Yagoto-Honmachi, Showa-ku, Nagoya, 466-8686, Japan. nakayama@mecl.chukyo-u.ac.jp.

‡Department of Economics, Gakushuin University, 1-5-1, Mejiro, Toshima-ki, Tokyo, 171-8588, Japan, andrew-uf@galaxy.plala.or.jp.

§Department of Mathematics, Corvinus University, Budapest, Fövám tér 8, 1093, Hungary, szidarka@gmail.com.

1 Introduction

In the literature of oligopoly theory a large variation of model variants has been developed and examined including models with and without product differentiation, multi-product and employee owned oligopolies among others. Important extensions include environmental regulations. The analysis of the effects of emission control has a long history and various kinds of regulations have been suggested (Downing and White, 1986; Jung et al., 1996, etc.). Montero (2002) examined the effects of R&D investments for different environmental policies in a duopoly market. Okuguchi and Szidarovszky (2002) considered oligopolies with pollution treatment cost sharing, while Okuguchi and Szidarovszky (2007) proved the existence of Cournot-Nash equilibrium, when the payoff of the firms included pollution control above an emission standard and pollution abatement R&D. They also examined their effects on the firms' profits. Chen et al. (2019) examined supply chains where penalizing each firm for the emission sometimes can lead to higher overall emission. Most of earlier studies assumed point source (PS) pollution, when the regulator was able to monitor the emission volume of each firm and regulations could be placed to each of them.

However considering environmental pollution today becomes much more complicated. Their source can be human or nature related. The negative health effects of air and water pollutions are well known and clearly demonstrated. More recently plastic pollution received high priority, since it can be found in almost everywhere, in national parks, rivers, lakes and oceans. It is also demonstrated that it has a huge effect on the health of animals and humans. In most cases it is impossible or unrealistic to find the individual pollutants, in which cases we talk about non-point source (NPS) pollutions. Wu and Chen (2013) compared the influence of PS and NPS pollutions on the water quality and determined that NPS pollution was the dominant contribution to nutrient loads. Segerson (1988) has proposed the monitoring of ambient concentrations of pollutants. To control pollutions the regulator defines an environmental standard level of pollution, and imposes uniform tax on the pollutants if the concentration is above the standard and gives uniform subsidies if it is below the standard. The main question arising in applying these ambient charges is to find conditions when pollution can be controlled. Ganguli and Raju (2012) considered and examined a Bertrand duopoly where an increased ambient charge could result in larger pollution. Raju and Ganguli (2013) investigated Cournot duopolies and numerically showed the effectiveness of ambient charges in a two-stage model. Sato (2017) explored this result in duopolies without product differentiation. In generalizing the earlier studies to n-firm oligopolies, Matsumoto et al. (2018) reexamined the static results and in a dynamic framework found stability and instability conditions. Ishikawa et al. (2019) demonstrated that in an n-firm Bertrand oligopoly the sign of the ambient charge effect depends on the number of firms, the degree of substitutability and the abatement technologies of the firms.

This paper introduces a three-stage Cournot duopoly with product differentiation, where the three decision variables are the production levels, the abatement technologies of the firms, and the ambient charge rate defined by the regulator. As usual, solving this three-stage game backwardly, we derive the sub-game perfect equilibrium of the game, that is, in the third stage the optimal output levels of the firms are determined with all other parameter being fixed and it is shown that higher ambient charges are effective in controlling the total emission of NPS pollutions. In the second stage the best ambient charge technologies of the firms are determined with optimal output levels. In the first stage the regulator determines the optimal rate of the ambient charge to maximize social welfare.

The paper is organized as follows. In Section 2 the optimal output levels of the firms are determined, in Section 3 the optimal abatement technologies are derived. In Section 4 the social welfare optimizing problem is solved. Conclusions and further research directions are given in Section 5.

2 Determinations in the Third-stage

Let us consider a Cournot duopoly market in which firm i produces a differentiated output q_i with a linear price function,

$$p_i = \alpha_i - q_i - \gamma q_j \tag{1}$$

where $\alpha_i > 0$ is the maximum price and $0 \leq \gamma \leq 1$ denotes the degree of product differentiation; two goods are substitutes if $0 < \gamma < 1$, homogeneous if $\gamma = 1$ and independent if $\gamma = 0$. Let ϕ_i denote the pollution abatement technology of firm i ($0 \leq \phi_i \leq 1$) with a pollution-free technology if $\phi_i = 0$ and a fully-discharge technology if $\phi_i = 1$. We make the following constraints on γ and ϕ_i to get rid of the extreme cases.

Assumption 1. (1) $0 < \gamma < 1$ and (2) $0 < \phi_i < 1$.

If firm i has the marginal cost c_i and the belief that the competitor's output will remain unchanged, then its profit at the third stage is

$$\pi_i(q_i, \phi_i) = (\alpha_i - q_i - \gamma q_j)\, q_i - c_i q_i - \theta(\phi_i q_i + \phi_j q_j - \bar{E}) \tag{2}$$

where $\bar{E}$ is the ambient standard and θ is the ambient tax rate.[1] To have positive profit in case of no pollutions, $\alpha_i > c_i$ is assumed. Differentiating the profit function of firm i with respect to q_i presents the first-order condition for an interior solution maximizing the profit of firm i,

$$\frac{\partial \pi_i}{\partial q_i} = \alpha_i - 2q_i - \gamma q_j - c_i - \theta\phi_i = 0. \tag{3}$$

[1] This rate is measured in some monetary unit per emission, it is positive and can be larger than unity (e.g., dollar/ton, yen/kg, etc.)

The first-order condition for firm j is similarly obtained. The optimal levels of outputs can be obtained by solving simultaneously the first-order conditions, which can be rewritten as

$$\begin{aligned} 2q_i + \gamma q_j &= \alpha_i - c_i - \theta\phi_i, \\ \gamma q_i + 2q_j &= \alpha_j - c_j - \theta\phi_j. \end{aligned} \tag{4}$$

Let $\beta_k = \alpha_k - c_k > 0$ for $k = i, j$ and assume the following for the sake of analytical simplicity[2].

Assumption 2. $\beta_i = \beta_j = \beta$.

