Chukyo University Institute of Economics

Discussion Paper Series

August 2016

No. 1609

Environmental Preservation Policy

Keiko Nakayama
Faculty of Economics, Chukyo University, JAPAN
Masatoshi Shirai
Faculty of Economics, Chukyo University, JAPAN
Akio Matsumoto
Faculty of Economics, Chuo University, JAPAN

Environmental Preservation Policy

Keiko Nakayama*, Masatoshi Shirai* and Akio Matsumoto* *
* Faculty of Economics, Chukyo University, JAPAN
** Faculty of Economics, Chuo University, JAPAN
nakayama@mecl.chukyo-u.ac.jp

Abstract

In our modern economy, we face serious environmental problems due to the pollution by the firms and households caused in the production and consumption process. The forest stock in many countries is functioning to purify. However, individuals usually do not cultivate since forest is public goods in nature. The forest stock is dying in many countries, and the world environment is rapidly deteriorating. The government must maintain the clean environment to attain the sustainable growth of the economy.

In this paper, we consider the economy in which one generation produces goods and pollution in the production process and the government plans to invest for cultivating the forest to preserve the environment for the next generation. We suppose that the government imposes taxes to finance the cultivating costs for environmental preservation.

We derive the optimal conditions for investment of capital and cultivating forests for future generation in the steady state economy. We analyze whether or not the government policy to invest for cultivation of forests, and financing by income tax and consumption tax, can attain the efficient state.

We conclude that in the case where the government uses consumption tax, the income level is lower than the level of income in the social optimal case. The steady state pollution level in this case is lower than the social optimal one, so the forest stock level is also lower than optimal.

Key Words: Forest Preservation Tax、Optimal Sustainable Growth of Forest

1. Model

We consider an economy in which individuals live for one period. We call individuals who live in t period the t generation. The number of individuals in each period is assumed to be constant. They produce goods using capital stock, left by the older generation. Individuals allocate the produced goods to their consumption, investment for capital stock and forest cultivation. We assume that the production process generates pollution, which damages individual life, and the forest stock can purify the pollution.

An individual of t generation is supposed to have a separable utility function,

$$u(c_t, Z_t) + (1+\delta)^{-1} u(c_{t+1}, Z_{t+1}) \quad (1)$$

where, c_t , and c_{t+1} are the amounts of consumption of generation t and $t+1$ in the period t and $t+1$, respectively. We assume that the utility function is invariant over generations.

We define the following variables, $y_t, I_t, I_t^M, K_t, M_t, \rho, \zeta$, and Z_t as products, investment, forest conservation investments, capital stock, forest stock, in the period t , and α, β, m as the rate of pollution produced by the capital stock used in the production process, the purification rate of pollution by the forest stock, and the rate of the forest growth by cultivation, respectively.

The following relations must be satisfied in any period:

$$c_t = y_t - I_t - I_t^M \quad (2)$$
$$y_t = f(K_t) \quad (3)$$
$$K_{t+1} = (1-\rho)K_t + I_t \quad (4)$$
$$M_{t+1} = (1-\zeta)M_t + mI_t^M \quad (5)$$
$$Z_t = \alpha K_t - \beta M_t \quad (6)$$

2. The optimization problem

We define the social welfare function as

$$W = \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})\,(1+\delta)^{-(t+\tau)} \quad (7)$$

where, δ is the social discount rate which is assumed to be equal to the individual's. Then the optimal problem is

$$\max_{I_t, I_t^M} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})\,(1+\delta)^{-(t+\tau)} \quad (8-1)$$

subject to (2) -(6).

