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Female Labor Supply, Fertility Rebounds, and Economic Development

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Abstract
We show that a fertility rebound can occur as the female wage rate rises with economic development. Under plausible conditions, capital accumulation raises the marginal product of labor and hence the female wage rate. Unless the economy is trapped in a lower equilibrium of economic development, the fertility rate starts to decline at a certain level of the female wage rate and then turns upward at a higher female wage, i.e., a fertility rebound. For such fertility rebounds to appear, the availability of external child-care at high female wages is crucial. Otherwise, the fertility rate might continue to decline.

Keywords: gender wage gap, child-care outside the home, female labor force participation
JEL classifications: D11, J13, J16, O11

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1. Introduction

Fertility rebounds have been observed at higher levels of per capital income in developed countries in recent decades, although the magnitude of rebounds varies from country to country.[1] This fact is in contrast to the so-called "stylized fact" that fertility declines at more advanced stages of economic development (e.g., Galor, 2005). One of convincing explanations of the rebounds is the effect of family policies especially undertaken after the 1990s (e.g., Luci-Greulich and Thévenon, 2003). This paper shows the possibility of fertility rebounds without such policies, focusing on time-allocation decisions of women and the availability of child-care outside the home.[2]

Galor and Weil (1996) assume a physical factor of male labor which is not in female labor. The male wage rate is higher than that of female. Therefore, women first spend their time on child rearing at home and then men might also do. Physical capital accumulation raises the both female and male wage rates. The higher opportunity cost of internal child-care at home induces couples to have a smaller number of children. Incorporating the limited availability of external child-care into Galor and Weil (1996), Morita and Yakita (2016) obtain not only the persistent declines in fertility but also shrinkages of the gender wage gap along physical capital accumulation paths.[3]

Iyigun and Walsh (2007) and Kemnitz and Thum (2015) emphasize the role of mothers in pregnancy, delivery and lactation period, rather than difference between female and male labor. Kemnitz and Thum (2015) also analyze the response of time allocation of mothers to female wage rates: At low female wage rates, the opportunity cost of child rearing at home is low enough for women to bear and rear their children at home; when the female wage rate is higher than the benefits from rearing children at home, women work outside the home and earn wage income; and when the female wage rate exceeds the price of external child-care, they purchase external child-care to rear children without spending their time on child rearing at home. However, Kemnitz and Thum

[1] The bottoms of the fertility rate were 1.40 for Canada in 2000, 1.73 for France in 1993, 1.24 for Germany in 1994, 1.19 for Italy in 1995, 1.26 for Japan in 2005, 1.63 for UK in 2001, and 1.74 for US in 1976. Actually, Myrskylä et al. (2009) report that the relation between fertility and the human development index (HDI) during the past decades can be described by a reversed-J shape, where Human Development Index is composed of per capita income, life expectancy and years of schooling. The fertility rates in France, UK and US are about 2 although those in Canada, Italy, Japan and Germany are still much lower than the replacement rate.

[2] Mizuno and Yakita (2013) and Hirazawa and Yakita (2017) also show that even without family policies, the endogenous retirement decisions of individuals can enable the fertility rate to turn up at higher per-capita income levels when life expectancy increases.

[3] In Morita and Yakita (2016), physical capital accumulation directly increases per capital income as in Galor and Weil (1996) although Galor (2005) emphasizes human capital accumulation induced by technological progress.

(2015) do not consider the dynamics of female wage rate and fertility.[4] In this paper we present the dynamics of female labor supply and fertility decisions in families.

For our purposes we assume a version of the Galor and Weil (1996) model, extended by introducing market child-care outside the home.[5] Each male worker supplies mental (or non-physical) and physical labor as a unit of labor indivisibly, whereas each female worker supplies only mental labor. Men work fulltime, but women allocate their time between market work and internal child-care at home.[6] Although the production of final goods uses labor, both female and male, and capital, the external child-care services production uses female labor and produced goods, where the goods input includes child-care equipment and facilities such as child-care centers and kindergartens.

The major result is the following. When the female wage rate is sufficiently low, mothers are willing to spend their time to bear and rear children at home because the marginal utility of time spent in child rearing is higher than that spent in market labor. Capital accumulation might raise the wage rates, increasing the opportunity cost of internal child-care of women. As the female wage rate rises, couples might start to decrease the number of children reared at home. As capital accumulates, the wage rates become high relative to the goods price, which in turn tends to lower the relative price of external child-care. When the female wage rate becomes higher than the price of external child-care, they start to increase the number of children by purchasing external child-care with higher wage income. Then, fertility rebound might appear as the female wage rate rises along economic development. The salient feature of this paper is to present the

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[4] The models in Iyigun and Walsh (2007) and Kemnitz and Thum (2015) are based on the collective approach. The main concern in Kemnitz and Thum (2015) is the effects of female wage rate in determining the bargaining power distribution within their families and on the number of their children. They concluded that endogenous bargaining between spouses leads to a downward bias in fertility choices of families.

