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Policy Implications of the Bargaining Family

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ABSTRACT

We consider a two-stage family game in which women and men choose education levels in stage 1 and choose the amount of contribution to family public goods in stage 2. If they cannot commit themselves to decisions of the provision of family public goods, the stage-2 decision might be made through bargaining. That possibility affects the stage-1 decision. We show that bargaining in stage 2 engenders over-investment in education and under-provision of family public goods. To achieve an efficient level of family public goods, government must rely on policies related to education choices rather than those related to contribution decisions.

Keywords: Nash bargaining; family public goods; family policy
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1. Introduction

This study analyzes the effects of strategic behaviors of individuals related to education and family public goods provision. Individuals in any society might be expected to behave cooperatively to maximize the future welfare of the society. Although this is true for decisions related to both premarital investment in human capital accumulation and the provision of family public goods, they might alter their behaviors to be strategic in providing family public goods such as child rearing after marriage. They might do so because investment in education is determined personally before marriage. They do not know who is a partner. However, after marriage, a woman and a man must share housework as a particular couple despite their potential inherent mutual differences. Spouses might wish to take advantage during bargaining related to family public goods provision between them. Therefore, without commitment, they must consider the possibility of bargaining even when choosing a level of investment in education before marriage. This paper presents an examination of the consequence of such a family bargaining game and then, comparing its sequence with the efficient solution, derives policy implications for family decisions. The efficient solution is obtainable by unitary cooperation with commitment throughout stages before and after marriage.

The possibility of no commitment in family decision-making has been emphasized by Kemnitz and Thum (2015), who described that family decisions made cooperatively at earlier stages might be altered by changes in the relative bargaining power of women and men. Such a change in bargaining powers engenders time inconsistency issues related to family decision-making.[1] Konrad and Lommerud (2000) considered that human capital investment decisions might be made non-cooperatively, although daily life decisions related to family public goods might be made through bargaining between spouses.[2] Rasul (2008) used the Malaysia Family Life Survey to show that spouses bargain without commitment. If couples bargain without commitment, then the influence of each spouse's preference on fertility outcomes depends on the relevant threat point in marital bargaining, and the distribution of bargaining power. Mazzocco (2007) used US data to test intra-family commitment and concluded that

[1] Basu (2006) asserted that household decisions affect the distribution of bargaining power between genders, which in turn alters household decisions. Iyigun and Walsh (2007b) analyzed effects of the difference in the relative population sizes of women and men on education investments of each gender before marriage, assuming similar feedback effects on relative bargaining power.

[2] Rainer (2008) extended Konrad and Lommerud (2000) by assuming that wives have a comparative advantage in the household activity to examine when and how couples can achieve effective outcomes in a self-enforcing manner.

non-commitment collective models might be appropriate for policy making. Without commitment, women (or wives) wish to supply labor to the market at a lower wage rate than with commitment in Kemnitz and Thum (2015), whereas women invest more in their human capital than in the efficient solution in Konrad and Lommerud (2000). In both studies, the change of the behavioral rule tends to lower family public goods provision.

We consider a two-stage game, i.e., before-marriage and after-marriage, as reported also by Konrad and Lommerud (2000). Women and men choose education levels in stage 1 and decide upon contributions to family public goods through Nash bargaining in stage 2.[3] The main result is that, without commitment, the possibility of family bargaining related to public goods in stage 2 engenders over-investment in education and under-provision of family public goods. This result is consistent with those described by Lundberg and Pollak (2003) and Pollak (2011). To achieve the efficient level of family public goods, government must rely on policies related to education choices rather than on decisions of contributions. The latter result has important implications for family policies in developed countries with low fertility. In this paper, family public goods is specified as child-rearing time of parents. The number of children the couple has depends on the sum of their rearing time.

The game examined herein can be regarded as an interesting extension of the analysis described by Konrad and Lommerud (2000). The next section introduces the model. Section 3 presents analysis of the two-stage game. Section 4 explains policy implications of the theoretical results. The last section concludes the paper.