Equations (4) can be compactly rewritten as

$$\begin{pmatrix} 2 & \gamma \\ \gamma & 2 \end{pmatrix} \begin{pmatrix} q_i \\ q_j \end{pmatrix} = \begin{pmatrix} \beta - \theta\phi_i \\ \beta - \theta\phi_j \end{pmatrix}. \tag{5}$$

Solving (5) gives the optimal levels of outputs,

$$\begin{aligned} q_i^*(\theta, \phi_i, \phi_j) &= \frac{(2-\gamma)\,\beta + \theta\left(\gamma\phi_j - 2\phi_i\right)}{4-\gamma^2}, \\ q_j^*(\theta, \phi_i, \phi_j) &= \frac{(2-\gamma)\,\beta + \theta\left(\gamma\phi_i - 2\phi_j\right)}{4-\gamma^2}. \end{aligned} \tag{6}$$

To have non-negative output, we have the following inequality conditions, first for $q_i^* \geq 0$,

$$\phi_j \geq \frac{2}{\gamma}\phi_i - \frac{(2-\gamma)\,\beta}{\gamma\theta} \tag{7}$$

and then for $q_j^* \geq 0$,

$$\phi_j \leq \frac{\gamma}{2}\phi_i + \frac{(2-\gamma)\,\beta}{2\theta}. \tag{8}$$

Since the ambient technologies and the ambient tax rate are given, we call the third-stage the *short-run*. Hence the non-negativity of output is summarized as follow:

Lemma 1 *In the short-run, the optimal Cournot outputs are non-negative if the following inequalities hold,*

$$\frac{2}{\gamma}\phi_i - \frac{(2-\gamma)\,\beta}{\gamma\theta} \leq \phi_j \leq \frac{\gamma}{2}\phi_i + \frac{(2-\gamma)\,\beta}{2\theta}. \tag{9}$$

[2] This is a rather strong assumption however slightly weaker than assuming $\alpha_i = \alpha_j = \alpha$, $c_i = c_j = c$ and $\alpha - c > 0$.

The total amount of NPS pollution at the Cournot equilibrium point[3] is

$$E^*(\theta) = \phi_i q_i^*(\theta) + \phi_j q_j^*(\theta) \tag{10}$$

for which we have the following:

Theorem 1 *In short-run, the ambient charge is effective in controlling the total emission of NPS pollutions,*

$$\frac{\partial E^*(\theta)}{\partial \theta} < 0.$$

Proof. Substituting the optimal outputs q_i^* and q_j^* into (10) and then differentiating it with respect to θ presents

$$\begin{aligned} \frac{\partial E^*(\theta)}{\partial \theta} &= \frac{-2}{4-\gamma^2}\left(\phi_i^2 + \phi_j^2 - \gamma\phi_i\phi_j\right) \\ &= -\frac{2}{4-\gamma^2}\left[\left(\phi_i - \phi_j\right)^2 + (2-\gamma)\phi_i\phi_j\right] < 0 \end{aligned}$$

where the inequality is due to Assumption 1. ■

3 Determinations in the Second-stage

In this section, we determine the optimal abatement technology for each firm. Substituting the optimal level of output in (6) into the profit function (2) and adding an installation cost of technology, $(1-\phi_i)^2$, yield the reduced form of the profit function of firm i,

$$\pi_i^*(\phi_i, \phi_j) = \left(\alpha_i - q_i^* - \gamma q_j^*\right) q_i^* - c_i q_i^* - (1-\phi_i)^2 - \theta\left[\sum_{k=i}^{j} \phi_k q_k^* - \bar{E}\right] \tag{11}$$

Differentiating the resultant expression with respect to ϕ_i yields the first-order condition for the profit maximization of firm i,

$$\frac{\partial \pi_i^*}{\partial \phi_i} = \frac{\partial \pi_i^*}{\partial q_i}\frac{\partial q_i^*}{\partial \phi_i} + \frac{\partial \pi_i^*}{\partial q_j}\frac{\partial q_j^*}{\partial \phi_i} + \left.\frac{\partial \pi_i^*}{\partial \phi_i}\right|_{q_i^*, q_j^* : const} = 0 \tag{12}$$

[3] The arguments of ϕ_i and ϕ_j in the optimal outputs defined in (6) is omitted only for the sake of notational simplicity.

where

$$\frac{\partial \pi_i^*}{\partial q_i} = \alpha_i - 2q_i^* - \gamma q_j^* - c_i - \theta\phi_i = 0,$$

$$\frac{\partial \pi_i^*}{\partial q_j} = -\gamma q_i^* - \theta\phi_j,$$

$$\frac{\partial q_i^*}{\partial \phi_i} = -\frac{2\theta}{4-\gamma^2}$$

$$\frac{\partial q_j^*}{\partial \phi_i} = \frac{\gamma\theta}{4-\gamma^2},$$

$$\left.\frac{\partial \pi_i^*}{\partial \phi_i}\right|_{q_i^*, q_j^*: const} = 2(1-\phi_k) - \theta q_i^*.$$

The second-order conditions (SOC henceforth) for firms i and j are the same,

$$\frac{\partial^2 \pi_i^*}{\partial \phi_i^2} = \frac{\partial^2 \pi_j^*}{\partial \phi_j^2} = \frac{8\theta^2}{\left(4-\gamma^2\right)^2} - 2 < 0$$

where the inequality holds if

$$\theta < \frac{4-\gamma^2}{2}. \tag{13}$$

Rearranging the terms in (12) simplifies the form of the first-order condition for firm i as

$$2\left[\left(4-\gamma^2\right)^2 - (2\theta)^2\right]\phi_i + \left(8-\gamma^2\right)\gamma\theta^2\phi_j = 4\left(\gamma-2\right)\beta\theta + 2\left(4-\gamma^2\right)^2.$$

In the same way, the first-order condition for firm j is obtained as

$$\left(8-\gamma^2\right)\gamma\theta^2\phi_i + 2\left[\left(4-\gamma^2\right)^2 - (2\theta)^2\right]\phi_j = 4\left(\gamma-2\right)\beta\theta + 2\left(4-\gamma^2\right)^2.$$

These two equations are put into a matrix form

$$\begin{pmatrix} 2\left[(4-\gamma^2)^2 - (2\theta)^2\right] & \left(8-\gamma^2\right)\gamma\theta^2 \\ \left(8-\gamma^2\right)\gamma\theta^2 & 2\left[(4-\gamma^2)^2 - (2\theta)^2\right] \end{pmatrix} \begin{pmatrix} \phi_i \\ \phi_j \end{pmatrix} = \begin{pmatrix} K \\ K \end{pmatrix} \tag{14}$$

with

$$K = 4\left(\gamma-2\right)\beta\theta + 2\left(4-\gamma^2\right)^2.$$

Then (14) is solved for optimal abatement technologies of firms i and j,

$$\phi_i^* = \phi_j^* = \phi^e(\theta) = \frac{2(2-\gamma)\left(2+\gamma\right)^2 - 4\beta\theta}{2(2-\gamma)\left(2+\gamma\right)^2 - (4-2\gamma-\gamma^2)\theta^2}. \tag{15}$$

Notice that each firm selects the same technology due to Assumption 2.