We define the state evaluation function:

$$J(K_t, M_t) = \max_{I_t, I_t^M}\{u(c_t, Z_t) + (1+\delta)^{-1} J(K_{t+1}, M_{t+1})\} \qquad (9-1)$$

We restrict our attention to a steady state path, assuming that an optimal choice exists and converges to a steady state. Then, we obtain

$$J_K = mJ_{M.} \qquad (10)$$

Furthermore, on the steady state path, the following relations must hold:

$$J_K = u_c f'(K_t) + u_Z \alpha + (1+\delta)^{-1} J_K (1-\rho) \qquad (11)$$

$$J_M = -u_Z \beta + (1+\delta)^{-1} J_M (1-\zeta) \qquad (12)$$

Using (10), we have the optimal condition in the steady state:

$$\frac{u_c f'(K) + u_Z \alpha}{\delta + \rho} = \frac{-u_Z m \beta}{\delta + \zeta} \qquad (13)$$

The left-hand side of (13) is the social marginal benefits of one unit of the investments in capital stock, and the right-hand side of (13) is one of the units of the investment in the forest stock.

3. Individual behavior and the government: the income tax case

In this section, we assume that the government imposes the income tax T_t to finance the cost of forest cultivation investment and the government determines the tax rate so as to maximize individual welfare subjected to individual behavior and the government budget equation.

$$T_t y_t = I_t^M \qquad (14)$$

We assume that the utility of the individual is the sum of his own utility and the discounted utility of his child. And that the individual chooses his own consumption and investment for the capital of his child to maximize his utility, subject to his budget and the existing stock of inherited capital and forest inherited by his parent.

Then we formalize his maximization problem of generation t as

$$\max_{I_t,} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})(1+\delta)^{-(t+\tau)} \qquad (8-2)$$

subject to

$$c_t = (1-T_t)y_t - I_t \qquad (2')$$

$$y_t = f(K_t) \qquad (3)$$

$$K_{t+1} = (1-\rho)K_t + I_t \qquad (4)$$

$$M_{t+1} = (1-\zeta)M_t + mT_t y_t \qquad (5')$$

$$Z_t = \alpha K_t - \beta M_t \qquad (6)$$

We define the evaluate function as:

$$J(K_t, M_t) = \max_{I_t}\{\, u(c_t, Z_t) + (1+\delta)^{-1} J(K_{t+1}, M_{t+1})\}. \qquad (9-2)$$

Then, the optimal condition of the individual restricting the case of the steady state is:

$$u_c = (1+\delta)^{-1} J_K \qquad (15)$$

The individual invests so as to satisfy (15). The government chooses the tax rate T so as to maximize the social welfare (7). The state evaluates the function of government in this case as

$$J(K_t, M_t) = \max_{T_t} u(c_t, Z_t) + (1+\delta)^{-1} J(K_{t+1}, M_{t+1}) \qquad (9-3)$$

The first conditions on the steady state path are:

$$J_K = (1+\delta)u_c \qquad (16)$$

$$mJ_M = (1+\delta)u_c \qquad (17)$$

From (16) and (17), we obtain

$$J_K = mJ_M \qquad (18)$$

Furthermore, on the steady state path, the following condition must hold:

$$J_K = u_c f'(K) + u_Z \alpha + (1+\delta)^{-1} J_K (1-\rho) \quad (19)$$

The conditions (18) and (19) are the same as the first best conditions (11) and (12) in the social optimization problem. Then, we conclude that the government tax policy, which levies the income tax to finance cultivation, can attain efficiency in the steady state.

4. The consumption tax case

In this section, we assume that the government imposes the consumption tax to finance the cost of forest cultivation investment. The government then determines the consumption tax rate to maximize individual welfare subject to individual behavior and the government budget equation:

$$(T_t - 1)c_t = I_t^M \quad (20)$$

The maximization problem of generation t in this case is

$$\max_{I_t,} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})(1+\delta)^{-(t+\tau)} \quad (8-3)$$

subject to

$$T_t c_t = y_t - I_t \quad (2")$$

$$y_t = f(K_t) \quad (3)$$

$$K_{t+1} = (1-\rho)K_t + I_t \quad (4)$$

$$M_{t+1} = (1-\zeta)M_t + m(T_t - 1)c_t \quad (5")$$

$$Z_t = \alpha K_t - \beta M_t \quad (6)$$

The first-order conditions of the individual maximization problem on the steady state path are,

$$-u_c/T + (1+\delta)^{-1} J_K = 0 \quad (21)$$

The government determines the tax rate to maximize the social welfare function with individual behavior given. The maximization problem of the government is:

$$\max_{T_t} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})(1+\delta)^{-(t+\tau)}, \qquad (8-4)$$

subject to the budget constraint (20) and individual behavior (21).