2. Model

We assume a variation of the two-stage family decisions model described by Konrad and Lommerud (2000), who compared the Nash bargaining outcome with a full non-cooperative game instead of full cooperation throughout two stages. Therefore, we make use of the same notation of variables unless it is necessary to distinguish them.

A family is assumed to consist of a woman and a man, i.e., a couple. Our main concern is the provision of family public goods in stage 2. Therefore, we assume equal sizes of women and men population to avoid the issue of matching individuals. The marriage matching process is also assumed to be exogenous. Each individual has a payoff function

[3] Lundberg and Pollak (1996) and Pollak (2011) describes that the Nash bargaining model have become the standard tool for analyzing intrafamily allocation.

$$u_i = c_i + G - a(g_i) - b(w_i). \quad (1)$$

Subscript i denotes the gender, $i = f, m$, where f and m represent female and male. We assume away discounting of persons in this paper for simplicity. G is the amount of family public goods, which is the sum of individual time contributions of the two, i.e.,

$$G = g_f + g_m. \quad (2)$$

Denoting the time endowment of a person by y, a person can allocate it between market labor, $y - g_i$, and the contribution to family public goods, g_i. Person i's consumption is given as

$$c_i = (y - g_i)w_i, \quad (3)$$

where w_i denotes the person i's market wage rate. The right-hand side of (3) is wage income of person i. Spouses do not pool their wage income.[4] The contribution to family public goods has a psychic cost that is measured using a strictly convex cost function $a(g_i)$, $a'(g_i) > 0$ and $a''(g_i) > 0$. Individuals also choose efforts on educational activities that increase their wage rate. The effort to gain wage w_i is expressed as a strictly convex function $b(w_i)$, $b'(w_i) > 0$ and $b''(w_i) > 0$.

In stage 1 of our two-stage game, individuals simultaneously choose their education levels and thereby their wage rates. In stage 2, individuals know the choices of wage rates in stage 1 and simultaneously decide how long they devote to provision of family public goods. We assume that women and men jointly make education decisions, although they might not want to commit to the joint decision related to family public goods provision to be made in stage 2. A choice of education is often made before making up a family. Therefore, if persons wish to be happy during marital time, then their decision-making might be done jointly in a society because each person might not know who is a partner. Once they are married as a particular couple, however, there can be conflicts between them in the allocation of contributions to family public goods for given

[4] Attanasio and Lechene (2002) tested and rejected the income pooling hypothesis using PROGRESA data of rural Mexico.

education choices.[5] Chiappori et al. (2009) reported that women increased their market work whereas men reduced their market work and increased their non-market work in the United States during 1975–2003. Women and men have come to behave more equally. Therefore, decisions related to contributions to family public goods (or market labor supply) might be regarded as the outcome of bargaining between spouses with equal bargaining power. For these analyses, we assume that the contribution decision is made through Nash bargaining.

We now solve such two-stage family problems backward. First, starting from a situation with given education choices of women and men (w_f, w_m), we calculate a Nash bargaining solution in stage 2. Because we are concerned with provision of family public goods, we assume here that couples do not legally divorce once they have married. Instead, they might divorce or separate within the home.[6] The non-cooperative Nash equilibrium is taken as the threat point of a Nash cooperative bargaining solution.[7] Next, we calculate the problem for a case in which both decisions related to the education and contribution choices are made unitarily between women and men. This unitarily cooperative case provides an efficient solution by definition. In the present setting with identical individuals, it can be justified that the social utility weight for each person is the same, which is normalized to one. Therefore, the efficient solution coincides with the first-best solution. Finally, assuming that family public goods provisions can be renegotiated and bargained between spouses in stage 2, we examine the outcome of the Nash bargaining over provision of family public goods. Hereinafter, the game with unitary cooperation in both stages is called game C, whereas the game with unitary cooperation in stage 1 and Nash bargaining in stage 2 is called game NB.[8] Therefore, the purpose of this paper can be redescribed as a comparison of the education and contribution levels in game NB with those in game C.

