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Educational Investment, Liquidity Constraint and Optimal Public Policy

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Abstract

In Japan and other East Asian societies, household educational expenditures per child (especially private tutoring expenditures) have increased sharply, perhaps to an excessive degree. This paper suggests a rationale for many families to invest extensively in education, while other relevant literature rarely addresses the possibility of excessive educational investment. Introducing altruism and liquidity constraints into a model in which parent and child interact for determining investment in the child's education, we show that educational investment may be excessive unless the family is profoundly liquidity-constrained. Our result extends previous findings incorporating the Samaritan's Dilemma (Buchanan, 1975; Lindbeck and Weibull, 1988). We also discuss public policy designed to remedy the inefficiency in educational investment.

JEL classification: I2; D1

Keywords: Altruism; Liquidity constraint; Education; Intergenerational transfers; Samaritan's Dilemma

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1. Introduction

Many societies now witness an aging population due to falling fertility rates and increasing longevity. Japan, in particular, provides an extreme case of fertility rates falling dramatically, and it is often pointed out that one crucial factor in this trend is the increasing cost of educating a child. As shown in Fig. 1, the ratio of Japanese households' educational expenditure to disposable income increased until 1990, stabilized during the 1990s and has subsequently increased since the early 2000s. At the same time, the number of children per household has fallen sharply since the 1980s. This implies that educational expenditures per child have increased greatly in Japan. In addition, expenditures on private supplementary tutoring accounts for a large share of educational expenditures. This phenomenon prevails elsewhere in East Asia including South Korea, Taiwan and Hong Kong (Bray and Kwok, 2003).[1] That private tutoring is freely chosen by households suggests that they invest actively in education. Thus, it is important to investigate the positive and normative aspects of households' decisions concerning private educational investment.

The rate of return on investment in education should provide useful information for evaluating whether households over-spend on private education. Based on Japanese cross-sectional data from 1986 to 1995, Arai (2001) found that the average internal rate of return on a university education is 5.93–6.42% for women and 4.81–5.36% for men; likewise for Japan, Cabinet Office (2005) has estimated that the rate of return on a university education for men born in 1975 was 5.7%. In other countries, many studies have been conducted since the late 1950s. Psacharopoulos and Patrinos (2004), who reviewed empirical results for a variety of countries, summarized that the world average rate of return on an additional year of schooling is 10%, which is above the average for the high-income countries of the OECD. According to cross-country analysis by Trostel et al. (2002), the rate of return on schooling was below 4% for several countries, including Germany (West), the Netherlands, Norway, Sweden and Canada. These results make it difficult to conclude whether the rate of return on investment in education is

[1] Tansel and Bircan (2006) state that there is a growing demand for private tutoring in Turkey as well as many other countries.