We have the following three optimal conditions by the same procedure as the previous section:

$$J_K = u_C f'(K) + u_z \alpha + (1+\delta)^{-1} J_K (1-\rho) \qquad (22)$$

$$J_M = -u_z \beta + (1+\delta)^{-1} J_M (1-\zeta) \qquad (23)$$

$$J_K = m J_M \qquad (24)$$

On the steady state, using (22)-(24), we have the optimal condition in the steady state in this case:

$$\frac{u_c f'(K)/T + u_Z \alpha}{\delta + \rho} = \frac{-u_Z m \beta}{\delta + \zeta} \qquad (25)$$

Since T is larger than 1, in the case of the government use of consumption tax to finance cultivating cost, the left-hand side of (25) is less than the right-hand side. Therefore, in the consumption tax case, the income level is lower than that of the social optimal case. The steady state pollution level in this case is lower than the social optimal case, so the optimal forest stock level is also lower than the optimal case.

Appendix.

We consider the same economy as before, except that goods are produced using forest stock, inherited from the older generation. We assume here that individuals allocate the produced goods to their consumption, and investment for forest cultivation.

A-1. The social optimization problem

We define the social welfare function as

$$W = \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})\,(1+\delta)^{-(t+\tau)} \qquad (A-1)$$

δ is the social discount rate which is assumed to be equal to that of the individual one. The optimization problem is to determine the allocation of forest inputs, M_t^P and cultivationI_t^M, subject to the following relations:

$$c_t = y_t - I_t^M \qquad (A-2)$$

$$y_t = f(M_t^P) \qquad (A-3)$$

$$M_{t+1} = (1-\zeta)(M_t - M_t^P) + mI_t^M \qquad (A-4)$$

$$Z_t = \alpha M_t^P - \beta(M_t - M_t^P) \qquad (A-5)$$

We define the state evaluation function:

$$J(M_t) = \max_{M_t^P, I_t^M}\{u(c_t, Z_t) + (1+\delta)^{-1} J(M_{t+1})\} \qquad (A-6)$$

subject to (A-1) -(A-5).

Driving the first-order conditions, we have

$$\frac{\partial u}{\partial c_t}\frac{\partial c_t}{\partial M_t^P} + \frac{\partial u}{\partial Z_t}\frac{\partial Z_t}{\partial M_t^P} + (1+\delta)^{-1} J_{M_{t+1}} \frac{\partial M_{t+1}}{\partial M_t^P} = 0 \qquad (A-7)$$

$$\frac{\partial u}{\partial c_t}\frac{\partial c_t}{\partial I_t^M} + \frac{\partial u}{\partial Z_t}\frac{\partial Z_t}{\partial I_t^M} + (1+\delta)^{-1} J_{M_{t+1}} \frac{\partial M_{t+1}}{\partial I_t^M} = 0 \qquad (A-8)$$

