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Abstract
We analyze fertility decisions of families using an intergenerational exchange model that might be applicable to less-developed economies. Parents who have children for elderly support increase the number of children they have when the probability of becoming dependent rises. Rises in the children's wage rates decrease the fertility rate when the elasticity of marginal utility of family care is low. In that case, children's care might be replaced eventually by market care insurance along with economic development. Fertility declines do not derive from a quantity–quality tradeoff of children but from decreases in children's needs for bequests with wage rises.

Keywords: bequests, elderly care, fertility, intergenerational exchange
JEL Classification: D13, D64, D74, J13, J14

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1. Introduction

Fertility rates have declined during recent decades not only in economically developed but also even in economically developing areas of the world along with economic development.[1] However, different reasons might underlie these fertility declines in such developed and developing areas. In economically developed areas, the opportunity costs of child-rearing have been increasing because the wage rate, especially the female wage rate, rises as the economy develops. Higher opportunity costs induce parents to reduce child-rearing time. They therefore rear fewer children. By contrast, in developing areas, and especially in such areas without an unfunded social security system, children might be regarded as their parents' "investment" for their own old age. Kagitcibasi (1982) reports that in Asian countries such as Indonesia, the Philippines, and Thailand, the proportions of parents who consider old age security as a reason for having children were approximately 80% and higher (see also Leroux and Pestieau, 2014). In contrast, the respective proportions in the U.S. and Germany were only about 8%.[2] Therefore, the intergenerational exchange model might be applicable to developing economies, especially in Asia. Most parents transfer more bequests to their children who provide more family care to them. In these areas, the fertility rates have also declined along with per-capita income growth. We analyze the fertility decisions of families in such an exchange model.

In a seminal paper, Becker and Tomes (1976) formalize a model in which children are "consumption goods" with a tradeoff between the quality and quantity of children. Most works in the literature of population economics have followed this strand of research. By contrast, few reports describe studies that have examined family fertility decisions in intergenerational exchange models, although an exchange model has been developed theoretically by Bernheim et al. (1985) and more recently, for example, by Chang and Weisman (2005).

Although fertility decisions are analyzed in an intergenerational exchange model, we do not assume intergenerational altruism of parents and children. As described herein, parents' strategic transfers and children's transfer-seeking competition are modeled using a contest success function (CSF) à la Chang and Weisman (2005). Presuming that such competition among siblings and between parents and the children take place after

[1] Total fertility rates have declined since about 1980, even in low-income countries (UN, 2017). In some high-income economies, however, fertility rebounds were observed recently (Myrskylä et al., 2009).

[2] Horioka (1984) reports that the proportion of such parents is 35.1% in Japan, although Horioka et al. (2014) describe that the proportion changes to 20.5% while the proportion in the US changes to 2.5%.

children grow up, we analyze the fertility decisions of parents in this paper. Our main concern is intergenerational exchange. Therefore, we assume that a couple comprising a woman and a man behaves unitarily as if they were a single economic unit.[3] For expositional simplicity, we designate a couple as "a parent" and a pair of a girl and a boy "a child" for these analyses.[4]

The main conclusion is that increases in the probability of parents' becoming dependent increase the fertility rate. However, the result also predicts that rises in the children's wage rates decrease the fertility rate if the parent's marginal utility of family long-term care declines moderately. Children come to need fewer transfers from their parents and are unwilling to provide long-term care to their parents if the wage rates are higher. The latter implies that the fertility rate declines along with economic growth, even in less-developed areas, as far as the parents desire to have family long-term care persistently during their years of old age. The latter condition is apparently satisfied in countries such as Indonesia, the Philippines and Thailand. In that case, family long-term care might eventually be replaced by market long-term care insurance as the wage rate grows.

The next section introduces a model of the game and analysis of a subgame perfect equilibrium. Section 3 presents analysis of the fertility decisions of families. Sections 2 and 3 assume away market long-term care insurance, another means of old age security. To clarify the role of intergenerational exchanges in fertility decisions along with economic development, Section 4 considers the availability of private long-term care insurance. The last section concludes the paper.

2. Model

The benchmark problem can be formalized in the following way. It might be considered plausible that parents play a game with their children after they grow up and that grown-up children compete for transfers from the parent. Therefore, in stage 0, parents decide how many children they have. Then a two-stage game between elderly parents and adult children will take place: Parents choose the allocation of income

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[3] We can instead consider women and men explicitly. In that case, spouses have equal probability of becoming dependent. The number of children must be considered as the sum of the numbers of girls and boys. Issues of marriage are assumed away. However, with specific examination of collective decisions between parents and their children in each family, we assume a parent in a family.