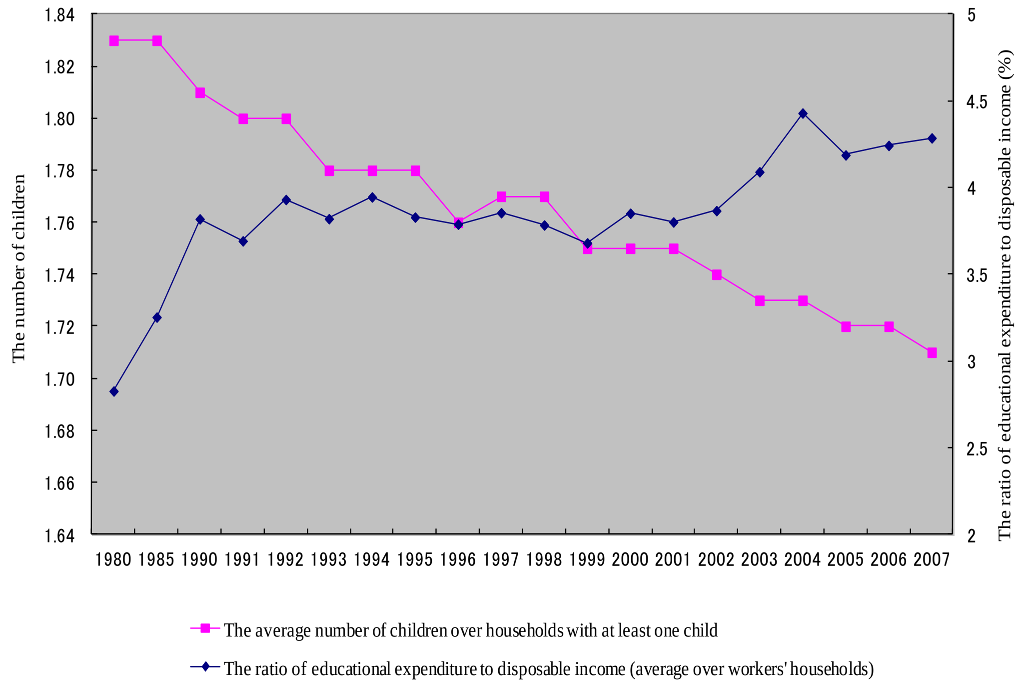

Fig. 1. Household educational expenditure and the number of children in Japan (Source: Ministry of Internal Affairs and Communications (Japan), *Family Income and Expenditure Survey*; Ministry of Health, Labor and Welfare (Japan), *Comprehensive Survey of Living Conditions*)

disproportionately high or low relative to investment in physical capital. However, we should note that several factors have been identified as causing upward bias in estimations of the rate of return on educational investment, such as the correlation between years of schooling and the innate ability to earn income, the effects of liquidity constraints on education decisions, positive spill-over effects from co-worker's education, and the direct costs of education (including private tutoring).[2] Further, the down trend in rate of return on education (Psacharopoulos and Patrinos, 2004; Cabinet Office, 2005) implies that children today may face lower rates of return than rates estimated in previous studies.

The purpose of this study is to explain the possibility of educational investment that is excessive relative to the family optimum, and provide a rationale for the family behavior

[2] In many studies (especially those using the Mincer specification), the cost of education is measured by forgone earnings alone.

on educational investments in practice. We develop a model of families with different incomes, each of which consists of one parent and one child who interact to determine investment in the child's education. More specifically, while children choose the level of their educational investment, its cost is shared between them and their parent, with the share determined by the parent. Who determines the level of educational investment in a family is a modelling issue. In the related literature, Balestrino (1997), De Fraja (2002), Anderberg and Balestrino (2003) and Cremer and Pestieau (2006) presume that parents decide; however, Barham et al. (1995) and Boldrin and Montes (2005) presume that the children do. In our model, parents influence their child's educational investment by setting their share of the cost, and the level of investment is determined as the outcome of the game between parent and child.

We also assume that children's ability to borrow in order to finance their education is constrained. Children's borrowing cannot exceed limits that differ, depending on their parents' income: children whose parents earn more can borrow more. It is important to consider differences in income and the existence of a liquidity constraint, which is a unique feature of our model.

Another important feature of our model is that we assume parents are purely altruistic towards their children, which motivates involvement in their children's education. Besides supporting their children's education financially, parents make financial transfers to their children after they begin to earn an income. These ex-post transfers, which allow parents to redistribute their income to their children at later stages of life, provide an incentive for children to over-consume in their youth so as to receive more parental transfers later, thus engendering the Samaritan's Dilemma (Buchanan, 1975; Lindbeck and Weibull, 1988). This incentive problem affects parents' decisions about the two kinds of transfers.

We obtain three results pertaining to three categories of families. Families in the first category are wealthy, and thus not liquidity-constrained in equilibrium; their educational investment is shown to equal or surpass the family optimum. Families in the second category are middle-class and liquidity-constrained; their educational investment surpasses the family optimum. Families in the third category are poor and profoundly liquidity-constrained; their educational investment is below the family optimum. These are new results in that they clarify the different behavior on educational investments among income categories, and enable us to explain why educational expenditure becomes excessive for many families.