[5] The unitary model has been recently rejected in empirical works. See, for example, Lundberg et al. (1997). However, to ease the exposition of the *intergenerational* dynamics of couples' fertility and female labor supply, we adopt a unitary model in this paper. Prettner and Strulik (2016) report that in case of identical preferences for the quantity and quality of children between women and men female empowerment has only weak and insignificant effects on economic development: if female and male preferences differ to a large extent, female empowerment can have a strong effect on economic development. Empirical studies sometimes only wheal an insignificant effects (Duflo, 2012). Mazzocco (2007) theoretically demonstrates that the unitary model can be a special case of the full-efficiency collective model.

[6] In modern economies, technological progress and capital accumulation complement mentally-intensive tasks in production, reducing the importance of physically-intensive ones. Actually, Kleven and Landais (2017) report that 80% of the currently remaining earnings gap between sexes in Denmark comes from the opportunity cost of child raising (called child penalty) faced by mothers.

dynamics of fertility along economic development by endogenizing women's time allocation between market labor supply and internal child-care.

The remainder of this paper is organized as follows. The next section introduced an overlapping generations model with endogenous time allocation of women. Section 3 presents the examination of the market equilibrium for various levels of the female wage rate and then analyzes the dynamics of the economic system. The last section concludes this paper.

2. Model

A family consists of a woman and a man, i.e., a couple.[7] Individuals live for two periods, first young working-period and the second old retired-period. The length of lifetime is certain and the length of each period is normalized to one. To avoid the matching issue in marriage we assume the number of women and men are the same and a couple has the same number of girls and boys. Each couple has a common utility from consumption and having children. Men supply labor inelastically to the labor market, although women choose the allocation of their time between child rearing at home and market labor. The production technology of consumption goods in the economy is represented by a constant-returns-to-scale (CRTS) production function of physical capital and labor. Physical capital in goods production depreciates in one period. Men are employed only in goods production. The market labor supplied by women can be employed in either goods production or child-care production. Market child-care services are produced using female labor and goods input.[8]

2.1 Family decisions[9]

The utility function of a couple in period t is represented by

$$u_t = \ln c_t^1 + \varepsilon \ln n_t + \rho \ln c_{t+1}^2, \tag{1}$$

[7] The period of childhood is not explicitly shown in this paper.

[8] Actually, the ratio of male worker in the child-care sector is significantly low. Even in Norway, male workers accounted for about 20% of all workers, such as child-care managers, in the child-care services sector in 2008, and were only 2.8% in Japan in 2010. See the Japan Gender Equality Bureau Cabinet Office (http//www.gender.go.jp/about_danjo/whitepaper/h26/zentai/html/column/clm_04.html: accessed 1 January 2015) and Japanese Embassy in Norway (http://www.no.emb-japan.go.jp/Japanese/Nikokukan/nikokukan_files/danjyobyoudou.pdf : accessed 1 January 2015).

[9] This subsection greatly draws on Kemnitz and Thum (2015), though their main concern is the downward bias to fertility caused by family bargaining.

where c_t^1 and c_{t+1}^2 are first- and second-period family consumption and n_t denotes the number of pairs of a boy and a girl. $\varepsilon > 0$ is the utility weight on having children and $\rho \in (0,1)$ stands for the time preference factor. Letting w_t^f, w_t^m and P_t be female and male wage rates and the price of external child-care, the first-period budget constraint of the couple can be written as

$$w_t^m + (1-\tilde{l}_t)w_t^f = c_t^1 + P_t x_t + s_t, \tag{2}$$

where $\tilde{l}_t$ denotes (internal) child-rearing time of a wife at home, x_t is the amount of external child-care purchased, and s_t is savings for their retirement. The second-period budget constraint is

$$r_{t+1}s_t = c_{t+1}^2, \tag{3}$$

where r_{t+1} stands for the gross interest rate in period $t+1$. The couple rears their children by using a wife's time and external child-care time. Letting $1/\phi$ be a required time input to rear a pair of children, the total time input required to rear n_t pairs of children is given by $n_t/\phi = \tilde{l}_t + x_t$, where $0 \le \tilde{l}_t \le 1$ and $x_t \ge 0$.[10]

We split the utility maximization problem of a couple into two steps: the cost minimization of child bearing and the utility maximization. The cost of rearing n_t pairs of children is given as $C = w_t^f\tilde{l}_t + P_t x_t = (w_t^f - P_t)\tilde{l}_t + P_t(n_t/\phi)$. Minimizing the cost for rearing n_t pairs of children subject to time constraint $0 \le \tilde{l}_t \le 1$, one can obtain the cost function:

[10] Under a logarithmic utility function, however, it is infeasible for utility maximization to have $\tilde{l}_t = 0$ *and* $x_t = 0$ simultaneously.