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[5] We assume away biological gender differences in this paper for analytical simplicity.

[6] Lundberg and Pollak (1993) proposed a separate spheres model in which the threat point from which cooperative Nash bargaining proceeds is not divorce, but a non-cooperative equilibrium within marriage. Our assumption of Nash bargaining is similar to the notion of the separate sphere in their model.

[7] Nash equilibrium does not necessarily mean a legal divorce because family public goods are commonly available to both once they are provided. The threat point is internal to the marriage in this noncooperative marriage. This model is called a noncooperative marriage in Lundberg and Pollak (1996).

[8] Game NB in this paper is the same as game C of Konrad and Lommerud (2000).

3. Family decisions related to the contribution to public goods decisions

3.1 Contribution decisions in stage 2

In this section, for given education choices (w_f, w_m), we first examine non-cooperative Nash equilibrium, then unitary cooperation, and finally the Nash bargaining solution. The Nash equilibrium (g_f^*, g_m^*) is obtainable as follows. Equilibrium g_i^* maximizes

$$u_i = (y - g_i)w_i + g_i + g_j^* - a(g_i) - b(w_i) \tag{4}$$

for $g_i \in [0, y]$ $(i, j = f, m$ and $i \neq j)$. Given the convexity of $a(g_i)$, the contribution can be determined uniquely. Assuming an interior solution, the equilibrium contribution is obtained using the first-order condition

$$-w_i + 1 - a'(g_i^*) = 0. \tag{5}$$

From (5) we obtain

$$g_i^{*\prime} \equiv dg_i^* / dw_i = -1/a''(g_i^*) < 0. \tag{6}$$

The sign of (6) is obtainable from the convexity of $a(g_i)$. The Nash-equilibrium utility levels in the stage-2 non-cooperative game are denoted as $u_i^* = u_i(g_f^*, g_m^*)$.

Next, we analyze the unitary cooperation in stage 2 for given (w_f, w_m). The game in stage 1 is also unitarily cooperative. Therefore, the solution gives the efficiency outcome. We assume that monetary and utility transfers between family members are allowed. The cooperative levels of contribution for public goods (g_f^e, g_m^e) are determined by the first-order conditions as

$$-w_f + 2 - a'(g_f^e) = -w_m + 2 - a'(g_m^e) = 0. \tag{7}$$

Here we assume an interior solution. From the convexity of $a(g_i)$, the contributions

are determined uniquely; they satisfy $g_i^e{}' \equiv dg_i^e / dw_i = -1/a''(g_i^e) < 0$. From (5) and (7), it follows that $g_i^e(w_i) > g_i^*(w_i)$. Because cooperative contribution includes the effect on the partner's utility, each person is willing to contribute more than in Nash equilibrium. This result of a "downward bias" caused by family bargaining is also obtained in Konrad and Lommerud (2000) and Kemnitz and Thum (2015).

We now turn to the Nash bargaining solution in stage 2, where the threat point is given as (u_f^*, u_m^*). Because efficiency contributions of g_f^e and g_m^e depend respectively only on w_f and w_m, the utility possibility frontier is $u_f = V - u_m$, with

$$\begin{aligned} V &= u_f(g_f^e, g_m^e) + u_m(g_f^e, g_m^e) \\ &= \sum_{i\in(f,m)} [(y - g_i^e)w_i - a(g_i^e) - b(w_i)] + 2(g_f^e + g_m^e). \end{aligned} \tag{8}$$

The utility possibility frontier is linear with slope of negative one. The Nash bargaining solution brings about utilities[9]

$$u_i^{NB} = \frac{V}{2} + \frac{u_i^* - u_j^*}{2} \quad \text{for} \quad i, j = f, m \quad \text{and} \quad i \neq j. \tag{9}$$

3.2 Education decisions in stage 1

Next we examine the choices on education levels in stage 1 when the provision of family public goods is chosen through Nash bargaining in stage 2. To compare the non-commitment solution with the efficient solution, we examine the case in which decisions about contributions to family public goods are also made unitarily in stage 2 because education choice depends on whether stage 2 is characterized by unitary cooperation or by Nash bargaining.