While families in the first and second categories may over-invest in their children's education, the mechanism between these two categories differs. For a family in the first category, if ex-post transfers are made and there is no financial support for the child's education, the child chooses the efficient level of educational investment because the liquidity constraint is not binding, but the Samaritan's Dilemma arises. On the other hand, if the parent makes sufficiently large transfers in the form of educational expenditures and does not provide ex-post transfers, efficient intertemporal allocation of consumption is achieved, but the child over-invests in education. Thus, the parent is forced to choose between these two types of inefficiency. For a family in the second category, the liquidity constraint is binding, and the child must marginally adjust her consumption allocation through the educational investment. Therefore, the level of educational investment that attains an efficient consumption allocation generally does not coincide with its efficient level (namely, where the marginal return on education equals the market interest rate). Specifically, under the efficient level of educational investment, the Samaritan's Dilemma still arises for families in the second category. Hence, parents behave so as to induce children to pursue education beyond the efficient level because educational investment reallocates resources forward and counteracts the Samaritan's Dilemma.

In the literature, it has been argued that private investment in education tends to be insufficient due to the external effect on economic growth, liquidity constraints (Barham et al., 1995; De Fraja, 2002; Fender and Wang, 2003), a self-enforcing 'family constitution' (Balestrino, 1997; Anderberg and Balestrino, 2003), strategic bequest motives (Cremer and Pestieau, 1992), imperfect compensation for ability (Blankenau and Camera, 2009) and intergenerational transmission of attitudes towards education (Kirchsteiger and Sebald, 2010). An exception is Cremer and Pestieau (2006), who considered the joy of giving as the motivation behind parental involvement in children's education. They show that if the joy-of-giving term is excluded from the social welfare function, parents may invest above the social optimum in their children's education, and taxation on investment in education may be necessary for social optimality. Our study is different from Cremer and Pestieau (2006) in that we clarify the effect of family income on educational investment. In Cremer and Pestieau, individuals are divided into two types, high and low productivity (thus income), and the joy-of-giving motive may induce excessive educational investment irrespective of the individual's productivity. On the other hand, we show the difference in educational investment behavior among income categories: families in the lowest income category invest too little in education, while families in other categories may invest too much. This result may explain the fact that some low-income families are reconciled to little or no investment in education due to liquidity constraints, even if educational investment is excessive on average.

We also discuss public policy which can achieve the family optimum for all families. Since standard education policies, such as government provision of education and a subsidy to educational investments, are intended for children to receive more education, they would not remedy the problem of excessive investments in education families in the first and second categories face. In our model, the inefficiency is caused by liquidity constraints and the Samaritan's Dilemma, both of which distort the inter-temporal consumption allocation. Therefore, we consider the government intervention on the loan market, because a policy that directly impinges upon borrowings would be required to achieve optimality. More specifically, we suppose that the government rations credit to each family: families are not allowed to borrow more than their ration, but are provided additional loans by the government if they cannot borrow as much as their ration from the loan market. We show that this can ensure the efficiency in both educational investments and inter-temporal consumption allocation is restored and that the family optimum can be attained for all families.

The paper is organized as follows. Section 2 describes the model. Section 3 derives the family optimum as a benchmark. Sections 4 and 5 analyse the game between parent and child. In Section 4, we derive sub-game perfect equilibrium in the case where the liquidity constraint is non-binding, and compare it to the family optimum. In Section 5, we examine the case where the liquidity constraint is binding. In Section 6, based on results obtained in Sections 4 and 5, we show the effect of a parent's income on investments in a child's education. In Section 7, we introduce the government intervention on the loan market into the model, and examine its effect on the family welfare. Section 8 summarizes the paper.

2. The model

Consider an economy that consists of two generations: a parent's generation and a child's. A parent lives for three periods of equal length: youth (period 0), middle age (period 1) and old age (period 2). A child also lives for three periods: youth (period 1), middle age (period 2) and old age (period 3), with an overlap of periods 1 and 2 between generations. Each member of the parent's generation is heterogeneous with respect to their income level. The population of the parent's generation is N, and each parent produces one child exogenously.