We now verify circumstances under which $0 < \phi^e < 1$ holds. The condition for $\phi^e > 0$ is that both of the denominator and the numerator of (15) are either positive or negative. The denominator is positive if

$$\theta < f_1(\gamma) = \sqrt{\frac{2(2-\gamma)(2+\gamma)^2}{(4-2\gamma-\gamma^2)}} \tag{16}$$

with

$$f_1(0) = 2 \text{ and } f_1(1) = 3\sqrt{2}\,(\simeq 4.243)$$

and negative if the inequality of (16) is reversed. The numerator is positive if

$$\theta < f_2(\gamma, \beta) = \frac{(2-\gamma)(2+\gamma)^2}{2\beta} \tag{17}$$

with

$$f_2(0, \beta) = \frac{4}{\beta},\ f_2(1, \beta) = \frac{9/2}{\beta} \text{ and } \frac{\partial f_2}{\partial \beta} < 0$$

and negative if the inequality of (17) is reversed.

For $\phi^e < 1$, we have two conditions as well. One of them is that in addition to the positivity of the denominator and the numerator (15), the denominator is larger than the numerator,

$$\theta < f_3(\gamma, \beta) = \frac{4\beta}{4-2\gamma-\gamma^2} \tag{18}$$

where

$$f_3(0, \beta) = \beta,\ f_3(1, \beta) = 4\beta \text{ and } \frac{\partial f_3}{\partial \beta} > 0.$$

The other condition is that both of the denominator and the numerator are negative and the absolute value of the denominator is larger than the absolute value of the numerator under which the inequality of (18) is reversed. Finally, to check the SOC, we denote the right hand side of (13) as

$$f_4(\gamma) = \frac{4-\gamma^2}{2} \tag{19}$$

with

$$f_4(0) = 2 \text{ and } f_4(1) = \frac{3}{2}.$$

In the special case of $\beta = 2$, we have the following relative size among f_k for $k = 1, 2, 3, 4$,

$$f_4(\gamma) < f_3(\gamma, \beta) < f_1(\gamma) < f_2(\gamma, \beta) \text{ for } \gamma \in (0, 1)$$

with

$$f_1(0) = f_2(0, \beta) = f_3(0, \beta) = f_4(0) = 2.$$

These results are represented in Figure 1 in which the blue and green curves describe the loci of $\theta = f_2(\gamma, \beta)$ and $\theta = f_3(\gamma, \beta)$ whereas the upward-sloping and downward-sloping black curves illustrate the loci of $\theta = f_1(\gamma)$ and $\theta = f_4(\gamma)$, respectively. The region below the $\theta = f_4(\gamma)$ curve is colored in yellow in which the SOC is fulfilled. Hence, for (γ, θ) in the yellow region,

$$\theta < f_2(\gamma, \beta) < f_1(\gamma) \text{ hold, implying } \phi^e > 0 \text{ due to (16) and (17)}$$

and

$$\theta < f_3(\gamma, \beta) \text{ holds, implying } \phi^e < 1 \text{ due to (18).}$$

We have, therefore, the benchmark result in which we call the second stage the *medium-run* in which θ is fixed.

Lemma 2 *In the medium-run with $\beta = 2$, the optimal level of the abatement technology is positive and less than unity (i.e., $0 < \phi^e < 1$) if the SOC is satisfied.*

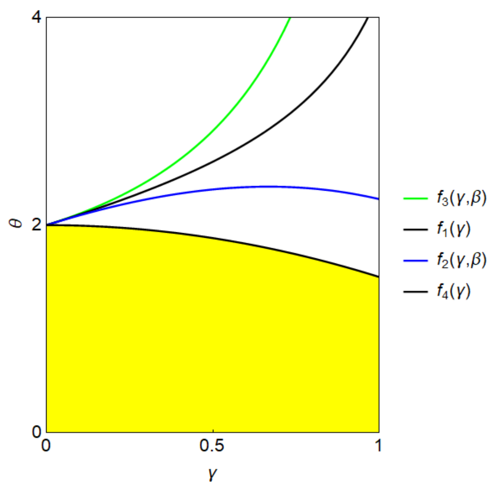


Figure 1. Relative location of f_k for $k = 1, 2, 3, 4$ and $\beta = 2$

We next validate the conditions for which $0 < \phi^e < 1$ holds when $\beta \neq 2$. The locations of the two black curves are independent from β whereas those of the green and blue curves depend on it (i.e., $\partial f_2/\partial\beta < 0$ and $\partial f_3/\partial\beta > 0$). Considering the directions of these derivatives, we turn attention to two cases, one with $\beta < 2$ and the other with $\beta > 2$.

Case I: $0 < \beta < 2$

Decreasing the value of β from 2 shifts the blue curve upward and the green curve downward. We can eliminate the blue curve out of considerations (i.e., positivity of ϕ^e is guaranteed). Since the green curve describes $\theta = f_3(\gamma, \beta)$ with which the optimal level of the abatement technology is unity, the yellow region of Figure 1 is divided into two subregions, one in which $\phi^e \geq 1$ and the other in which $\phi^e < 1$. The former violates Assumption 1(2) and should be discarded. To see this division more clearly, we first solve $f_3(1, \beta) = f_4(1)$ for β to have the critical value $\beta_a = 3/8 (= 0.375)$. Then we can confirm the followings:

(i) if $\beta_a < \beta < 2$, then the green curve crosses the lower black curve locus for some value $0 < \tilde{\gamma}(\beta) < 1$ at which $f_3(\tilde{\gamma}(\beta), \beta) = f_4(\tilde{\gamma}(\beta))$ holds;[4]

(ii) if $0 < \beta \leq \beta_a$, then the green curve does not cross the lower black curve for $0 < \gamma < 1$ and hence $\tilde{\gamma}(\beta) \geq 1$.