[4] This point is analogous to a convention adopted by Basu and Van (1998). This assumption enables us to avoid conflict with the recent empirical rejection of a unitary model. We also assume that economic behaviors of a girl and a boy are identical.

between their own consumption and bequests to their children in stage 1. Parents promise their children to transfer bequests if they are cared for by the children when becoming dependent. Then, with the promised bequest, children decide how much attention and care to provide when their parents become dependent in their old age, competing for parental transfers among siblings in stage 2. Only after the provision of attention and care by children are revealed are the bequests actually transferred. Transfers of bequests can be a threat by assuring children's attention and care for parents.[5] However, even though parents do not become independent, children will receive the bequests.

The subgame perfect equilibrium game of the last two stages is solvable through backward induction in this section. Decision-making in stage 0 is analyzed as described in the next section.[6]

2.1 Attention and care provision of children in stage 2

In this stage, we first assume that a parent bequeaths transfers to $n(>1)$ children according to a contest success function. Regarding fertility decisions, we also assume that children are identical. The expected after-transfer income of child i, y_i, can be written as

$$y_i = \pi(1-a_i)w_i + (1-\pi)w_i + p_i B, \qquad (1)$$

where π denotes the probability that parents become dependent, a_i stands for a child's attention and care extended to the parent, w_i represents the wage rate, and B signifies the bequest from the parent. The third term $p_i B$ on the right-hand side of (1) denotes the transfer which the child receives from the parent. $p_i = a_i / \sum_h^n a_h$ represents the contest success function. The time endowment of children is assumed to be one. If the parent becomes dependent in old age, then the child's labor supply is $1-a_i$ because she provides attention and care time to her parent. If the parent is autonomous, then the child does not provide attention and care. In these analyses, children receive

[5] If a parental couple has only one child, then transferring a bequest cannot be a credible threat to the child. The child knows that the parent, if being dependent, must need the child's attention and care in this model. Therefore, we assume that more than two children compete for transfers from the parent by promising to provide attention and care to them. Bernheim et al. (1985) provide a similar argument. However, because of wage income, it might be only an enticement in this paper.

[6] Mathematically, the optimization problem of parents can be solved for the number of children and bequests simultaneously. However, as described in the previous section, it is apparently natural to consider that a game with respect to transfers and care is played between elderly parents and grown-up children.

bequests from parents even when parents are autonomous.

For a given bequest B, the child chooses the amount of attention and care to maximize the after-transfer income (1). The first-order condition is

$$\frac{dy_i}{da_i} = -\pi w_i + \frac{dp_i}{da_i} B = 0 . \tag{2}$$

Therein, we have $dp_i / da_i = [(\sum_h^n a_h) - a_i] / (\sum_h^n a_h)^2$. From (2) we obtain

$$a_i = (\sum_h^n a_h) - \frac{\pi w_i (\sum_h^n a_h)^2}{B} . \tag{2'}$$

The amount of attention and care of each child depends on the received bequest, the parent's probability of becoming dependent, the wage rate, and the siblings' amounts of attention and care.

The Nash equilibrium of transfer-seeking competition among children is obtainable from (2') as

$$a_i = \frac{(n-1)B}{\pi} \frac{(\sum_h^n w_h) - (n-1) w_i}{(\sum_h^n w_h)^2} . \tag{3}$$

Assuming that children are identical, one can obtain attention and care per child as

$$a = \frac{(n-1)B}{\pi w n^2} \equiv a(B, n; \pi, w) . \tag{4}$$

From (4) we have the following inequalities:

$$\frac{\partial a}{\partial B} > 0 , \quad \frac{\partial a}{\partial \pi} < 0 , \quad \frac{\partial a}{\partial w} < 0 \text{, and} \tag{5}$$

$$\frac{\partial a}{\partial n} = \frac{B(2-n)}{\pi w n^3} \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 0 \quad \text{as} \quad 2 \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} n . \tag{6}$$

The population growth rate is plausibly less than 100%. Therefore, we can readily assume that $n < 2$. Therefore, we have $\partial a / \partial n > 0$ from (6).[7] More attention and care will be provided with a greater bequest received. Children will at least partly offset the decrease in the expected after-transfer income by reducing attention and care because

[7] Although n represents the number of children, it is natural to consider that $n = 2$ means 4 children, 2 girls and 2 boys, when a couple consists of a woman and a man as in this paper. That is, $n \geq 2$ signifies that the total fertility rate is higher than 4. It is apparently too high to be plausible because, except for low-income countries, total fertility rates in the world have been below 4 after 1995–2000 (UN, 2017). Therefore, condition $n < 2$ seems plausible, although it is not necessarily so.

the increased probability reduces the expected after-transfer income. Children will reduce attention and care when the wage rate rises because the need for bequest, i.e., the relative weight of bequest in post-transfer income, becomes small. In addition, an increase in the number of siblings might enable each child to reduce attention and care per child without reducing the total attention and care provided to the parent. The increased number of siblings itself decreases a per-sibling receipt of bequests, giving each child an incentive to increase attention and care.