$$C(n)=\begin{cases}(i) & \dfrac{P_t n_t}{\phi} & when\ \ w_t^f > P_t \\ (ii) & \dfrac{n_t}{\phi}w_t^f & when\ \ w_t^f \le P_t\ \ and\ \ \dfrac{n_t}{\phi}<1 \\ (iii) & w_t^f + P_t(\dfrac{n_t}{\phi}-1) & when\ \ w_t^f \le P_t\ \ and\ \ \dfrac{n_t}{\phi}\ge 1\end{cases}. \quad (4)$$

In case (i), the couple chooses $\tilde{l}_t = 0$ and purchases $x_t = n_t / \phi$ of external child-care to have n_t pairs of children. The wife supplies full time of labor to the market. In case (ii), the wife allocate the time endowment between internal child-care, $0 < \tilde{l}_t = n_t / \phi < 1$, and market labor, $0 < 1 - \tilde{l}_t < 1$; and in case (iii), the couple chooses to rear n_t pairs of children at home and the wife does not supply labor to the market, i.e., $\tilde{l}_t = 1$. In cases (ii) and (iii), they are not willing to purchase external child-care, i.e., $x_t = 0$.

As the second step, the couple chooses the number of children, n_t, and consumption during two periods, c_t^1 and c_{t+1}^2, to maximize family utility subject to the following intertemporal budget constrain:

$$w_t^m + w_t^f = c_t^1 + \frac{c_{t+1}^2}{r_{t+1}} + C(n_t). \quad (5)$$

The first-order conditions are

$$1/c_t^1 = \lambda_t, \quad (6a)$$

$$\varepsilon / n_t = \lambda_t C'(n_t), \quad (6b)$$

$$\rho / c_{t+1}^2 = \lambda_t / r_{t+1}, \quad (6c)$$

and the budget constraint (5). Using the cost function (4), and from (5) and (6), we obtain the following solutions. In case (i) in which $w_t^f > P_t$, we have

$$n_t = \frac{\varepsilon\phi}{1+\varepsilon+\rho}\frac{w_t^f + w_t^m}{P_t}, \tag{7a}$$

$$s_t = \frac{\rho}{1+\varepsilon+\rho}(w_t^f + w_t^m). \tag{7b}$$

In case (ii) in which $w_t^f \le P_t$ and $n_t/\phi < 1$, we have

$$n_t = \frac{\varepsilon\phi}{1+\varepsilon+\rho}\frac{w_t^f + w_t^m}{w_t^f}, \tag{8a}$$

$$s_t = \frac{\rho}{1+\varepsilon+\rho}(w_t^f + w_t^m). \tag{8b}$$

In this case, for the optimal plan (8a) to be consistent with condition $n_t/\phi < 1$, one must have $w_t^f / w_t^m > \varepsilon/(1+\rho)$. In case (iii) in which $w_t^f \le P_t$ and $n_t/\phi \ge 1$, if the optimal plans are interior solutions to the problem, we have

$$n_t = \frac{\varepsilon\phi}{1+\varepsilon+\rho}\frac{P_t + w_t^m}{P_t}, \tag{9a}$$

$$s_t = \frac{\rho}{1+\rho}w_t^m. \tag{9b}$$

However, for n_t in (9a) to be consistent with condition $n_t/\phi \ge 1$, one must have $P_t / w_t^m \le \varepsilon/(1+\rho)$. Because $w_t^f \le P_t$, this means that $w_t^f / w_t^m \le \varepsilon/(1+\rho)$. In this case, because couples are not willing to purchase external child-care, we have a corner solution, i.e., $n_t/\phi = \tilde{l}_t = 1$ if men are employed on full time. Therefore, when $w_t^f / w_t^m \le \varepsilon/(1+\rho)$, we have

$$n_t = \phi. \tag{9a'}$$

2.2 Goods production sector

The production technology of consumption goods is assumed to be given by the following a constant-returns-to-scale production function.

$$Y_t = F(K_t, L_t) + bL_t^m, \tag{10}$$

where Y_t represents aggregate output, K_t is aggregate physical capital, L_t is aggregate labor and L_t^m is aggregate male labor in period t. $F(K,L)$ is a homogeneous function in physical capital K and labor L, and $b>0$ is the constant marginal productivity of male physical labor. We assume here that capital accumulation increases the labor productivity, i.e., $F_{LK}>0$. Aggregate labor L_t is the sum of non-physical labor of female and male workers, $L_t = L_t^m + L_t^{fY}$, where L_t^{fY} denotes female non-physical labor employed in the goods production sector. Letting N_t be the number of couples in period t, the production function can be rewritten in per couple terms as

$$Y_t / N_t = F(k_t, 1+l_t^{fY}) + b, \tag{10'}$$

where $k_t = K_t / N_t$, $L_t^m / N_t = 1$ and $l_t^{fY} = L_t^{fY} / N_t$. The zero-profit conditions are

$$F_K(k_t, 1+l_t^{fY}) = r_{t+1}, \tag{11a}$$

$$F_L(k_t, 1+l_t^{fY}) = w_t^f, \tag{11b}$$

$$F_L(k_t, 1+l_t^{fY}) + b = w_t^m. \tag{11c}$$

The gender wage gap is given by $(w_t^m - w_t^f)/w_t^f = b / F_L(k_t, 1+l_t^{fY})$. It is noteworthy that even if women do not supply market labor as in case (iii), i.e., even when $l_t^{fY} = 0$, the *potential* female wage rate can be given by the marginal product.