The following relations are satisfied:

$$\frac{\partial c_t}{\partial M_t^P} = f'(M_t^P) \qquad (A-9)$$

$$\frac{\partial c_t}{\partial I_t^M} = -1 \qquad (A-10)$$

$$\frac{\partial Z_t}{\partial M_t^P} = \alpha + \beta \qquad (A-11)$$

$$\frac{\partial Z_t}{\partial I_t^M} = 0 \qquad (A-12)$$

$$\frac{\partial M_{t+1}}{\partial M_t^P} = -(1-\varsigma) \qquad (A-13)$$

$$\frac{\partial M_{t+1}}{\partial I_t^M} = \mathrm{m} \qquad (\mathrm{A}-14)$$

Substituting (A-10)-(A-14) into (A-7) and (A-8), respectively, we have

$$u_c f'(M_t^P) + u_Z(\alpha+\beta) - (1+\delta)^{-1} J_{M_{t+1}}(1-\varsigma) = 0 \qquad (\mathrm{A}-15)$$

$$-u_c + (1+\delta)^{-1} m J_{M_{t+1}} = 0 \qquad (\mathrm{A}-16)$$

In the steady state, (A-15) and (A-16) become

,

$$u_c f'(M_t^P) + u_Z(\alpha+\beta) - (1+\delta)^{-1} J_M(1-\varsigma) = 0 \qquad (\mathrm{A}-17)$$

$$(1+\delta)u_c = mJ_M \qquad (\mathrm{A}-18)$$

By substituting (A-16) into (A-15), we have,

$$u_c f'(M_t^P) + u_Z(\alpha+\beta) = (1-\varsigma)u_c/m \qquad (\mathrm{A}-19)$$

We restrict our attention to a steady state path, assuming that an optimal choice exists and converges to a steady state. On the steady state path, the following conditions are satisfied:

$$J_M = u_c \frac{\partial c_t}{\partial M_t} + u_Z \frac{\partial Z_t}{\partial M_t} + (1+\delta)^{-1} J_M \frac{\partial M_{t+1}}{\partial M_t} \qquad (A-20)$$

Where,

$$\frac{\partial c_t}{\partial M_t} = 0 \qquad (A-21)$$

$$\frac{\partial Z_t}{\partial M_t} = -\beta \qquad (A-22)$$

$$\frac{\partial M_{t+1}}{\partial M_t} = 1 - \zeta \qquad (A-23)$$

We obtain the following equations in the steady state,

$$J_M = -u_Z\beta + (1+\delta)^{-1}J_M(1-\zeta) \qquad (A-24)$$

$$-u_Z\beta = \frac{\delta+\zeta}{1+\delta}J_M \qquad (A-25)$$

Substituting (A-18) into (A-25), we have

$$u_c = -\frac{u_Z m\beta}{\delta+\zeta} \qquad (A-26)$$

Then, using (14), we have the optimal condition for M_t^P in the steady state:

$$u_c f'(M_t^P) + u_Z(\alpha+\beta) = -\frac{u_Z\beta}{\delta+\zeta} \qquad (A-27)$$

The left-hand side of the equation (A-27) is the social marginal benefits of one unit of the forest used in the production, and the right-hand side of the equation is the marginal benefit of one unit of the investment in the forest stock received by future generations.

A-2. Government and individual behavior

In this section, we assume that the government imposes the income tax T_t to finance the cost of forest cultivation investment and that the government determines the tax rate so as to maximize social welfare subject to individual behavior and the government budget equation.

$$T_t y_t = I_t^M \qquad (\mathrm{A}-28)$$

We assume that the utility of the individual is the sum of his own utility and the discounted utility of his child. And that the individual chooses his own consumption and investment for the capital of his child to maximize his utility, subject to his budget and the existing stock of inherited capital and forest inherited by his parent. Then his maximization problem of generation t is:

$$\max_{M_t^P} \sum_{\tau=0}^{\infty} u(c_{t+\tau}, Z_{t+\tau})(1+\delta)^{-(t+\tau)} \qquad (A-29)$$

subject to

$$c_t = (1-T_t)y_t \qquad (A-30)$$

$$y_t = f(M_t^P) \qquad (A-31)$$

$$Z_t = \alpha M_t^P - \beta(M_t - M_t^P) \qquad (A-32)$$

$$M_{t+1} = (1-\zeta)M_t + mT_t y_t \qquad (A-33)$$

We define the evaluation function as:

$$J(M_t) = \max_{M_t^P} u(c_t, Z_t) + (1+\delta)^{-1} J(M_{t+1}) \qquad (A-34)$$

Then, the optimal condition of individual is,

$$u_c \frac{\partial c_t}{\partial M_t^P} + u_Z \frac{\partial Z_t}{\partial M_t^P} + (1+\delta)^{-1} J_{M_{t+1}} \frac{\partial M_{t+1}}{\partial M_t^P} = 0 \qquad (\mathrm{A}-35)$$

$$u_c(1-T_t)f'(M_t^P) + u_Z(\alpha+\beta) + (1+\delta)^{-1} J_{M_{t+1}} m T_t f'(M_t^P) = 0 \qquad (\mathrm{A}-35')$$

For the state variable M_t, we obtain

$$J_{M_t} = u_c \frac{\partial c_t}{\partial M_t} + u_Z \frac{\partial Z_t}{\partial M_t} + (1+\delta)^{-1} J_{M_{t+1}} \frac{\partial M_{t+1}}{\partial M_t} \qquad (A-36)$$

$$J_{M_t} = -\beta u_Z + (1+\delta)^{-1} J_{M_{t+1}} (1-\zeta) \qquad (A-36')$$

We restrict the case of the steady state. The eq. (A-35') and ($A-36$') become:

$$u_c(1-T)f'(M^P) + u_Z(\alpha+\beta) + (1+\delta)^{-1} J_M m T f'(M^P) = 0 \qquad (A-37)$$

$$J_M = -\beta u_Z + (1+\delta)^{-1} J_M (1-\zeta) \qquad (A-38)$$

$$-\beta u_Z = \frac{\delta + \zeta}{1 + \delta} J_M \qquad (A-38')$$

Considering the individual's behavior (A-37) and (A-38), the government chooses the tax rate T to maximize the social welfare (A-29). The state evaluation function of government in this case is,

$$J(M_t) = \max_{T_t} u(c_t, Z_t) + (1 + \delta)^{-1} J(M_{t+1}) \qquad (A-39)$$

The first-order conditions are:

$$-u_c y_t + (1 + \delta)^{-1} J_{M_{t+1}} \frac{\partial M_{t+1}}{\partial T_t} = 0 \qquad (A-40)$$

$$-u_c + (1 + \delta)^{-1} J_{M_{t+1}} m = 0 \qquad (A-41)$$

For state variables, the following conditions are also satisfied:

$$J_{M_t} = -u_Z \beta + (1 + \delta)^{-1} J_{M_{t+1}} (1 - \zeta) \qquad (A-42)$$

We restrict the steady state path, and then have:

$$-u_c + (1 + \delta)^{-1} J_M m = 0 \qquad (A-43)$$

(A-37) is:

$$u_c f'(M^P) + u_Z(\alpha + \beta) - (u_c + (1 + \delta)^{-1} J_M) m T f'(M^P) = 0 \qquad (A-44)$$

The last term in $(A-39)$ is 0 from $(A-38)$ and $(A-39)$ is for any T:

$$u_c f'(M^P) + u_Z(\alpha + \beta) = 0 \qquad (\mathrm{A}-45)$$

The efficiency case requires that this term should be positive as (A-27) shows. Then, we conclude that any income tax on income to finance cultivation cannot attain efficiency. This condition means that the marginal benefits of consumption equal to the marginal disutility of pollution of present generation are equal.

References

Hanley, N., Shogren J. F., & White B.(1997), *Environmental Economics in Theory and Practice*, Macmillan Press.

Nakayama, K.. & Shirai.,M.(2009), Burden of public investment costs for forest conservancy in preservation of water supply sources –Analysis of the effects of environmental taxes on the destruction of forests-, *Studies in Regional Science*, Vol.39, No.3, 553-565.

Pittel,K.,(2002), *Sustainability and Endogenous Growth,* Edward Elgar Publishing Limited.

The authors are grateful to the Chukyo University Research Fund for financial assistance with this research.