From (5) we also obtain

$$\frac{dy}{dB} = \pi w(-\frac{\partial a}{\partial B}) + \frac{1}{n} = \frac{1}{n^2} > 0, \tag{7}$$

where we use $p = 1/n$. Therefore, each child has an incentive to acquire more bequests from the parent by providing more attention and care.

2.2 Bequests from parents in stage 1

Parents are assumed to choose allocation between their own consumption and bequest to children to maximize utility from both. The utility function of a parent is assumed to be linearly separable, following the literature such as Chang and Weisman (2005):

$$U = [Y - B - \phi(n)] + \pi H(na). \tag{8}$$

Therein, Y represents the pre-transfer income of the parent, $\phi(n)$ denotes the child-rearing cost of n children, and $H(na)$ represents the utility from the total elderly attention and care provided by n children.[8] We consider here that the child-rearing cost is incurred before transferring a bequest to the children (and after receiving bequests from their parents), i.e., in stage 0. Function $H(.)$ is assumed to be strictly concave and $H(na)$ goes to $-\infty$ as $na \to 0$, whereas function $\phi(.)$ can be either concave or convex. The welfare of an elderly parent is lower when the parent becomes dependent without attention and care from children than when the parent is independent. We also assume here that the consumption level of the parent is independent of whether they becomes dependent in old age, or not.[9]

If the parent bequeaths B to transfer-seeking children, then the total elderly attention and care provided by children is obtained from (4) as

$$na = \frac{(n-1)B}{\pi wn}. \tag{9}$$

[8] Child-rearing costs are measured in terms of consumption goods in this setting. We can instead assume that parents spend time in child rearing. In that case, the cost is measured in terms of foregone income i.e., the wage rate times the child-rearing time.

[9] Unless parents are sure to be autonomous in their old age, parents are assumed to have children for security during their old age.

Substituting (9) into utility function (9), we obtain the first-order condition with respect to B as

$$1 = \frac{n-1}{nw} H'\left(\frac{(n-1)B}{\pi wn}\right), \tag{10}$$

from which it follows that

$$B = \frac{\pi wn}{n-1} H'^{-1}\left(\frac{wn}{n-1}\right) \equiv B(n;\pi,w). \tag{11}$$

The amount of the bequest depends on the number of children, the children's wage rate, and the probability of becoming dependent. From (10) we obtain

$$\frac{\partial B}{\partial n} = -\frac{H'\pi w}{H''(n-1)^2}(1-\eta) \gtreqless 0 \quad \text{as} \quad 1 \gtreqless \eta, \tag{12}$$

$$\frac{\partial B}{\partial \pi} = \frac{B}{\pi} > 0, \tag{13}$$

$$\frac{\partial B}{\partial w} = \frac{H'\pi n}{H''(n-1)}(1-\eta) \lesseqgtr 0 \quad \text{as} \quad 1 \gtreqless \eta, \tag{14}$$

where $\eta \equiv -[\frac{(n-1)B}{\pi wn} H'']/H' > 0$ is the elasticity of the marginal utility of elderly attention and care from children. The comparative static results can be interpreted as follows. Increases in the number of children increase the total elderly attention and care if per-child attention and care remains constant. When the marginal utility of elderly care declines greatly with elderly care, i.e., if η is large, then parents reduce bequests, affecting the children's provision of attention and care negatively, and increase their own consumption (see (12)). With a higher probability of dependence, parents increase bequests, thereby increasing children's elderly care (see (13)). With a higher wage rate of children, when η is large, parents increase bequests to offset, at least partly, decreases in the children's provision of attention and care because higher children's wage rates reduce their needs for transfers from parents (see (14)).