2.3 Child-care production sector

We assume that the production of child-care needs goods input $A>0$ per unit of child-care output as well as female labor. Because the goods input includes equipment and facilities for child rearing such as child-care centers and kindergartens, it can be assumed to depend on the level of child-care service output. The greater the number of children cared in the sector is, the more the input of goods is needed to care them. The production technology of child-care can be written as $X_t = \mu L_t^{fX}$, where X_t represents aggregate product of child-care, L_t^{fX} is labor employed and μ stands for the labor productivity in the sector. We assume that $\mu > 1$, because each child-care worker is expected to care more than one child at a time.[11] The profit of the child-care production sector is given as

$$\pi_t^X = P_t X_t - w_t^f L_t^{fX} - AX_t, \tag{12}$$

where AX_t stands for the total goods cost. The zero-profit condition in this sector can be written as

$$P_t \mu - w_t^f - A\mu = 0. \tag{13}$$

Therefore, we have $P_t \overset{>}{\underset{<}{=}} w_t^f$ as $\frac{\mu A}{\mu - 1} \overset{>}{\underset{<}{=}} w_t^f$. When the female wage rate is sufficiently low relative to the per-unit goods cost, the price of external child-care is higher than the female wage rate; and *vice versa*. It is noteworthy that because external child-care services are not demanded, the services will not be produced in cases (ii) and (iii).

3. Dynamics

3.1 Market equilibrium

We examine the dynamics of the system for each of three cases, which is determined by the equilibrium in the capital market

$$K_{t+1} = s_t N_t, \tag{14}$$

where s_t is given by (7b), (8b) and (9b) for each case.

The other important market is the labor market. Assuming that men are employed on full time, the equilibrium condition can be explained by female labor. In case (i), because both spouses of each couple supply full-time labor to the market and purchase

[11] The Japanese national standards for the number of children per childcare worker in childcare centers are about 3 for those under age one and about 6 for those aged 1 and 2.

external child-care, one has $L_t^f = L_t^m = N_t$ and $\tilde{l}_t = 0$, where L_t^f denotes female market labor. Because couples do not purchase external child-care in cases (ii) and (iii), we have $L_t^{fX} = 0$. Wives supply market part-time labor in case (ii), whereas they do not supply market labor in case (iii). Therefore, we have

$$L_t^f = N_t = L_t^{fY} + L_t^{fX}, \text{ and } L_t^{fY}, L_t^{fX} > 0 \text{ in case } (i), \tag{15a}$$

$$L_t^f = (1-\tilde{l}_t)N_t = L_t^{fY} \text{ in case } (ii), \tag{15b}$$

$$L_t^f = 0 \text{ in case } (iii). \tag{15c}$$

The aggregate labor employed in goods production is $L_t = L_t^m + L_t^{fY}$ and the employed labor in external child-care production is L_t^{fX}.

Finally, the equilibrium condition in the child-care market is $X_t = n_t N_t$ in case (i).[12] Because the production of child-care services is linear in female labor, the supply of child-care is determined to be equal to the demand which is described by the right-hand of the equilibrium condition.

Now we examine the dynamics in each case in turn:

Case (i) From (7a), (11b), (11c), and (13), we have

$$n_t = \frac{\varepsilon\phi}{1+\varepsilon+\rho}\frac{2F_L(k_t, 2-n_t/\mu\phi)+b}{F_L(k_t, 2-n_t/\mu\phi)/\mu + A}, \tag{16}$$

from which we obtain $n_t = n(k_t)$. Differentiating (16) with respect to k_t, we obtain

[12] In case (i), from (7a), (11b), and (11c), one has $n_t = n(l_t^{fY}; k_t)$. The equilibrium condition in external child-care market can be rewritten as $n_t = \mu(1-l_t^{fY})$ by using the labor market equilibrium condition (15a). Thus, one can obtain $l_t^{fY} = l^{fY}(k_t)$. Therefore, variables in period t, w_t^f, w_t^m, r_{t+1}, P_t, l_t^{fY}, l_t^{fX}, Y_t, X_t, n_t, and s_t, are determined for a given k_t.