First, we consider game C with unitary cooperation in stage 2, which leads to an efficient solution. Individual i's problem is to choose a wage rate w_i that maximizes

[9] The solution can be obtained by maximizing $(u_f - u_f^*)(u_m - u_m^*)$ subject to the utility possibility frontier.

$$\sum_{i\in(f,m)}[(y-g_i^e)w_i - a(g_i^e) - b(w_i)] + 2(g_f^e + g_m^e), \tag{8'}$$

where $g_i^e(w_i)$ is determined in (7). The first-order condition is

$$y - g_i^e(w_i^C) - b'(w_i^C) = 0. \tag{10}$$

In deriving (10), we used (7).

Next, we consider a case of Nash bargaining within the family in stage 2, i.e., game NB. Because individuals are symmetrical in this paper, the equilibrium wage rate of individual i can be formalized as the choice of w_i that maximizes[10]

$$\begin{aligned} u_i^{NB} = \frac{1}{2}\{ & \sum_{i\in(f,m)}[(y-g_i^e)w_i - a(g_i^e) - b(w_i)] + 2(g_f^e + g_m^e) \\ & + [(y-g_f^*)w_f + g_f^* + g_f^* - a(g_f^*) - b(w_f)] \\ & - [(y-g_m^*)w_m + g_f^* + g_m^* - a(g_m^*) - b(w_m)]\}. \end{aligned} \tag{11}$$

The first-order condition for $i = f$ is

$$y - g_i^*(w_f^{NB}) - b'(w_f^{NB}) - g_f^{*\prime}(w_f^{NB}) = 0, \tag{12}$$

where we use (5) and (7). Conditions (5) and (7) hold in each type of games, respectively, taking the respective wage rate as given.[11] The last term on the right-hand side of (12) represents the redistribution according to the Nash bargaining game. Because of the increased wage rate, the individual will marginally change her contribution to family public goods production by $g_f^{*\prime}(w_f)$. This change in turn reduces the male's utility by the same amount.

Now we compare the education level in game NB with that in game C. Differentiating (11) with respect to w_f and evaluating at w_f^C, we obtain

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[10] If asymmetry between genders is assumed, then the analysis is more complicated. Rainer (2008) specifically addressed the differences in ability between genders.

[11] Because we assume symmetric spouses in a couple, the same arguments can apply to the husbands as well. In the following we proceed with the analysis by concentrating on women.

$$\frac{\partial u_f^{NB}}{\partial w_f} = \frac{1}{2}[y - g_f^*(w_f^C) - b'(w_f^C) - g_f^{*\prime}(w_f^C)]. \tag{13}$$

Therefore, from (12) we have[12]

$$w_f^{NB} \underset{<}{>} w_f^C \quad \text{as} \quad \frac{\partial u_f^{NB}}{\partial w_f} \underset{<}{>} 0. \tag{14}$$

Because $b'(w_f^C) = y - g_f^e(w_f^C)$ from (10), we can rewrite (13) as

$$\left.\frac{\partial u_f^{NB}}{\partial w_f}\right|_{w_f = w_f^C} = \frac{1}{2}[g_f^e(w_f^C) - g_f^*(w_f^C) - g_f^{*\prime}(w_f^C)]. \tag{15}$$

Because $g_i^e(w_i) - g_i^*(w_i) > 0$ and $g_i'(w_i) < 0$, it follows that $\left.\partial u_f^{NB} / \partial w_f\right|_{w_f = w_f^C} > 0$ and that we therefore have $w_f^{NB} > w_f^C$, which leads to the following proposition.