We focus on periods in which both generations overlap, i.e. periods 1 and 2. In period 1, the parent in family i allocates income $Y_{p,i}$, which is determined by the educational investment made in period 0 (and thus exogenous in period 1), among consumption $C^1_{p,i}$, savings S_i and financial contributions to the child's education. We assume that investment in the child's education k_i is partly financed by the parent and the child finances the rest. In period 2, the parent observes the child's income and allocates savings from period 1 between her consumption $C^2_{p,i}$ and ex-post transfers to her child $A_i (\geq 0)$. Thus, the parent's budget constraints in periods 1 and 2 are

$$Y_{p,i} = C^1_{p,i} + S_i + p_i k_i, \qquad (i = 1, \ldots N)$$

$$(1+r)S_i = C^2_{p,i} + A_i, \qquad (i = 1, \ldots N)$$

where p_i $(0 \leq p_i \leq 1)$ is the parental share of educational expenditure and r is the interest rate, determined exogenously.

The child has no income in period 1 and must borrow from the market to finance consumption $C^1_{k,i}$ and education $(1 - p_i)k_i$. In period 2, the child receives her income $Y_{k,i}$, which is a function of k_i satisfying $Y'_{k,i}(k_i) > 0$, $Y''_{k,i}(k_i) < 0$ and $\lim_{k_i \to 0} Y'_{k,i}(k_i) = \infty$.

She repays the borrowings from the sum of her income and ex-post transfers from her parent, and allocates the rest between consumption $C^1_{k,i}$ and savings $S_{k,i}$. Thus, the child's budget constraints in periods 1 and 2 are

$$D_i = C^1_{k,i} + (1 - p_i)k_i, \qquad (i = 1, \ldots N)$$

$$Y_{k,i}(k_i) - (1+r)D_i + A_i = C^2_{k,i} + S_{k,i}, \qquad (i = 1, \ldots N)$$

where D_i is the child's borrowings. Without a loss in generality of the model, we can neglect the child's old age (period 3). Namely, $S_{k,i}=0$ is assumed hereafter. Further, we assume that the amount the child may borrow has an upper limit $\bar{D}_i$, set by her parent's income:

$$D_i \le \bar{D}(Y_{p,i}), \qquad \bar{D}'(Y_{p,i})>0, \qquad (i=1,...N) .$$

The parent is altruistic toward the child, and her utility function is given by

$$U_{p,i}=u_p(C^1_{p,i})+v_p(C^2_{p,i})+\delta U_{k,i},$$

where δ is the weight attached to the child's utility $U_{k,i}$. We assume that $u_p'>0$, $u_p''<0$, $\lim_{C^1_{p,i}\to 0} u_p'(C^1_{p,i})=\infty$, $v_p'>0$, $v_p''<0$ and $\lim_{C^2_{p,i}\to 0} v_p'(C^2_{p,i})=\infty$.

The child cares only about her own consumption, and her utility function is given by

$$U_{k,i}=u_k(C^1_{k,i})+v_k(C^2_{k,i}) .$$

We assume that $u_k'>0$, $u_k''<0$, $\lim_{C^1_{k,i}\to 0} u_k'(C^1_{k,i})=\infty$, $v_k'>0$, $v_k''<0$ and $\lim_{C^2_{k,i}\to 0} v_k'(C^2_{k,i})=\infty$. We hereafter omit the subscript i wherever it does not cause any misunderstanding.

The timing of the game is as follows: (i) the parent chooses C^1_p, S and p; (ii) the child chooses C^1_k, D and k; (iii) the child's income Y_k is realized, and the parent chooses C^2_p and A. (As a result, C^2_k is determined.)