$$\frac{dn_t}{dk_t}=\frac{(2-\frac{1+\varepsilon+\rho}{\varepsilon\phi\mu}n_t)F_{LK}}{\frac{1+\varepsilon+\rho}{\varepsilon\phi}(\frac{F_L}{\mu}+A)+\frac{F_{LL}}{\phi\mu}(2-\frac{1+\varepsilon+\rho}{\varepsilon\phi\mu}n_t)}. \tag{17}$$

The sign of dn_t/dk_t depends on the signs of $F_L+F_{LL}(2-\frac{n_t}{\phi\mu})$ and $2-\frac{(1+\varepsilon+\rho)n_t}{\varepsilon\phi\mu}$. We can show that $F_L+F_{LL}(2-\frac{n_t}{\phi\mu})>0$ holds if $F(K,L)$ is the CES function as in Galor and Weil (1996). Using (13) and $w_t^m=w_t^f+b$, we can obtain $dn_t/dw_t^f=P_t[2-\frac{(1+\varepsilon+\rho)n_t}{\varepsilon\phi\mu}]>0$ because children are normal goods.[13] It seems plausible in this case that parents take the price changes into account in purchasing external child-care services. Therefore, we plausibly have $dn_t/dk_t>0$.

The equilibrium condition in the capital market can be written as

$$n(k_t)k_{t+1}=\frac{\rho}{1+\varepsilon+\rho}[2F_L(k_t,2-\frac{n_t}{\phi\mu})+b], \tag{18}$$

from which we obtain

$$\frac{dk_{t+1}}{dk_t}=\frac{\rho}{\varepsilon\phi\mu}(F_{LK}-F_{LL}\frac{1}{\phi\mu}\frac{dn_t}{dk_t})>0, \tag{19}$$

and

$$\frac{dw_t^f}{dk_t}=(F_{LK}-F_{LL}\frac{1}{\phi\mu}\frac{dn_t}{dk_t})>0. \tag{20}$$

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13 Assuming $\rho=0.3$ as in de la Croix and Michel (2002) and $\varepsilon=0.271$ as in de la Croix and Doepke (2003), we have $2-(1+\varepsilon+\rho)n_t/(\varepsilon\phi\mu)>0$ if $(n_t/\phi)/\mu=l_t^{fX}$ ≤ 0.345. Actually, in Japan, for example, the number of workers, even both female and male, in the child-care sector, e.g., kindergartens and child-care centers, is less than 0.5 million (both female and male) in 2016, whilst the number of female workers is 27.6 million in 2016 in Japan (Ministry of Education, Culture, Sports Science, and Technology of Japan, 2016, http://www.e-stat.go.jp/SG1/estat/List.do?bid=000001079859&cycode=0; Ministry of Internal Affairs and Communication of Japan 2016, http://www.stat.go.jp/data/roudou/longtime/03roudou.htm#hyo_1; and Ministry of Health, Labour and Welfare, 2015). More 0.07 million of child-care workers will be needed to achieve the "Zero Children Waiting List" in Japan by the end of 2017 (Ministry of Health, Labour and Welfare, 2015). Therefore, we can assume that l_t^{fX} is sufficiently small. Therefore, the condition is plausibly satisfied.

Case (ii) From (11b) and (11c) we can rewrite (8a) as

$$n_t = \frac{\varepsilon\phi}{1+\varepsilon+\rho}[2+\frac{b}{F_L(k_t, 2-n_t/\phi)}], \tag{21}$$

from which we can obtain $n_t = n(k_t)$. We can readily show that

$$\frac{dn_t}{dk_t} = \frac{-\phi F_{LK}}{\frac{1+\varepsilon+\rho}{\varepsilon}\frac{(F_L)^2}{b} - F_{LL}} < 0. \tag{22}$$

In this case, wives allocate their time between internal child-rearing ($\tilde{l}_t = n_t/\phi < 1$) and market (part-time) labor $(1-\tilde{l}_t)$. Because couples do not demand external child-care, we obtain $l_t^{fY} = 1-\tilde{l}_t = 1-n(k_t)/\phi \equiv l^{fY}(k_t)$, where $dl_t^{fY}/dk_t > 0$. Capital accumulation demands for more female labor in the goods production.

The equilibrium condition in the capital market is given as

$$k_{t+1} = \frac{\rho}{\varepsilon\phi}F_L[k_t, 1+l^{fY}(k_t)], \tag{23}$$

from which we can obtain

$$\frac{dk_{t+1}}{dk_t} = \frac{\rho}{\varepsilon\phi}(F_{LK} + F_{LL}\frac{dl^{fY}}{dk_t}) = \frac{\rho F_{LK}}{\varepsilon\phi}[1-\frac{bF_{LL}}{(F_L)^2}\frac{\varepsilon}{1+\varepsilon+\rho}]^{-1} > 0, \tag{24}$$