3 Fertility decisions (stage 0)

In this section, we present analyses of parents' fertility decisions. Parents choose the number of children knowing that they will play an intergenerational exchange game with their children after children are born and have grown up. The utility of a parent can be rewritten as

$$U(n) = [Y - B(n) - \phi(n)] + \pi H\left(\frac{(n-1)B(n)}{\pi wn}\right). \qquad (15)$$

Bequests the children receive and elderly care provided by children are regarded as functions of the number of children. The first-order condition of the parent's utility maximization with respect to the number of children is given as

$$\frac{dU}{dn} = -\frac{\partial B}{\partial n} - \phi' + \pi H'\left(\frac{B}{\pi wn^2} + \frac{n-1}{\pi wn}\frac{\partial B}{\partial n}\right) = 0. \qquad (16)$$

Making use of (10), the optimal number of children must satisfy the following condition.

$$\phi' = \frac{B}{wn^2}H'. \qquad (17)$$

This condition gives the optimal number of children implicitly. However, we cannot solve for the optimal number of children explicitly because both sides of (17) include n explicitly and implicitly. Condition (17) means that the optimal fertility rate depends on the probability of the parents' dependence and children's wage rates, i.e., $n(\pi, w)$.

To explore some properties of the fertility rate, by differentiating (17) totally, we have

$$Ddn = \frac{BH'}{\pi wn^2}d\pi - \frac{1}{\eta}\frac{BH'}{w^2n^2}(1-\eta)dw, \qquad (18)$$

where $D = -d^2U/dn^2$, which is positive because we assume that the second-order condition is satisfied.[10] Therefore, from (18), we have

$$\frac{dn}{dw} \gtreqless 0 \text{ as } \eta \gtreqless 1\text{, and} \qquad (19a)$$

$$\frac{dn}{d\pi} > 0. \qquad (19b)$$

A rise in the children's wage rate might raise or reduce the fertility rate depending upon the elasticity of the marginal utility of family attention and care provided by children: an increase in the probability that parents become dependent in old age increases the number of children per parent[11]. The result shown in (19a) has not been

[10] If $H(.)$ is logarithmic and if $\phi(.)$ is convex, i.e., $\eta = 1$ and $\phi'' > 0$, then the condition is satisfied. Zhang and Zhang (1998) assume condition $\phi'' > 0$ for the second-order condition for the individual's lifetime utility maximization to hold in their model without elderly care provision.

[11] The probability of being dependent can be considered to a proportion of the time period between the longevity and healthy life expectancy relative to longevity, although the length of lifetime is normalized in this paper. Mayhew (2011), among others, observes from UK data during 1980–2005 that a healthy lifetime increases at a slower pace than longevity. Most models in the preceding long-term care literature assume that the probability is given exogenously.

described in the literature. An increase in the children's wage rate lowers the fertility rate if the elasticity of the marginal utility of family care by children is smaller than one. It is noteworthy that the children's wage rates are measured in terms of parents' consumable income. Therefore, increases in the children's wage income relative to the parents' income might be regarded as consequences of economic development.[12]

These results can be interpreted in the following way. An increase in the children's wage rate reduces per-child attention and care. Condition (19a) implies that if the elasticity of the parent's marginal utility of family attention and care is one ($\eta = 1$), then the children's wage rate does not affect the number of children. If the elasticity of the marginal utility of elderly care η is small, i.e., if the marginal utility of family care decreases only moderately, then parents have fewer children with higher wage rates. A rise in the children's wage rate reduces the per-child attention and care provision for a level of parental bequest because the children's need for income transfers from parents become small.[13] Therefore, parents might decrease the number of children to avoid or mitigate child-rearing costs. By contrast, if η is great, i.e., if the marginal utility of family attention and care decreases rapidly, then it is optimal for parents to increase the number of children to secure greater family attention and care during old age. The increased number of children tends to increase the total family attention and care through sibling competition for transfers. Therefore, we have the following proposition.

Proposition 1. *Assume that parents have children for elderly support. If the fertility rate declines along with economic development, then the parent's marginal utility of children's long-term care declines only moderately, i.e.,* $\eta < 1$*, in the economies.*

Condition (19b) is apparently foreseeable and is apparently not surprising. However, the probability of elderly dependence has two-fold effects on fertility. First, a higher probability of becoming dependent increases the parent's utility of elderly care provided by children. If the elderly care each child provides remains constant, then more children raise the parent's utility from total elderly care because the marginal child-rearing cost is independent of the probability. Second, a higher probability of becoming dependent affects the total family long-term care. Increasing the number of children might not be good for parents because of child-rearing costs: When the elasticity of the marginal utility of elderly care η is great (small), more children might induce parents to reduce

[12] Even if the income level grows at a constant rate, the difference between parents' income and children's income becomes wider.
[13] See Appendix A1.

(increase) bequests (see (12)). Therefore, it is ambiguous whether changes in bequests consequently increase children's elderly care or not. Nevertheless, on balance, condition (19b) implies that a higher fertility rate must increase the total amount of children's attention and care as a result of the game.