Proposition 1. *Presuming that contributions to family public goods are bargained cooperatively between women and men, then education levels are always higher than those that would be obtained by unitary cooperation in both stages.*

It is noteworthy that the education level in game NB is higher than the education level in game C, irrespective of whether function $a'(g_i)$ is convex or concave. Nash bargaining between spouses in stage 2 invariably results in overinvestment in education. In Konrad and Lommerud (2000) with Nash equilibrium in stage 1, whether the education level chosen in Nash bargaining in stage 2 with the education level determined non-cooperatively in stage 1 is greater than, equal to, or less than that obtained in Nash equilibrium depends on the characteristics of function $a'(g)$. In contrast, in this paper, education choices are jointly made. A marginal increase in her wage at w_f^C might change her utility at a threat point although it does not change her utility, u_f, at w_f^C. Because of the changed threat-point utility, she will marginally

[12] Function u_i is a concave function of w_i.

reduce her contribution to public goods. Not only the lower contribution increases her labor income, but also it lowers the marginal psychic cost. These effects can be represented by the terms on the right-hand side of (15). The first two terms in bracket corresponds to the efficiency effect and the last term is the redistribution effect. Both effects are positive in our setting in which investment in education is determined by unitary cooperation in stage 1. Therefore, in the case of symmetric individuals, each spouse has an incentive to invest more in game NB than in game C. The lack of commitment to family public goods provision distorts upward the incentives to invest in education in stage 1.

Because $g_i'(w_i) < 0$, we have $g_i^{NB} = g_i(w_i^{NB}) < g_i^C = g_i(w_i^C)$.[13] The level of family public goods in game NB is lower than that in game C. g_i^C is obtainable by unitary cooperation in both stages. Therefore, Nash bargaining in stage 2 also renders the level of family public goods provision lower than in the efficient solution. It is noteworthy, however, that given education levels of spouses, the bargaining in stage 2 engenders the efficient outcome *per se*.[14]

3.3 Policy implications for fertility decision

The previous subsection shows that Nash bargaining in stage 2 affects the choice of education levels in stage 1. Child rearing can be regarded as a family public good.[15] Therefore, the number of children might depend on the total contribution to family public goods.[16] Following Galor and Weil (1996), the number of children is proportional to the time spent by parents, i.e., $n = (g_f + g_m)/z \equiv \nu(g_f + g_m)$, where n denotes

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[13] Assuming interior solutions, we have $g_i^{NB}(w_i) = g_i^C(w_i)$ because (7) holds for both games C and NB.

[14] This result is consistent with Lundberg and Pollak (2003) and Pollak (2011), who suggest, referring to two-stage cooperative bargaining models, that when the spouses cannot make binding commitments, the first-stage decision may be an inefficient allocation. Kemnitz and Thum (2015) also show, in a collective model, that a non-commitment child-care choice game engenders a downward bias in fertility choice.

[15] We are concerned here with child-rearing time although family public goods include other goods and services such as houses, gardens and housework.

[16] Although Iyigun and Walsh (2007a) emphasized a biological difference between genders, Gupta and Smith (2002) reported that there is no indication that rearing children had any long-term negative effects on the earning potential of their mothers in Denmark during 1980–1995. If child rearing has no effect on mothers earning potentials, the absence of mothers' specific contributions to child rearing might be negligible in formulating lifetime earnings.

the number of children in the family and z is the cost in time of raising one child.[17] With Nash bargaining in stage 2, the chosen number of children is smaller than that obtained under the efficient solution. In the literature related to family economics, it is often suggested that governments of nations that are affected by low fertility should pursue some family policy such as a child allowance or child-care policy. Such policies have actually been implemented in many countries (see, for example, Luci-Greulich and Thévenon, 2013). The theoretical result obtained in the previous section implies that, if fertility decisions are made by family bargaining, then education policy rather than family policy is necessary to increase the number of children and thereby increase the fertility rate.