and

$$\frac{dw_t^f}{dk_t} = F_{LK} + F_{LL}\frac{dl_t^{fY}}{dk_t} > 0. \tag{25}$$

Case (iii) From (9a') and (9b), we have

$$k_{t+1} = \frac{\rho}{\phi(1+\rho)}[F_L(k_t, 1) + b], \tag{26}$$

from which we obtain

$$\frac{dk_{t+1}}{dk_t} = \frac{\rho}{\phi(1+\rho)}F_{LK} > 0, \tag{27}$$

and

$$\frac{dw_t^f}{dk_t} = F_{LK} > 0.^{14} \tag{28}$$

3.2 Dynamics of the development path

In this section we examine the time paths of the female wage rate and the fertility rate, assuming that the initial per-couple capital stock is sufficiently small. For expositional purpose we assume that the economy does not fall into a trap of long-term equilibrium in cases (ii) and (iii). We also assume $\frac{b\varepsilon(\mu-1)}{\mu(1+\varepsilon+\rho)} < A$.[15] The latter assumption means that child-care production requires sufficient goods inputs in addition to female labor, so that there is some range of the female wage rate in which women wish to rear children at home rather than to spend a high price of external child-care.

Presuming first that the initial female wage rate is low enough to satisfy $w_t^f / w_t^m \le \varepsilon/(1+\rho)$ or, equivalently, $w_t^f \le \varepsilon b/(1+\rho-\varepsilon)$, then the economy is in case (iii) in which the fertility rate is constant i.e., $n_t = \phi$. The female wage rate is too low for women to supply labor to the market. If the marginal utility of having children is still high at $\tilde{l}_t = 1$, i.e., if the women is in the corner solution, the couple gives up the utility from having more children.

From (27) and (28), capital accumulates and the female wage rate rises as time passes. After the female wage rate becomes high enough to satisfy $w_t^f / w_t^m > \varepsilon/(1+\rho)$ or, equivalently, $w_t^f > \varepsilon b/(1+\rho-\varepsilon)$, the economy goes into the phase of case (ii).[16] At

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[14] Although no women work in the market, the potential female wage rate is given by the marginal product of labor in the goods production sector.

[15] If this assumption is not satisfied, the fertility rate might not decrease with the female wage rate. However, in actuality, not only developed but also even developing countries have experienced fertility declines in the past decades. Therefore, this assumption is apparently plausible.

[16] We cannot *a priori* rule out the possibility that the economy is trapped in a lower equilibrium in case (iii). In this case, the long-term per-couple capital is given by $k_{(iii)} = [\rho/\phi(1+\rho)][F_L(k_{(iii)},1)+b]$, where $F_L(k_{(iii)},1) < b\varepsilon/(1+\rho-\varepsilon)$ and the fertility rate is $n_{(iii)} = \phi$.

$w_t^f(k_t) = b\varepsilon/(1+\rho-\varepsilon)$, we have $n_t = \phi$ and $k_{t+1} = \rho b/[\phi(1+\rho-\varepsilon)]$. Therefore, the paths of n_t and k_{t+1} are continuous in k_t at $w_t^f(k_t) = b\varepsilon/(1+\rho-\varepsilon)$.

In phase (ii), the fertility rate decreases although the female wage rate increases (see (22)). Since having too many children makes the opportunity cost of child-rearing time at home higher, couples reduces the number of children until the marginal utility of having children equals the opportunity cost. Women reduce internal child-rearing time at home and increase the labor supply to the market. The increased female wage income increases per-couple capital, which in turn increases the wage rates as can be seen from (24) and (25). However, unless the female wage rate becomes higher than the price of external child-care, couples are not willing to purchase external child-care.

After the female wage rate becomes high enough to satisfy $w_t^f \geq \mu A/(\mu-1)$, the economy moves into the phase of case (i) in which $w_t^f \geq P_t$. [17] Substituting $w_t^f(k_t) = \mu A/(\mu-1)$ into (16), (18), (21), and (23), we can show that the time paths of n_t and k_{t+1} are continuous in k_t at $w_t^f(k_t) = \mu A/(\mu-1)$, at which per couple capital level the values of the fertility rate and per couple capital stock are $n_t = \varepsilon\phi[2+b(\mu-1)/(\mu A)]/(1+\varepsilon+\rho)$ and $k_{t+1} = \rho\mu A/[\varepsilon\phi(\mu-1)]$ for both cases (ii) and (i). Therefore, the time paths are also continuous in k_t at $w_t^f(k_t) = \mu A/(\mu-1)$. At this point, women start to supply full time to the labor market, purchasing external child-care to have children. The female market labor is allocated between goods production and external child-care production sector so as to equate the female wage rate in both sectors although mothers rear their children at home just before the wage rate satisfies $w_t^f = P_t$. It is noteworthy that the marginal product of female labor in goods production is decreasing whilst that in the child-care production is constant.