3. The family optimum

As a benchmark, we start by deriving the optimal allocation for the parent. Since the parent is altruistic toward the child, the parental optimum can be regarded as the family optimum. The parent, who implements the optimal allocation with respect to $\left\{C^1_p,\ C^2_p,\ C^1_k,\ C^2_k,\ k\right\}$, maximizes her utility subject to the overall feasibility constraint of her family:

$$\max_{C^1_p, C^2_p, C^1_k, C^2_k, k} \ u_p(C^1_p)+v_p(C^2_p)+\delta[u_k(C^1_k)+v_k(C^2_k)]$$

(1) $$s.t.\ C_p^1 + \frac{C_p^2}{1+r} + C_k^1 + \frac{C_k^2}{1+r} + k = Y_p + \frac{Y_k(k)}{1+r}.$$

The first-order conditions (FOCs) for this problem are given by

(2) $$u_p'(C_p^1) = \delta u_k'(C_k^1),$$

(3) $$v_p'(C_p^2) = \delta v_k'(C_k^2),$$

(4) $$\frac{u_p'(C_p^1)}{v_p'(C_p^2)} = \frac{u_k'(C_k^1)}{v_k'(C_k^2)} = 1+r,$$

(5) $$Y_k'(k) = 1+r.$$

The optimality conditions (2)–(5) and the feasibility condition (1) determine the optimal allocation for the family.[3]

4. Families with non-binding liquidity constraint

From now on, we examine the behavior of families in the equilibrium of the game described in Section 2. Since parental income Y_p differs with each family, we can consider two types of families: one with non-binding liquidity constraints and the other with binding liquidity constraints. In this section, we deal with families whose liquidity constraint is non-binding.

4.1. Ex-post transfers, borrowings and educational investments

We first examine the optimizing behavior of the parent at the third stage of the game. In period 2, given k, D and S, the parent chooses transfers A so as to maximize $v_p((1+r)S - A) + \delta v_k(Y_k(k) - (1+r)D + A)$ subject to the non-negativity constraint on A. FOC is

(6) $$-v_p'((1+r)S - A) + \delta v_k'(Y_k(k) - (1+r)D + A) \le 0 \quad \text{(with equality if } A > 0\text{)},$$

[3] In this study, we do not consider social optimal allocation as a benchmark. With a utilitarian social welfare function where the weight to the children's generation equals the parent's degree of altruism, while (2)–(5) are the necessary conditions for the social optimum, it further requires the optimal consumption allocation between different families. The concern of this study is not the distribution effect of private education but the efficiency of the parent-child interaction, as argued in the literature regarding the Rotten Kid Theorem (Becker, 1974).

which yields the parent's reaction function:

$$(7)\qquad A = A(k,\ D,\ S) = \begin{cases} A^{+}(k,\ D,\ S), & \text{if (6) holds with equality,} \\ 0, & \text{if (6) holds with strict inequality.} \end{cases}$$

From (6) we also have $A_k^{+} \equiv \partial A^{+}/\partial k = -\eta Y_k'(k) < 0$, $A_D^{+} \equiv \partial A^{+}/\partial D = \eta(1+r) > 0$ and $A_S^{+} \equiv \partial A^{+}/\partial S = (1-\eta)(1+r) > 0$, where $\eta \equiv \delta v_k''/(v_p'' + \delta v_k'')$ $(0 < \eta < 1)$, which is assumed to be constant. While $A_k^{+} < 0$ implies that an increase in educational investment leads to higher income for the child and thus provides incentive to decrease transfers, $A_D^{+} > 0$ implies that an increase in borrowings reduces the child's disposable income in period 2 and thus provides incentive to increase transfers.

Next, we examine the second stage of the game. In period 1, anticipating the parent's reaction function (7), the child chooses educational investment k and borrowings D. This amounts to solving the following problem, given p and S:

$$\max_{D,\,k}\ u_k(D-(1-p)k) + v_k(Y_k(k) - (1+r)D + A(k,\ D,\ S))$$
$$s.t.\ \ D \le \bar{D}(Y_p).$$