In this section, to examine policy implications for family decisions, we analyze the effects of changes in education and family policies. It is noteworthy that the tax and subsidy policies are not applied to all levels of family public goods provision but only influence a threat point of Nash bargaining in stage 2.[18,19]

For instance, presuming that a subsidy in support of contributions to family public goods at rate s is financed through lump-sum taxes on individuals, then the terms in the brackets on the right-hand side of (15) become[20]

$$g_f^e(w_f^C) - g_f^*(w_f^C) - (1+s)g_f^{*\prime}(w_f^C) > 0 . \qquad (16)$$

The subsidy might increase the difference between education levels in game C and NB. Therefore, family policy related to child bearing cannot alleviate inefficiency in fertility decisions by itself. This result is in contrast to the result of Boadway et al.

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[17] In this case, representing children n as family public goods, the payoff of individual can be written as $u_i = (y - g_i)w_i + v(g_f + g_m) - a(g_i) - b(w_i)$. For analytical simplicity, we assume away inherent gender differences in the roles of raising children between women and men in this paper. Although only mothers can actually generate children, both spouses can provide internal child care even in the event of divorce within the home, i.e., at the threat point of this game. Though, even with respect to legal divorce, Alesina and Giuliano (2006) among others reported that the introduction of unilateral divorce does not seem to affect total marital fertility by encouraging women planning to have children marry more easily.

[18] Konrad and Lommerud (1995) pointed out that if the non-cooperative equilibrium only serves as a threat point for cooperative bargaining outcomes, it matters for the impact of a policy who would gain more in the non-cooperative equilibrium from the shift in the threat point due to the policy.

[19] We assume that government can commit itself to implementing these tax/subsidy policies certainly, i.e., government is not a player of the Nash equilibrium game. Basu (2011) might doubt such an assumption.

[20] Even though child-care activities might not be observable to governments, the number of children can be regarded as the tax base if it is positively related to child-rearing time spent by parents.

(1989) and Konrad and Lommerud (1995), who argued that lump-sum-tax-financed subsidies for the public goods reduce the under-provision problem by increasing the incentives for private provision of the public goods. The difference from them mainly stems from endogenous education investment in stage 1 in our model.[21] The subsidy policy rather aggravates the inefficiency issue by affecting a threat point for cooperative bargaining.[22] In this model, even publicly-provided child care cannot raise the fertility because the first-order conditions for education investment is not affected by the provision policy.

To increase the fertility rate, therefore, the government must rely on education policy even if education decisions are made through unitary cooperation in stage 1. A government might impose a tax on education investment in the situation of Nash bargaining of game NB, although taxes are not imposed in the case of unitary cooperation over two stages. Letting τ be the tax rate, the terms in the brackets on the right-hand side of (15) is

$$[g_f^e(w_f^C) - g_f^*(w_f^C) - g_f^{*\prime}(w_f^C)] - \tau[y - g_f^e(w_f^C)]. \tag{17}$$

The second term on the right-hand side of (17) is negative. The first term is positive. The tax policy can alleviate the inefficiency issue by shifting a threat point. Therefore, the tax might achieve the efficiency education level under certain conditions. If the policy reduces the education levels of individuals in stage 1 to the efficient level, then public goods provisions might increase toward the efficiency level.[23]

Proposition 2. *Presuming that the contributions to family public goods are negotiated cooperatively between women and men, then a positive tax on educational expenditure might achieve the efficient level of family public goods provision.*

The intuition behind the results is the following: Subsidies for provision of public goods are expected to induce both spouses to increase public goods supply by lowering the cost

[21] Kemnitz and Thum (2015) show that family policies such as maternal care benefits have the potential to correct the inefficiency in fertility choice caused by bargaining about child-care organization in a two-stage collective model which fertility choices are determined in stage 1. In contrast, the number of children is bargained in stage 2 in this paper.