The fertility rate increases with per-couple capital, i.e., $dn_t/dk_t > 0$, as in (17). Since

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[17] We cannot also *a priori* rule out the possibility that the economy is trapped in a lower long-term equilibrium in case (ii), in which the per-couple capital is given by $k_{(ii)} = (\rho/\varepsilon\phi)F_L[k_{(ii)}, 1+l^{fY}(k_{(ii)})]$, where $F_L[k_{(ii)}, 1+l^{fY}(k_{(ii)})] < \mu A/(\mu-1)$. In this case the economy does not have a fertility rebound.

the female wage rate increases with per-couple capital (see (20)), the fertility rate rises as the female wage rate increases through the income effect. As can be seen from (13), the price of external child-care becomes lower relative to the female wage rate as the female wage rate increases. Therefore, couples can afford to purchase more external child-care as the female wage rate rises.

However, because per-couple capital and hence the female wage rate stay constant in the long-term equilibrium, the fertility rate also becomes constant in the long term, as can be seen from (18). If the stability condition is satisfied, we have the long-term per-couple capital satisfying[18]

$$k = \frac{\rho}{\varepsilon\phi\mu}\{F_L[k, 1 + l^{fY}(k)] + \mu A\}, \tag{29}$$

and the long-term fertility rate satisfying

$$n = \frac{\varepsilon\phi}{1+\varepsilon+\rho}\frac{2F_L(k, 2 - n/\mu\phi) + b}{F_L(k, 2 - n/\mu\phi)/\mu + A}. \tag{30}$$

The long-term fertility rate might be higher than, equal to, or lower than ϕ, depending upon parameters of the economy. The gender wage gap still remains even in the long term, but it is smaller when the long-term per couple capital is greater.

The time path of the fertility rate is illustrated in Figure 1, where the horizontal line measures the level of per-couple capital stock. The path of the fertility rate shows a reverse-J-shape or U-shape in the female wage rate (and per-couple capital stock).[19] The result of a fertility rebound in this paper is in contrast to the result in Morita and Yakita (2016) and Galor and Weil (1996), in which a certain fraction of child-care must be provided at home by mothers although the price of external child-care is always lower than the female wage rate without policies.[20] In their setting, therefore, increases in female wage raises the opportunity cost of internal child-care proportionally. In this paper, contrastingly, women endogenously reduce internal child-care, increasing market labor supply. As the female wage rate is higher beyond a certain level, the price of external child-care becomes lower relative to the opportunity cost of internal child-care. Consequently, capital accumulation rises the female wage rate, which in turn increases the fertility rate.[21] However, it is noteworthy that because the external child-care

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[18] The stability condition is given by $dk_{t+1}/dk_t < 1$.

[19] Figure 1 illustrates a case in which mothers care their children at home in case (ii), i.e., $\tilde{l}_t < 1$, even when the female wage rate approaches $w_t^f(k_t) = \mu A/(\mu - 1)$. If $\tilde{l}_t = 1$ at $w_t^f(k_t) = \mu A/(\mu - 1)$, the time path might have a flat part correspondingly.

[20] In Galor and Weil (1996), the fraction is zero.

[21] We cannot *a priori* rule out the case in which the long-term fertility rate might be

demand splits off a fraction of the resources from the goods production sector to child-care production sector, the speed of capital accumulation might be slowed down in case (i) as compared to case (ii) (see Appendix).

4. Concluding remarks

We have shown that a fertility rebound can occur as the female wage rate rises along economic development. Under plausible conditions, capital accumulation raises the marginal product of mental labor and hence the female wage rate. Unless the economy is trapped in a low equilibrium of economic development, the fertility rate starts to decline first but then turns upward at higher levels of the female wage rate. Eventually, the fertility rate might approach a constant rate as the economy converges to the long-term equilibrium. It is noteworthy, therefore, that the fertility rebound can occur even without family policy changes.

There are two factors to be considered in future research. First, there might be such social norms that men should work outside the home whilst the activities of married women are best confined to the home and family. Goldin et al. (2006) report that the ratio of the college freshmen women in US approved this notion drastically fell from 41 percent in 1967 to 17 percent in 1973, although 51.6 percent of people did not oppose the notion in Japan even in 2012 (Cabinet Office, Government of Japan, 2012). In modern developed economies, such social factors can affect female labor supply rather than the biological heterogeneity of labor between sexes does. Second, child-care policy might actually affect fertility and labor supply behaviors of couples. Blau and Robins (1988) and Connelly (1992) use US data in 1980 and 1988, respectively, to demonstrate that mothers' labor force participation of was sensitive to the subsidy policy for child-care. They also emphasized the effects of informal child-care mostly provided by a relative on the probability of employment of mothers.

higher than ϕ.