To sum up, we have the following in case of $\beta < 2$:

Lemma 3 *Given $\gamma \in (0, 1)$, the optimal level of the abatement technology satisfies the condition, $0 < \phi^e < 1$, if $\beta_a < \beta < 2$ and $\theta < \min[f_3(\gamma, \beta), f_4(\gamma)]$ or if $0 < \beta \leq \beta_a$ and $\theta < f_3(\gamma, \beta)$.*

As an example, we take $\beta = 3\sqrt{2}/4 \,(\simeq 1.061) > \beta_a$ in Figure 2(A) in which the solid green curve crosses the downward sloping black curve at $\tilde{\gamma} \simeq 0.627$. The feasible region of (γ, θ) is constructed by

$$\theta < f_3(\gamma, \beta) \text{ and } 0 < \gamma < \tilde{\gamma} \text{ and } \theta < f_4(\gamma) \text{ and } \tilde{\gamma} < \gamma < 1$$

where the less-than-unity condition is violated in the yellow region with slants between $\theta = f_3(\gamma, \beta)$ and $\theta = f_4(\gamma)$. Notice that the dotted green curve illustrates the $\theta = f_3(\gamma, \beta)$ locus with $\beta = \beta_a$. For $\beta < \beta_a$, the feasible region of (γ, θ) is below the $\theta = f_3(\gamma, \beta)$ locus that is located below the dotted green curve.

Case II: $\beta > 2$

Increasing the value of β from 2 shifts the green curve upward and the blue curve downward. The roles of the blue and green curves in Case I is interchanged in Case II. Since the blue curve describes $\theta = f_2(\gamma, \beta)$ for which the optimal level of abatement technology is zero, the yellow region of Figure 1 is divided into two subregions as well, one in which $\phi^e \leq 0$ and the other in which $\phi^e > 0$. The former violates Assumption 1(2) and should be discarded. To see this division more clearly, we first solve $f_2(1, \beta) = f_4(1)$ for β to have the critical value $\beta_b = 3$. The dotted blue curve is the locus of $\theta = f_2(\gamma, \beta_b)$. Hence we can confirm the followings:

[4] Based on the definitions of $f_3(\gamma, \beta)$ and $f_4(\gamma)$, it is seen that $\tilde{\gamma}(\beta)$ is a solution of a quartic equation. It is possible to have its explicit form. Neverthless, it is not presented here as it is long and clumsy.

(iii) if $2 < \beta < \beta_b$, then the blue curve crosses the lower black curve locus for some value $0 < \bar{\gamma}(\beta) < 1$ at which $f_2(\bar{\gamma}(\beta), \beta) = f_4(\bar{\gamma}(\beta))$ holds;[5]

(iv) if $\beta_b \geq \beta$, then the blue curve does not cross the lower black curve for $0 < \gamma < 1$, implying that $\bar{\gamma}(\beta) \geq 1$.

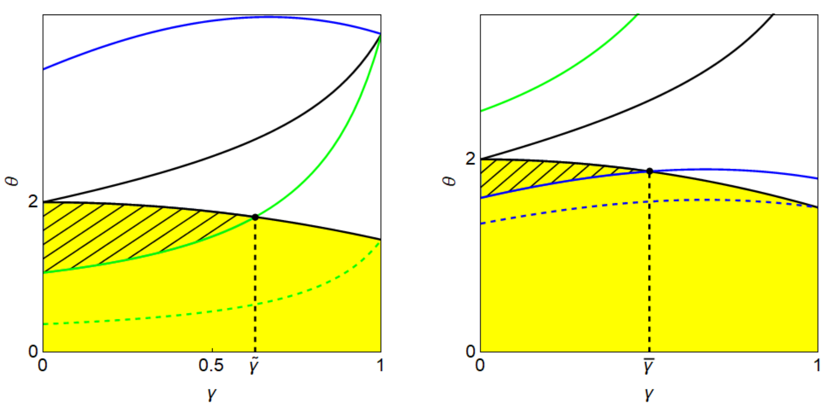
To sum up, we have the followings in Case 2.

Lemma 4 *Given $\gamma < (0, 1)$, the optimal level of the abatement technology satisfies $0 < \phi^e < 1$ if $2 < \beta < \beta_b$ and $\theta < \min[f_2(\gamma, \beta), f_4(\gamma)]$ or if $\beta > \beta_b$ and $\theta < f_2(\gamma, \beta)$.*

As an example, we take $\beta = 5/2 < \beta_b$ in Figure 2(B) in which the solid blue curve crosses the downward sloping black curve at $\bar{\gamma} \simeq 0.5$. Notice that (γ, θ) in the feasible region satisfies

$$\theta < f_2(\gamma, \beta) \text{ and } 0 < \gamma < \bar{\gamma} \text{ and } \theta < f_4(\gamma) \text{ and } \bar{\gamma} < \gamma < 1$$

where the positivity condition is violated in the yellow region with slants between $\theta = f_2(\gamma, \beta)$ and $\theta = f_4(\gamma)$. Notice that the dotted blue curve illustrates the $\theta = f_2(\gamma, \beta_b)$ locus. For $\beta > \beta_b$, the feasible region is below the $\theta = f_2(\gamma, \beta)$ locus that is also located below the dotted blue curve.



(A) $\beta = 3\sqrt{2}/4$ (B) $\beta = 5/2$

Figure 2. The feasible region of (γ, β) for $0 < \phi^e < 1$

From these lemmas, we have the following:

[5] Based on the definitions of $f_3(\gamma, \beta)$ and $f_4(\gamma)$, it is seen that $\tilde{\gamma}(\beta)$ is a solution of a quartic equation. It is possible to have its explicit form. Neverthless, it is not presented here as it is clumsy.