[22] Because of the lack of observability of child-care activities, Konrad and Lommerud (1995) regarded the labor income tax as a second-best policy. However, they also reported that it is when there is no third activity for which time can be used, e.g., leisure, that a lump-sum redistribution tax on labor income has the same effect. In contrast, we can show that the labor income tax cannot alleviate the inefficiency in our setting.

[23] For derivation, see Appendix.

of public goods provision. Therefore, each spouse is likely to increase educational expenditure to take an advantage over the other spouse in bargaining rather than to increase the contribution to family public goods. Taxes on educational expenditure induce each individual to reduce the expenditure because of higher costs. The lowered educational expenditure increases the level of public goods provision in Nash equilibrium as a threat point for cooperative Nash bargaining.

Two remarks follows: First, although the policy arguments resemble those presented by Konrad and Lommerud (2000) who assume Nash equilibrium in stage 1, the education tax policy might not be used because whether over-investment or under-investment occurs depends on the curvature of function $a'(g)$. In contrast, the tax on education investment must be positive because of excessive education investment in our game NB. Second, Konrad and Lommerud (2000) concluded that encouraging family public goods provision can never lead to a first-best situation; discouraging education can do, because the over-investment in education is the model's basic distortion caused by Nash equilibrium in stage 1. In contrast, in our model, the inefficiency caused by Nash bargaining in providing public goods in stage 2 might be alleviated by discouraging education investment.

Nevertheless, it is noteworthy that our result does not mean that investment in education must always be taxed. It is the case only when the chosen investment level is higher than the efficient solution level. Without commitment, spouses want to obtain advantageous benefits in bargaining family public goods provision by bearing additional efficiency costs of excessive education activities.

4. Conclusion

We have presented an examination of a two-stage game of unitary cooperation in stage 1, i.e., before marriage, and Nash bargaining in stage 2, i.e., after marriage; then it was compared with the education and contribution-to-family public-goods decisions with the efficient solution. This game was not analyzed in a study reported by Konrad and Lommerud (2000). The results presented in this paper therefore extend their results. The Nash-bargained contributions to family public goods are lower than those under the efficient solution. The education levels obtained through Nash bargaining in stage 2 are higher than those of the efficient solution.

The main message presented in this paper is the following. We might consider a case in which the provision of family public goods such as child rearing at home is determined without *ex ante* commitment, although education decisions are made

through unitary cooperation, i.e., efficiently. We instead assume Nash bargaining in stage 2. In our case, the level of family public goods chosen will be lower than that of the efficient solution. The policy related to the education decisions, but not to the contribution-to-family public-goods decisions, should be undertaken to increase the number of children and the fertility rate.

From the analysis described in this paper, given that education taxes are absent, the fertility rebounds observed in developed countries might be interpreted as a consequence of possible shifts from family bargaining to unitary cooperation between spouses.[24] Actually, gender-equality policies have recently been adopted in developed countries (Olivetti and Petrongolo, 2017).[25] If the economic situations of spouses become equal, then cooperation rather than bargaining might be plausible.[26] Next, although Chiappori et al. (2009) reported that the gender gaps in the amount of time spent in non-market work declined during 1975–2003, significant gender wage gaps persist, but education levels of women are higher than those of men in several developed countries. This gender twist between wage rates and education levels might affect individuals' decisions on education and fertility. This issue is left as a subject of future research.

Appendix

Proof of Proposition 2

We consider the effects of policies on education investment in game NB. First, we consider a subsidy in relation to children. It is not possible to grasp psychic costs for policy purposes. Therefore, we consider subsidies to family public goods provision g_i at rate s:

[24] Myrskylä et al. (2009) demonstrated that the relation between the total fertility rate and the human development index (HDI) changed from negative to positive using long-term data of more than 100 countries.