Appendix

To show the dynamics of the system in terms of per-couple capital stock, we assume a Cobb-Douglas production function:

$$Y_t = BK_t^{\alpha} L_t^{1-\alpha} \quad (B > 0;\ 0 < \alpha < 1),$$

or in per couple terms,

$$(Y_t / N_t) = Bk_t^{\alpha}(1 + l_t^{fY})^{1-\alpha}.$$

Equation (27) can be rewritten as

$$\frac{dk_{t+1}}{dk_t} = \frac{\rho}{\phi(1+\rho)}(1-\alpha)\alpha Bk_t^{\alpha-1}, \tag{27'}$$

where $l_t^{fY} = 0$ in case (iii). Equation (24) in case (ii) becomes

$$\frac{dk_{t+1}}{dk_t} = \frac{\rho}{\varepsilon\phi}(1-\alpha)\alpha Bk_t^{\alpha-1}(2-\frac{n_t}{\phi})^{-\alpha-1}[(2-\frac{n_t}{\phi}) - \frac{k_t}{\phi}\frac{dn_t}{dk_t}], \tag{24'}$$

and (19) in case (i) becomes

$$\frac{dk_{t+1}}{dk_t} = \frac{\rho}{\varepsilon\phi\mu}(1-\alpha)\alpha Bk_t^{\alpha-1}(2-\frac{n_t}{\phi\mu})^{-\alpha-1}[(2-\frac{n_t}{\phi\mu}) - \frac{k_t}{\phi\mu}\frac{dn_t}{dk_t}]. \tag{19'}$$

As long as ε is smaller than one, the right-hand side of (27') is less than that on (24') because $dn_t / dk_t < 0$ in case (ii). Next, from (19') and (24'), one has

$$\left.\frac{\left.\frac{dk_{t+1}}{dk_t}\right|_{(19')}}{\left.\frac{dk_{t+1}}{dk_t}\right|_{(24')}}\right|_{w_t^f = \frac{\mu A}{\mu-1}} = \frac{1}{\mu}\left(\frac{2-\frac{n_t}{\phi}}{2-\frac{n_t}{\phi\mu}}\right)^{\alpha} \frac{[1-(\frac{k_t}{\phi\mu}\left.\frac{dn_t}{dk_t}\right|_{(19')})/(2-\frac{n_t}{\phi\mu})]}{[1-(\frac{k_t}{\phi}\left.\frac{dn_t}{dk_t}\right|_{(24')})/(2-\frac{n_t}{\phi})]} < 1, \tag{A1}$$

because $dn_t / dk_t > 0$ in (19') of case (i), $dn_t / dk_t < 0$ in (24') of case (ii), and $\mu > 1$.

Therefore, the right-hand side of (24') is greater than that of (19') at $w_t^f = \mu A/(\mu - 1)$.

As shown in the text, we have $dk_{t+1} / dk_t > 0$ for any k_t below the steady state in case (i). Figure A1 illustrates the dynamics, assuming that the economy is not trapped on the way to the steady state k_∞. It is worthy to note that per couple capital in steady state of case (ii) might be higher than that of case (i). A part of female labor is allocated to the child-care production and the proportion increases with fertility.

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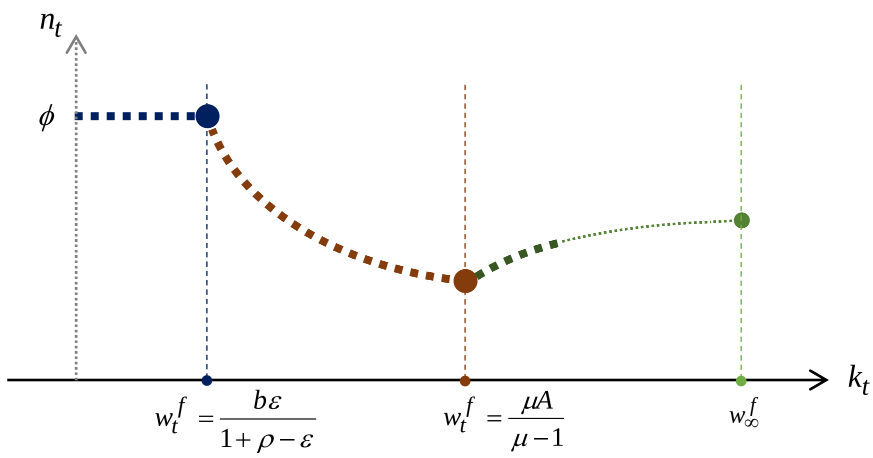
Figure 1 Changes in the fertility rate



l^{fY}	0	$0 < l^{fY} < 1$	$0 < l^{fY} < 1$
l^{fX}	0	0	$0 < l^{fX} < 1$
l^f	0	$0 < l^f = l^{fY} < 1$	$l^f = l^{fY} + l^{fX} = 1$
$\tilde{l}^f$	1	$0 < \tilde{l}^f < 1$	0

where $1 = \tilde{l}_t + l_t^f = \tilde{l}_t + l_t^{fY} + l_t^{fX}$