Theorem 2 *In the medium-run in which θ is given, the optimal abatement technology $\phi^e(\theta)$ is positive and less than unity if one of the following four conditions is satisfied for $0 < \gamma < 1$,*

(i) $0 < \beta \leq \beta_a$ *and* $\theta < f_2(\gamma)$,

(ii) $\beta_a < \beta < 2$ *and* $\theta < \min[f_3(\gamma, \beta), f_4(\gamma)]$,

(iii) $2 < \beta < \beta_b$ *and* $\theta < \min[f_2(\gamma, \beta), f_4(\gamma)]$,

(iv) $\beta_b \leq \beta$ *and* $\theta < f_2(\gamma)$.

The optimal production given in (6) is reduced to

$$q_k^*(\theta, \phi_i^*(\theta), \phi_j^*(\theta)) = q_k^*(\theta, \phi^e(\theta), \phi^e(\theta)) = q^e(\theta) \text{ for } k = i, j$$

where the reduced form of the optimal production is

$$q^e(\theta) = \frac{\beta - \theta\phi^e(\theta)}{2 + \gamma}. \tag{20}$$

and the corresponding prices are

$$p_i^e(\theta) = \alpha_i - (1 + \gamma)q^e(\theta). \tag{21}$$

It can be shown that the price is positive when $q^e(\theta) > 0$. Substituting (20) into (21) and arranging the terms yield

$$p_i^e(\theta) = \frac{\alpha_i + (1 + \gamma)\left[c_i + \theta\phi^e(\theta)\right]}{2 + \gamma} > 0.$$

The total amount of NPS pollution is

$$E^e(\theta) = 2\phi^e(\theta)q^e(\theta). \tag{22}$$

Differentiating (22) with respect to θ and then arranging the terms yield[6]

$$\frac{\partial E^e}{\partial \theta} = -\frac{4\left[\ell(\gamma, \theta)\beta^2 - m(\gamma, \theta)\beta + n(\gamma, \theta)\right]}{\left[2(2 - \gamma)(2 + \gamma)^2 - (4 - 2\gamma - \gamma^2)\theta^2\right]^3} \tag{23}$$

where the inequalities of the following forms are due to Assumption 1(1),

$$\ell(\gamma, \theta) = 4(2 - \gamma)^2(2 + \gamma)^3 + \theta^2\left[24(4 - \gamma^2) + (4 - 2\gamma - \gamma^2)\gamma\theta^2\right] > 0$$

$$m(\gamma, \theta) = (2-\gamma)(2+\gamma)\theta\left[2(2 - \gamma)(2 + \gamma)^2(12 - 2\gamma - \gamma^2) + (4 - 2\gamma - \gamma^2)(4 + \gamma^2 + 2\gamma)\theta^2\right] > 0$$

and

$$n(\gamma, \theta) = (2 - \gamma)^2(2 + \gamma)^3\left[2(2 - \gamma)(2 + \gamma)^2 + 3(4 - 2\gamma - \gamma^2)\theta^2\right] > 0.$$

[6] The calculations have been done with Mathematica, version 11.

The denominator is positive if the SOC is fulfilled. The expression in the square brackets of the numerator is a quadratic polynomial in β and its discriminant is

$$\begin{aligned} \boldsymbol{D} &= [m(\gamma,\theta)]^2 - 4\ell(\gamma,\theta)n(\gamma,\theta) \\ &= (4-\gamma^2)^2\left[-2\gamma^3 + \left(4-\gamma^2\right)\left(4-\theta^2\right) + 2\gamma(4+\theta^2)\right]^2\psi(\gamma,\theta) \end{aligned} \tag{24}$$

where

$$\psi(\gamma,\theta) = \left(16 - 8\gamma + 4\gamma^3 + \gamma^4\right)\theta^2 - 8(2-\gamma)(2+\gamma)^2.$$

In the last line of (24), its right hand side is described by the product of three factors and the first two factors are clearly positive while the sign of the last factor $\psi(\gamma,\theta)$ seems to be ambiguous, however, it can be shown to be negative. To this end, we rewrite the form of $\psi(\gamma,\theta)$ as

$$\left(16 - 8\gamma + 4\gamma^3 + \gamma^4\right)\left(\theta + \sqrt{\frac{8(2-\gamma)(2+\gamma)^2}{16-8\gamma+4\gamma^3+\gamma^4}}\right)\left(\theta - \sqrt{\frac{8(2-\gamma)(2+\gamma)^2}{16-8\gamma+4\gamma^3+\gamma^4}}\right)$$

where the first two factors are positive and the last factor is shown to be negative if the SOC is satisfied. Hence, we have the following:

Lemma 5 *$\psi(\gamma,\theta) < 0$ for the values of γ and θ satisfying the SOC of the optimal production.*

Proof. Using the square of $f_4(\gamma)$, we have

$$\frac{8(2-\gamma)(2+\gamma)^2}{16-8\gamma+4\gamma^3+\gamma^4} - \left(\frac{4-\gamma^2}{2}\right)^2 = \frac{(2-\gamma)\gamma(2+\gamma)^2\left(32 - 8\gamma - 8\gamma^2 + 2\gamma^3 + \gamma^4\right)}{4\left(16-8\gamma+4\gamma^3+\gamma^4\right)} > 0$$

that implies

$$f_4(\gamma) = \frac{4-\gamma^2}{2} < \sqrt{\frac{8(2-\gamma)(2+\gamma)^2}{16-8\gamma+4\gamma^3+\gamma^4}}.$$

Since $\theta < f_4(\gamma)$ to satisfy the SOC, we have

$$\theta < \sqrt{\frac{8(2-\gamma)(2+\gamma)^2}{16-8\gamma+4\gamma^3+\gamma^4}}.$$

This inequality leads to $\psi(\gamma,\theta) < 0$. ■

Since $\psi(\gamma,\theta) < 0$, the discriminant (24) is negative, implying that the numerator of (22) is positive. Therefore we have the following:

Theorem 3 *In the medium-run, the ambient charge is effective in controlling the total amount of NPS pollution,*

$$\frac{dE^e}{d\theta} < 0.$$

4 Determinations in the Third-stage

It is assumed that the regulator determines the optimal rate of the ambient charges to maximize the social welfare

$$W = CS + PS + T - D \tag{25}$$

where CS, PS, T and D stand for consumer surplus, producer surplus, tax collected with pollution emission and the damage caused by NPS pollutions, respectively.[7]