4. Fertility decisions in intergenerational exchanges – availability of private long-term insurance

To consider the relation between parents' fertility decisions and economic development in an intergenerational exchange setting, we assume that private long-term elderly care insurance is available to parents. Presuming that the insurance is actuarially fair, the utility of parents can be rewritten as

$$U(n) = [Y - B(n) - I - \phi(n)] + \pi H\left(\frac{I}{\pi} + \frac{(n-1)B(n)}{\pi wn}\right), \qquad (20)$$

where I denotes an insurance premium and I/π represents the insurance benefit. The first-order condition for optimal choice of the insurance is

$$\frac{\partial U}{\partial I} = -1 + H' \le 0 \text{, where the equality holds for } I > 0. \qquad (21)$$

From (10) and (17), if parents choose to have children and give them transfers, it holds that

$$1 = \frac{n-1}{wn}H' = \frac{1}{\phi'}\frac{B}{wn^2}H' \text{ or } H' = \frac{wn}{n-1} = \phi'\frac{wn^2}{B}, \qquad (22)$$

from which one can say that the optimal n and B satisfy relation $n(n-1)\phi' = B$.[14] Therefore, we have two cases:[15]

(i) if $\frac{wn}{n-1} > H' = 1$, then $I > 0$, and (23a)

(ii) if $\frac{wn}{n-1} = H' < 1$, then $I = 0$. (23b)

In case (i), we have a corner solution, $B = 0$, violating condition (10).[16] In the present setting we also have $n = 0$ because parents obtain no benefit from having children

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[14] In these analyses, because parents have children only for supporting their old age, condition (22) holds if they have transfer-seeking children. From (10), if $B > 0$, then we must have $n > 1$. See also the argument in footnote 5.

[15] When $wn/(n-1) = H' = 1$, parents are indifferent between having children and purchasing private insurance. For exposition, we include this case as case (i).

[16] Condition (16) for an interior solution is also violated.

when $a = 0$.[17] The left-hand inequality condition in (23a) becomes meaningless in this case because $B > 0$ cannot hold when $n = 0$. Parents are provided long-term care solely by virtue of the insurance. In other words, no exchange occurs between parents and children in this case. By contrast, in case (ii), parents do not purchase the long-term care insurance and instead have children and give them bequest in exchange for family long-term care. In this case, we have $w < (n-1)/n < 1$, i.e., the wage rate is lower than unity because $n-1 > 0$.

It is noteworthy that n in conditions (23a) and (23b) is determined endogenously, depending upon the model parameters. Therefore, the condition in (23a) ((23b)) should be read as stating that the wage rate is sufficiently high (low, respectively). Intergenerational exchanges might occur when the wage rate of children is sufficiently low. As described in the Introduction, the fertility rates decline along with economic development in Asian middle-income countries such as Indonesia, the Philippines, and Thailand. Therefore, from (19a), one might surmise that case (ii) holds, i.e., intergenerational exchanges occur, for low wage rates when $\eta < 1$ (proposition 1). The prediction of our intergenerational exchange model is that decrements in fertility that occur along with economic development might derive from parents' persistent desire for family long-term care from children.

However, because the fertility rate decreases with the wage rate, term $wn/(n-1)$ might become greater than unity as the wage rate increases. Therefore, after the wage rate becomes sufficiently high, such intergenerational exchanges will play no role in determining the number of children, i.e., the fertility rate.[18] In that stage of economic development, market long-term care insurance plays an important role as an old age security for parents instead of family care, as described for case (i). Parents purchase long-term care insurance from the market instead of giving bequests to their children.

Summing up the arguments, we have the following proposition.

Proposition 2. *Presume that $\eta < 1$ holds. As the wage rate rises along with economic development, intergenerational exchanges between parents and children will be replaced by market long-term care insurance for parents' elderly support.*

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[17] Although we abstract from parent's utility of having children for analytical purposes, we can introduce it as a "consumption" good into the model. In that case, the number of children parents have would be positive.

[18] From (12) and (14), decreases in the number of children might induce parents to increase bequests to their children as long as the number is positive. Once it lapses into a corner solution of zero, bequests also become zero, possibly discontinuously. In the present setting, parents obtain no benefit from having children per se.

5. Conclusion

We have analyzed the fertility decisions of families in an intergenerational exchange model presuming that parents have no utility of having children. Parents who choose to have children for the purpose of deriving support from them during old age increase the number of children when the probability of becoming dependent when old rises. Although this result is intuitively foreseeable, it is ambiguous whether elderly care provision per child increases along with the parental bequest to children. Per-child attention and care might decrease with a bequest from the parent. By contrast, increases in the children's wage rates decrease the number of children, i.e., the fertility rate, although each child's attention and care might increase if the elasticity of the marginal utility of family care is sufficiently low.