[25] The French government introduced fathers' paid child-care leave in 2002, but in Sweden, child-care leave has been compensated with income transfers since 2007. The Swedish government started to award bonuses if parents take child-care leave equally in 2008. As Rainer (2008) concluded, sharing rules such as "equal sharing" can be maintained when women and men have equal opportunities in the labor market. Marriage might not be formal or legal in many economically developed countries.

[26] However, if contributions to family public goods are bargained even with paid leaves for both spouses, over-investments in education might still be a result.

$$u_i^{NB} = \frac{1}{2}\{ \sum_{i\in(f,m)}[(y-g_i^e)w_i - a(g_i^e) - b(w_i)] + 2(g_f^e + g_m^e)$$

$$+[(y-g_f^*)w_f + (1+s)(g_f^* + g_f^*) - a(g_f^*) - b(w_f) - T]$$

$$-[(y-g_m^*)w_m + (1+s)(g_f^* + g_m^*) - a(g_m^*) - b(w_m) - T]\}, \qquad (11')$$

where T is a lump-sum tax. Individuals know that family public goods provision is subsidized only in Nash bargaining case, i.e., at the threat point. From maximization of u_i^{NB}, we obtain (16).

Next, if a tax is imposed on education investment at rate τ, then the education decision in game NB is obtainable from maximizing

$$u_i^{NB} = \frac{1}{2}\{ \sum_{i\in(f,m)}[(y-g_i^e)w_i - a(g_i^e) - b(w_i)] + 2(g_f^e + g_m^e)$$

$$+[(y-g_f^*)w_f + g_f^* + g_f^* - a(g_f^*) - (1+\tau)b(w_f) + B]$$

$$-[(y-g_m^*)w_m + g_f^* + g_m^* - a(g_m^*) - (1+\tau)b(w_m) + B]\} \qquad (11'')$$

for w_f, where B denotes lump-sum transfers from the government. Individuals are assumed to know that education investment is taxed only in Nash bargaining, i.e., at the threat point. From the first-order condition and evaluating at w_f^C, we obtain

$$\frac{\partial 2u_f^{NB}}{\partial w_f} = [g_f^e(w_f) - g_f^*(w_f) - g_f^{*\prime}(w_f)] - \tau[y - g_f^e(w_f)]. \qquad (17')$$

The second term on the right-hand side of (17') is negative. The first term is positive. Therefore, the tax might achieve the efficiency education level under certain conditions. In that case, the optimal tax rate is given by setting (17') equal to zero as

$$\tau = \frac{g_f^e(w_f^C) - g_f^*(w_f^C) - g_f^{*\prime}(w_f^C)}{y - g_f^e(w_f^C)} > 0. \qquad (A1)$$

If the tax rate is less than one, then a tax policy would not achieve efficiency. Because individuals are symmetric, the same argument is applicable to men. Therefore,

$w_i^{NB} = w_i^C$ with such a tax. When the education level is efficient, the level of family public goods is also efficient, i.e., $g_i(w_i^{NB}) = g_i(w_i^C)$. □

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Supplementary note [NOT TO BE PUBLISHED]

Letting t be the wage income tax rate and $Z[=t(y-g_f^*)w_f]$ be a lump-sum transfer from government, we have

$$u_i^{NB}=\frac{1}{2}\{\sum_{i\in(f,m)}[(y-g_i^e)w_i-a(g_i^e)-b(w_i)]+2(g_f^e+g_m^e)$$

$$+[(1+t)(y-g_f^*)w_f+g_f^*+g_f^*-a(g_f^*)-b(w_f)+Z]$$

$$-[(1+t)(y-g_m^*)w_m+g_f^*+g_m^*-a(g_m^*)-b(w_m)+Z]\},\qquad (11^{(3)})$$

from which we obtain

$$\frac{\partial 2u_f^{NB}}{\partial w_f}t[y-g_f^*(w_f)]+[g_f^e(w_f)-g_f^*(w_f)-g_f^{*\prime}(w_f)]>0.\qquad (17'')$$

Therefore, the wage income tax policy cannot alleviate the inefficiency.