The representative consumer maximizes

$$U(q_i, q_j) - \sum_{k=i}^{j} p_k q_k$$

where the utility function U is assumed to be quadratic and strictly concave,

$$U(q_i, q_j) = \alpha_i q_i + \alpha_j q_j - \frac{1}{2}\left(q_i^2 + 2\gamma q_i q_j + q_j^2\right) + m$$

with m being the numeraire good. Maximizing this utility gives rise to the inverse demand functions in (1). Hence the consumer surplus is

$$CS(\theta) = \left[U(q^e(\theta), q^e(\theta)) - U(0,0)\right] - \left[p_i^e(\theta) + p_j^e(\theta)\right] q^e(\theta)$$

where the first term indicates the total increase of utility caused by consuming $q^e(\theta)$ of each good and the second term is the total expenditure by the consumers. The producer surplus is the sum of the profits gained by the firms

$$\begin{aligned} PS(\theta) &= \pi_i^e(\theta) + \pi_j^e(\theta) \\ &= \sum_{k=i}^{j} \left(p_k^e(\theta) - c_k\right) q^e(\theta) - 2(1-\phi^e(\theta))^2 - 2\theta\left[E^e(\theta) - \bar{E}\right]. \end{aligned}$$

The net tax revenue is

$$T(\theta) = 2\theta\left[E^e(\theta) - \bar{E}\right].$$

The damage is a linear function of the total amount of NPS pollution where $\delta = 1$ is assumed for analytical simplicity,[8]

$$D(\theta) = \delta E^e(\theta)$$

Hence the form of the welfare function is reduced to

$$W(\theta) = 2\beta q^e(\theta) - (1+\gamma)\left[q^e(\theta)\right]^2 - 2(1-\phi^e(\theta))^2 - 2\phi^e(\theta) q^e(\theta). \tag{26}$$

[7] The emission standard $\bar{E}$ is another direct regulation of the regulator. Its level should be determined so as to maximize the net social gains that will be considered in a future study.

[8] Notice first that the qualitative aspects of the results to be obtained are not affected by $\delta = 1$. Notice second that $E^e(\theta)$ could be negative, depending on a choice of the parameter values, γ and β. Hence, if the damage is assumed to be quadratic, which is often adoped in the literature, the welfare function could involve the exaggerated damage.

Differentiating (26) with respect to θ and arranging the terms yield the following form,

$$\frac{dW(\theta)}{d\theta} = \frac{\varphi(\gamma,\theta,\beta)}{\left[2(2-\gamma)(2+\gamma)^2 - (4-2\gamma-\gamma^2)\theta^2\right]^3} \tag{27}$$

where

$$\varphi(\gamma,\theta,\beta) = 4\left[\alpha_0(\gamma,\theta)\beta^2 + \alpha_1(\gamma,\theta)\beta + \alpha_2(\gamma,\theta)\right]$$

with

$$\begin{aligned}\alpha_0(\gamma,\theta) &= 8(2-\gamma)^2(2+\gamma)^3 - 16(2-\gamma)\gamma(2+\gamma)^2\theta + 48(2-\gamma)(2+\gamma)\theta^2 - \\ &\quad 8(24-8\gamma-6\gamma^2+\gamma^3)\theta^3 + 2\gamma(4-2\gamma-\gamma^2)\theta^4,\end{aligned}$$

$$\begin{aligned}&\alpha_1(\gamma,\theta) = -2(2-\gamma)(2+\gamma)^2 - 4(2-\gamma)^2(2+\gamma)^3(12-2\gamma-\gamma^2)\theta + \\ &24(2-\gamma)(2+\gamma)^3(3-\gamma)\theta^2 - 2(2-\gamma)(2+\gamma)(4-2\gamma-\gamma^2)(4+2\gamma+\gamma^2)\theta^3 + \\ &(4-2\gamma-\gamma^2)(32-12\gamma-8\gamma^2+\gamma^3)\theta^4\end{aligned}$$

and

$$\begin{aligned}\alpha_2(\gamma,\theta) &= -2(2-\gamma)^2(2+\gamma)^3 + 2(2-\gamma)^2(2+\gamma)^2(1+\gamma)\theta - \\ &\quad 3(2-\gamma)(2+\gamma)(4-2\gamma-\gamma^2)\theta^2 + (4-2\gamma-\gamma^2)(10-3\gamma-3\gamma^2)\theta^3.\end{aligned}$$

It is already confirmed that the denominator of (27) is positive as far as the SOC is satisfied. The numerator $\varphi(\gamma,\theta,\beta)$ has a long and clumsy form and seems analytically intractable. Hence in the following, we numerically consider the determination of the optimal rate of the ambient charge. Notice that $\varphi(\gamma,\theta,\beta) = 0$ implies $dW(\theta)/d\theta = 0$. In the following, we divide the feasible interval of β into three subintervals, $0 < \beta < 1$, $1 \leq \beta < 2$ and $2 \leq \beta$ and then determine the optimal θ in each of these intervals.

Interval I: $0 < \beta < 1$

Figure 3 presents two examples for determining the optimal tax rate θ under $0 < \beta < 1$. The first example assumes $\beta = 0.9$ with which the gently sloping solid red curve, the steeper positive-sloping dashed red curve and the flatter positive-sloping dotted red curve illustrate, respectively, the loci of $0 = \varphi(\gamma,\theta,\beta)$ in (27), $\theta = f_3(\gamma,\beta)$ in (18) and $0 = q^e(\gamma,\theta,\beta)$ in (20).[9] Rearranging the numerator of (20), we can rewrite the zero-production curve as

$$\beta\gamma\theta^2 - 2(2-\gamma)(2+\gamma)\theta + 2(2-\gamma)(2+\gamma) = 0$$

that is solved for θ to have a smaller root,

$$\theta = f_5(\gamma,\beta) = \frac{4-\gamma^2 - \sqrt{(4-\gamma^2)(4-\gamma^2-2\gamma\beta^2)}}{\beta\gamma}. \tag{28}$$

[9] To clearify the parameter dependency, $q^e(\theta)$ is replaced with $q^e(\gamma,\theta,\beta)$ in this section.