The last possible negative effect of the wage increase suggests that the fertility rate might decrease along with economic growth, especially unexpected economic growth. In fact, during the 1970s and 1980s, when the fertility rates declined rapidly in Indonesia, the Philippines, and Thailand, their per-capita economic growth rates were rather high. Higher wage income decreases children's needs for transfers from parents, inducing children to provide less family care. The results presented herein indicate that the parent's marginal utility of family care increases only moderately in these countries. However, intergenerational exchanges for parents' elderly support might be replaced by market long-term care insurance as the wage rates become higher along with economic development.

It is noteworthy that the possibility of decreasing fertility does not derive from a tradeoff between the quantity and quality of children but from declines in children's needs for bequests from parents as income compensation to poor young people. The child labor literature also predicts that economic development reduces fertility (Hazan and Berdugo, 2002; Chakraborty and Das, 2005). However, the logic differs from ours. In models of the child labor literature, in which parents typically control children's time, economic development induces parents to substitute child education for child labor, i.e., quality of children is substituted for quantity.

So far we have not considered policy effects. Child allowances, for instance, can be regarded as affecting the fertility rate. In the present model setting, child allowances might affect only the fertility decisions of parents in stage 0. The introduction or extension of allowances increases the number of children in families if they are financed through lump-sum taxes on parents. The effect of increases in bequests depends on the elasticity of the marginal utility of elderly care. The increased number of children makes competition for transfer-seeking increasingly severe. Therefore, it increases each child's attention and care to parents.[19] However, various patterns of intergenerational exchanges can exist in reality. Our setting is merely a model. Especially, policy design must consider wider patterns of intergenerational exchanges.

Appendices

A.1 Effects on parental bequests and children's elderly care

This appendix presents an analysis of the effects of the children's wage rate and the probability of parental dependence on children's attention and care and on parental bequest. In a Nash equilibrium, we can express a parental bequest and per-child elderly care as

$$B = B(n(\pi, w); \pi, w) \quad \text{and} \quad a = a(B(\pi, w), n(\pi, w); \pi, w). \tag{A1}$$

Differentiating (A1) and from (5), (6), (12), (13), and (14), we obtain the following results:

$$\frac{dB}{d\pi} = \frac{\partial B}{\partial n}\frac{\partial n}{\partial \pi} + \frac{\partial B}{\partial \pi} > 0 \text{ as } 1 \geq \eta \text{, undetermined otherwise,} \tag{A2}$$

$$\frac{dB}{dw} = \frac{\partial B}{\partial n}\frac{dn}{dw} + \frac{\partial B}{\partial w} \lesseqgtr 0 \text{ as } 1 \gtreqless \eta, \tag{A3}$$

$$\frac{da}{d\pi} = \frac{\partial a}{\partial B}\frac{dB}{d\pi} + \frac{\partial a}{\partial n}\frac{dn}{d\pi} + \frac{\partial a}{\partial \pi} \text{ undetermined,} \tag{A4}$$

$$\frac{da}{dw} = \frac{\partial a}{\partial B}\frac{dB}{dw} + \frac{\partial a}{\partial n}\frac{dn}{dw} + \frac{\partial a}{\partial w} < 0 \text{ as } 1 \geq \eta \text{, undetermined otherwise.} \tag{A5}$$

In the case of a log-linear utility function of elderly care, we have $dB/d\pi > 0$, $dB/dw = 0$, and $da/dw < 0$ because $\eta = 1$.

A2. Child allowance policy

Presuming that a child-rearing subsidy per child is β, then the subsidy is financed by lump-sum taxes on parents. Let T be a lump-sum tax. The utility of a parent can be written as

$$U = Y - B - \phi(n) + \beta n - T + \pi H\left(\frac{(n-1)B}{\pi w n}\right). \tag{A6}$$

The first-order condition for utility maximization is

$$\frac{\partial U}{\partial n} = -\phi' + \beta + \frac{BH'}{wn^2} = 0, \tag{A7}$$

[19] See Appendix A2.

from which we can obtain

$$\frac{dn}{d\beta} = D^{-1} > 0\,. \tag{A8}$$

We also have

$$\frac{dB}{d\beta} = \frac{\partial B}{\partial n}\frac{dn}{d\beta} \ \substack{>\\=\\<}\ 0 \quad \text{as} \quad 1 \ \substack{>\\=\\<}\ \eta\,, \tag{A9}$$

$$\frac{da}{d\beta} = \frac{\partial a}{\partial n}\frac{dn}{d\beta} > 0\,. \tag{A10}$$

An increase in the child-rearing subsidy increases the number of children a parent has, intensifying the competition for transfers among children. Therefore, it increases per-child attention and care, thereby increasing the total family long-term care for a given bequest. However, the amount of the bequest depends on the elasticity of the marginal utility of elderly care provided by children. If the elasticity is sufficiently high, then parents might increase consumption of their own, thereby reducing their bequest to children.

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The author has no conflict of interest, financial or otherwise, related to this study.

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Supplementary notes

S1. Second-order condition of the parent's utility maximization

In relation to footnote 10, the second-order condition for parent's utility maximization is examined.

From (16) and using the definition of η, we have

$$\frac{d^2U}{dn^2} = -\phi'' + \frac{(2n-1)B}{n^2(n-1)^2}(\frac{1}{\eta}-1) - \frac{2H'}{wn^3}\frac{B}{\eta} + \frac{H''}{w^2n^4\pi}\frac{B^2}{\eta}$$

$$-\{\frac{1}{n(n-1)}(\frac{1}{\eta}-1) - \frac{1}{wn^2\eta}[H' + H''\frac{(n-1)B}{\pi nw}]\}\frac{\partial B}{\partial n}$$

$$= -\phi'' + \frac{(2n-1)B}{n^2(n-1)^2}(\frac{1}{\eta}-1) - \frac{2H'}{wn^3}\frac{B}{\eta} + \frac{H''}{w^2n^4\pi}\frac{B^2}{\eta}$$

$$-\frac{1}{n(n-1)}(1-\frac{n-1}{wn}H')(\frac{1}{\eta}-1)\frac{\partial B}{\partial n}. \quad \text{(S1)}$$

From (11) in the text, it follows that

$$\frac{d^2U}{dn^2} = -\phi'' + \frac{(2n-1)B}{n^2(n-1)^2}(\frac{1}{\eta}-1) - \frac{2H'}{wn^3}\frac{B}{\eta} + \frac{H''}{w^2n^4\pi}\frac{B^2}{\eta}. \quad \text{(S2)}$$

The last two terms on the right-hand side are negative. The coefficient of the second term is positive. Therefore, a sufficient condition for $d^2U/dn^2 < 0$ is a combination of $\phi'' \geq 0$ and $1 \leq \eta$.

S2. Simultaneous decision of the number of children and bequests

As described in footnote 6, the family game can be formalized in a two-stage game, in which parents choose the number of children and bequests to them simultaneously.

The utility function of a parent is

$$U(n) = [Y - B - \phi(n)] + \pi H(\frac{(n-1)B}{\pi wn}). \quad \text{(S3)}$$

The first-order conditions for utility maximization are

$$-\phi' + H'\frac{B}{wn^2} = 0, \quad \text{(S4)}$$

$$-1 + H'\frac{n-1}{wn} = 0. \quad \text{(S5)}$$

These conditions are identical to (17) and (10), respectively. Assuming an interior solution, we have from (S4) and (S5)

$$\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}\begin{pmatrix} dn \\ dB \end{pmatrix} = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} dw + \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} d\pi \tag{S6}$$

where

$$G_{11} = -\phi'' - \frac{2B}{wn^3}H' + \frac{B^2}{\pi w^2 n^4}H'', \tag{S7a}$$

$$G_{12} = \frac{1}{wn^2}H' + \frac{(n-1)B}{\pi w^2 n^3}H'' = \frac{H'}{wn^2}(1-\eta), \tag{S7b}$$

$$G_{21} = \frac{1}{wn^2}H' + \frac{(n-1)B}{\pi w^2 n^3}H'' = \frac{H'}{wn^2}(1-\eta), \tag{S7c}$$

$$G_{22} = \frac{(n-1)^2}{\pi w^2 n^2}H'', \tag{S7d}$$

$$W_1 = \frac{B}{w^2 n^2}H' + \frac{(n-1)B^2}{\pi w^3 n^3}H'' = \frac{BH'}{w^2 n^2}(1-\eta), \tag{S7e}$$

$$W_2 = \frac{n-1}{w^2 n}H' + \frac{(n-1)^2 B}{\pi w^3 n^2}H'' = \frac{(n-1)H'}{w^2 n}(1-\eta), \tag{S7f}$$

$$P_1 = \frac{(n-1)B^2}{\pi^2 w^2 n^3}H'' = \frac{BH'}{\pi wn^2}(-\eta), \tag{S7g}$$

$$P_2 = \frac{(n-1)^2 B}{\pi^2 w^2 n^2}H'' = \frac{(n-1)H'}{\pi wn}(-\eta). \tag{S7h}$$