This is an alternative form of the dotted curve. Given β, the regulator determines the optimal ambient tax rate by solving $0 = \varphi(\gamma, \theta, \beta)$ for $0 < \gamma < 1$. However, it is not sure whether the firms also can make the optimal choices under the selected θ. Since $\theta < f_2(\gamma, \beta) < f_1(\gamma)$ hold for $\beta = 0.9$, the optimal technology satisfying $0 < \phi^e < 1$ is obtained in the region under the dashed curve in which $\theta < f_2(\gamma, \beta)$. The optimal output is positive in the region under the dotted curve that crosses the solid curve at point $a = (\gamma_a, \theta_a)$ with

$$\gamma_a \simeq 0.745 \text{ and } \theta_a \simeq 0.997.$$

Hence if θ is selected from the segment ab of the solid curve against $\gamma \in (\gamma_a, 1)$, then firm i can determine the optimal technology and produce positive output $q^e(\gamma, \theta, \beta) > 0$ and sell it with positive price $p_i^e(\gamma, \theta, \beta) > 0$.

The length of the feasible segment of the solid curve depends on the values of the parameters. The second example decreases the value of β to $\beta_L \simeq 0.796$ for which such a feasible segment shrinks. In Figure 3, the gently sloping solid blue curve, the steeper positive-sloping dashed blue curve and the flatter positive sloping dotted blue curve present, respectively, the loci of $0 = \varphi(\gamma, \theta, \beta)$, $\theta = f_3(\gamma, \beta)$ and $0 = q^e(\gamma, \theta, \beta)$ with this new value of β. As can be seen, the solid curve crosses the dotted curve at point $c = (1, \theta_c)$ with $\theta_c \simeq 0.905$ on the vertical line at $\gamma = 1$. Alternatively put, β_L together with θ_c solves $0 = \varphi(1, \theta, \beta)$ and $0 = \varphi(1, \theta, \beta)$ simultaneously. It can be observed that decreasing β from β_L shifts the solid curve upward and the dotted curve downward. As a natural consequence, the crossing of these two curves does not occur for $\gamma \in (0, 1)$ if $\beta < \beta_L$, implying $q^e(\gamma, \theta, \beta) < 0$ along the $0 = \varphi(\gamma, \theta, \beta)$ curve. Roughly speaking, when $0 < \beta < \beta_L$, the optimal tax rate is too strict for the firms to produce positive outputs. We summarize these results as follows:

Lemma 6 *For $\beta_L < \beta < 1$, the firms produce positive production $q^e(\gamma, \theta, \beta) > 0$ if $0 = \varphi(\gamma, \theta, \beta)$ and $\theta < f_5(\gamma, \beta)$.*

Lemma 7 *For $0 < \beta \leq \beta_L$, the optimal production is non-positive for γ and θ satisfying $0 = \varphi(\gamma, \theta, \beta)$.*

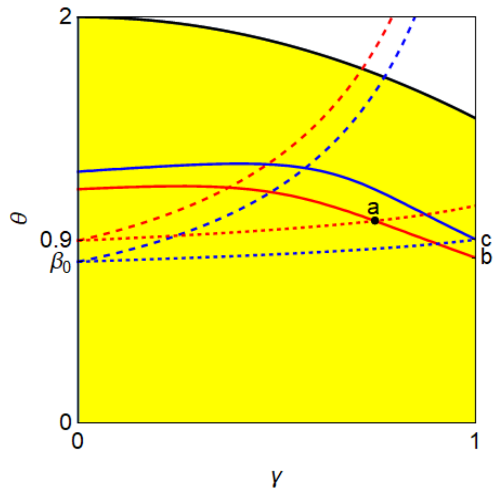


Figure 3. Optimal θ under $0 < \beta < 1$

Interval II: $1 \leq \beta < 2$

Figure 4 provides three examples. As in Figure 3, the solid curve, the dashed curve and the dotted curve correspond to the loci of $0 = \varphi(\gamma, \theta, \beta)$, $\theta = f_3(\gamma, \beta)$ and $0 = q^e(\gamma, \theta, \beta)$. The red, blue and green curves are illustrated with $\beta = 1$, $\beta = 1.3$ and $\beta = 1.6$. It is seen that the dashed and dotted curves are located above the corresponding solid curves in the same color. This implies that $0 < \phi^e(\gamma, \theta, \beta) < 1$ and $q^e(\gamma, \theta, \beta) > 0$ are guaranteed if the optimal θ is selected from the solid curve against $\gamma \in (0, 1)$. To put it in another way, the solid curve that determines the optimal θ is located below the $\theta = 1$ line and $\beta > 1$. In consequence, $\beta/\theta > 1$ and thus $q^e(\gamma, \theta, \beta) > 0$ from (20).

Lemma 8 *For $1 \leq \beta < 2$, the firms can select the optimal abatement technology and produce positive productions under the optimal ambient tax rate.*

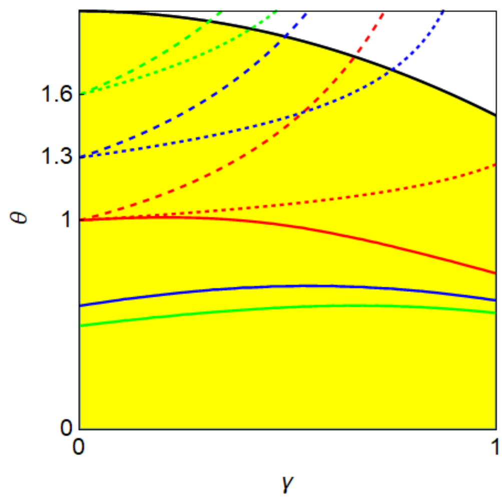


Figure 4. Optimal θ under $1 \leq \beta < 2$

Interval III: $\beta \geq 2$

For $\beta \geq 2$, $\theta < f_3(\gamma, \beta)$ always in the yellow region. Hence it is replaced with $\theta < f_2(\gamma, \beta)$ and the locus of $\theta = f_2(\gamma, \beta)$ is described by the dashed curve, instead. In Figure 5, the red, blue and green curves take, respectively, $\beta = 2$, $\beta = 3$ and $\beta = 4.5$. The dotted curve that guarantees the positivity of the optimal output are located far above Figure 5 and thus are not depicted. Since the solid red curve of $0 = \varphi(\gamma, \theta, \beta)$ with $\beta = 2$ does not cross the dashed red line, the optimal ambient tax rate can be determined for each value of $\gamma \in (0, 1)$. On the other hand, for $\beta = 3$, the solid blue curve meets the dashed blue curve at point $a = (\gamma_a, \theta_a)$ where

$$\gamma_a \simeq 0.449 \text{ and } \theta_a \simeq 1.551.$$

The optimal θ that is feasible to the firms is determined against $\gamma \in (\gamma_a, 1)$ along the segment ab of the solid blue curve. In the third example, $\beta_U = 4.5$ is selected to make the green solid and dashed curves meet at point $c = (1, \theta_c)$ with $\theta_c = 1$. Here the firms take the zero level of the abatement technology under θ_c at point c. Further, increasing β from β_U shifts the solid curve upward and the dashed curve downward. No intersection of these curves occur for $\beta > \beta_U$. In consequence, the firms select the negative abatement technology with the optimal θ satisfying $0 = \varphi(\gamma, \theta, \beta)$, however such a selection is assumed away in this paper.