From these, we obtain

$$\tilde{D}\frac{dn}{dw} = \frac{(n-1)(H')^2}{w^3 n^3}(\eta - 1) \gtreqless 0 \quad \text{as} \quad \eta \gtreqless 1, \tag{S8}$$

and

$$\tilde{D}\frac{dn}{d\pi} = (P_1 G_{22} - P_2 G_{12}) = \frac{(n-1)(H')^2}{\pi w^3 n^3}\eta > 0, \tag{S9}$$

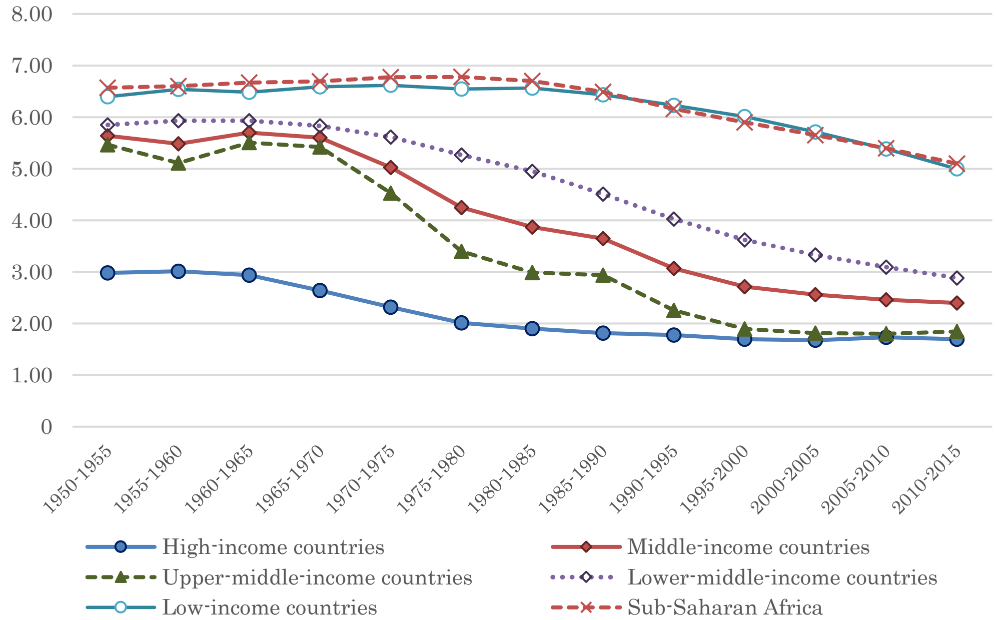
where $\tilde{D} = G_{11}G_{22} - G_{12}G_{21} > 0$ from the second-order condition. Results (S8) and (S9) are respectively identical to those in the text: (19a) and (19b).

S3. Fertility and economic development in some Asian countries

Figure S1 illustrates the fertility rates around the world during 1950–2015. Although the fertility rates in low-income countries still remain high, those in middle-income countries have declined about after the 1970s. After about the latter 1990s, the fertility rates in high-middle income have been close to those in high-income countries.

From the data described in Kagitcibasi (1982), the intergenerational exchange hypothesis is expected to plausibly hold for some Asian countries such as Indonesia, the Philippines, Singapore, Thailand, and Turkey. The fertility rates of those countries during 1950–2015, whereas the five-year moving averages of per-capita GDP of those countries after the 1960s are illustrated in Figure S2. After the 1970s, per-capita GDP growth were relatively high, especially for middle-income countries. The fertility rates of those middle-income countries have declined around the 1970s as illustrated in the lower part of Figure S2. Roughly speaking, some negative relation between fertility and per-capita GDP growth apparently exists for these countries characterized by possible intergenerational exchanges between grown-up children and elderly parents.

Fig. S1. Total fertility rates around the world.



Source: United Nations, 2017. World Population Prospects: The 2017 Revision

Table S1. Old age security as a reason to have children

(% women and men who answered "very important")

Country	Women	Men	Income class in World Bank (2018)
Javanese Indonesia	93	89	Lower middle
Sundanese Indonesia	98	94	Lower middle
Republic of Korea	54	40	High
Philippines	89	86	Lower middle
Singapore	51	44	high
Taiwan	79	72	-
Thailand	79	71	Upper middle
Turkey	77	77	Upper middle
Federal Republic Germany	8	-	High
USA	8	7	High
Japan*	46.5	34.0	High

Source: Kagitcibasi (1982); Horioka (1984)

Fig S2. Growth rates and total fertility rates.