Lemma 9 *For $\beta > \beta_U$, the firms are unable to select the optimal abatement technology under the optimal ambient tax rate.*

Lemma 10 *For $2 \leq \beta < \beta_U$, the firms can select the optimal technology and positive production if $0 = \varphi(\gamma, \theta, \beta)$ and $\theta > f_3(\gamma, \beta)$.*

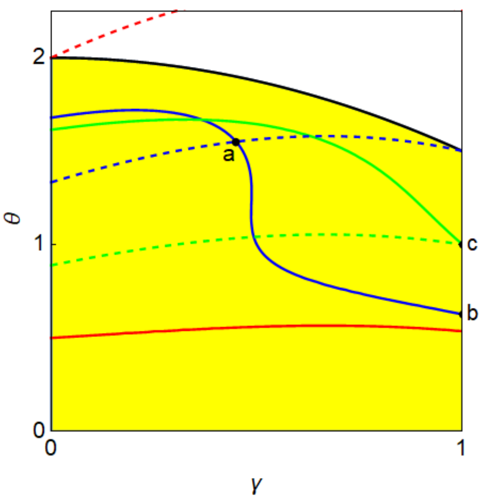


Figure 5. Optimal θ under $\beta \geq 2$

Here β_L and β_U are the lower and upper bounds in the sense that no positive production takes place for $\beta \leq \beta_L$ and no positive level of the abatement technology is adopted for $\beta \geq \beta_U$. From Lemmas 6-10, we arrive at the conclusion at the first stage that we call the *long-run* because the tax rate is a selection variable:

Theorem 4 *In the long run, the firms can select the optimal abatement technology* $0 < \phi^e(\gamma, \theta, \beta) < 1$ *and produce positive production* $q^e(\gamma, \theta, \beta) > 0$ *under the optimal ambient charge if one of the following three conditions is fulfilled,*

(i) $1 \leq \beta < 2$ *and* $0 = \varphi(\gamma, \theta, \beta)$,

(ii) $\beta_L < \beta < 1,\ 0 = \varphi(\gamma, \theta, \beta)$ *and* $\theta < f_5(\gamma, \beta)$,

(iii) $2 < \beta < \beta_U,\ 0 = \varphi(\gamma, \theta, \beta)$ *and* $\theta < f_2(\gamma, \beta)$.

5 Concluding Remarks

This paper considers NPS pollutions in Cournot duopolies. The firms' objective is to maximize profits, and the regulator wants to maximize social welfare. A three-stage model is developed where in the third stage the optimal output levels of the firms are determined. In the second stage the optimal abatement technologies of the firms are derived. In both stages the firms maximize their profits and the effectiveness of the ambient charge on the total emission level of NPS pollutions is proved. In the third stage the optimal ambient tax rate is found which maximizes social welfare. Linear price and cost functions were assumed. Further research will consider other function forms, including hyperbolic prices, and nonlinear costs. It is also a challenging task to generalize the findings of this paper to n-firm Cournot and Bertrand oligopolies. These topics will be the subjects of our next projects.

References

Chen, X., Benjaafar, S., and Elomri, A., On the effectiveness of emission penalties in decentralized supply chains, *European Journal of Operational Research*, 274, 1155-1167, 2019.

Downing, P. and White, L., Innovation in pollution control, *Journal of Environmental Economics and Management*, 13, 18-29, 1986.

Ganguli, S., and Raju, S., Perverse environmental effects of ambient charges in a Bertrand duopoly, *Journal of Environmental Economics and Policy*, 1, 289-296, 2012.

Ishikawa, T., Matsumoto, A., and Szidarovszky, F., Regulation of non-point source pollution under n-firm Bertrand competition, *Environmental Economics and Policy Studies*, 21, 579-597, 2019.

Jung, C., Krutilla, K. and Boyd, R., Incentives for advanced pollution abatement technology at the industry level: An evaluation of policy alternatives, *Journal of Environmental Economics and Management*, 30, 95-111, 1996.

Matsumoto, A., Szidarovszky, F., and Yabuta, M., Environmental effects of ambient charges in Cournot oligopoly, *Journal of Environmental Economics and Policy,* 7, 41-56, 2018.

Montero, J., Permits, standards, and technology innovation, *Journal of Environmental Management and Economics*, 44, 23-44, 2002.

Okuguchi, K. and Szidarovszky, F., Existence of Nash-Cournot equilibrium in Oligopoly games with pollution treatment cost-sharing, *Pure Mathematics and Applications*, 13, 455-462, 2002.

Okuguchi, K. and Szidarovszky, F., Environmental R&D in Cournot oligopoly with emission or performance standard, *Pure Mathematics and Applications*, 18, 111-118, 2007.

Raju, S., and Ganguli, S., Strategic firm interaction, returns to scales, environmental regulation and ambient charges in a Cournot duopoly, *Technology and Investment*, 4, 113-122, 2013.

Sato, H., Pollution from Cournot duopoly industry and the effect of ambient charge, *Journal of Environmental Economics and Policy*, 6, 305-308, 2017.

Segerson, K., Uncertainty and incentives in nonpoint pollution control, *Journal of Environmental Economics and Management*, 15, 87-98, 1988.

Wu, Y. and Chen, J., Investigating the effect of point source and nonpoint source pollution on the water quality of the East River (Dangjiang) in South China, *Ecological Indicators*, doi.org/10.1016/j.ecolind.2013.04002, 2013.