Since we assume liquidity constraints are non-binding in this section, FOCs for this problem are given by

$$(8)\qquad u_k'(D-(1-p)k) - v_k'(Y_k(k)-(1+r)D + A(k,\ D,\ S))\cdot\left[(1+r) - \frac{\partial A}{\partial D}\right] = 0,$$

$$(9)\qquad -u_k'(D-(1-p)k)\cdot(1-p) + v_k'(Y_k(k)-(1+r)D + A(k,\ D,\ S))\cdot\left[Y_k'(k) + \frac{\partial A}{\partial k}\right] = 0.$$

From (8) and (9), we obtain

$$(10)\qquad k = k(p,\ S) = \begin{cases} k^{+}(p,\ S), & \text{if } \partial A/\partial k = \partial A^{+}/\partial k, \\ k^{0}(p,\ S), & \text{if } \partial A/\partial k = 0, \end{cases}$$

with $k_p^{+} \equiv \partial k^{+}/\partial p > 0$, $k_S^{+} \equiv \partial k^{+}/\partial S = 0$, $k_p^{0} \equiv \partial k^{0}/\partial p > 0$ and $k_S^{0} \equiv \partial k^{0}/\partial S = 0$.[4] The implication of $k_p^{+} > 0$ (or $k_p^{0} > 0$) is that an increase in the parental share of educational expenditure lowers the marginal cost for the child and thus stimulates the educational investment.

Equations (8) and (9) also yield

[4] See Appendix.

(11) $$D = D(p,\ S) = \begin{cases} D^{+}(p,\ S), & \text{if } \partial A/\partial D = \partial A^{+}/\partial D, \\ D^{0}(p,\ S), & \text{if } \partial A/\partial D = 0, \end{cases}$$

with $D_S^{+} \equiv \partial D^{+}/\partial S > 0$ and $D_S^{0} \equiv \partial D^{0}/\partial S = 0$. The sign of D_p ($\equiv \partial D/\partial p$) is indeterminate in general because the direct effect has the opposite sign to the indirect effect via the change in k.[5] Given k, a rise in p increases the amount transferred to the child and thus induces the child to borrow less. On the other hand, k increases in response to the rise in p ($k_p > 0$), and this may lead to an increase in borrowing.

When $A > 0$, from (8) and $\partial A/\partial D = \partial A^{+}/\partial D > 0$, we obtain

(12) $$u_k'(C_k^1) - (1+r)v_k'(C_k^2) < 0,$$

which means that the marginal rate of substitution of C_k^1 for C_k^2 is smaller than the gross interest rate. We thus obtain the following proposition:

Proposition 1. *In families with non-binding liquidity constraints, if $A > 0$, the child over-consumes in period 1 and the Samaritan's Dilemma arises in equilibrium.*

Further, (8) and (9) imply

(13) $$Y_k'(k) - (1+r)(1-p) = 0,$$

which implies that the marginal rate of return to educational investments is not greater than the gross interest rate, and derives the following proposition:

Proposition 2. *If $p = 0$, the child chooses the optimal level of educational investment for the family. If $p > 0$, the child chooses excessive educational investments relative to the family's optimal level.*

4.2. Parental share of educational expenditures

At the first stage, the parent chooses savings S and the parental share of educational expenditures p so as to maximize

5 See Appendix.

$$
(14) \quad \begin{aligned} U_p = & \, u_p[Y_p - S - pk(p,\ S)] \\ & + v_p[(1+r)S - A(k(p,\ S),\ D(p,\ S),\ S)] \\ & + \delta\{u_k[D(p,\ S) - (1-p)k(p,\ S)] \\ & + v_k[Y_k(k(p,\ S)) - (1+r)D(p,\ S) + A(k(p,\ S),\ D(p,\ S),\ S)]\}. \end{aligned}
$$

Using the envelope theorem, we obtain FOC with respect to S as

$$
(15) \quad -u_p' + v_p' \cdot [(1+r) - \frac{\partial A}{\partial D}\frac{\partial D}{\partial S}] = 0,
$$

from which we derive the following proposition:

Proposition 3. *If* $A > 0$*, the parent over-consumes in period 1 relative to the optimal allocation.*

Proposition 3 suggests that the child's strategic behavior distorts her own as well as her parent's consumption allocation.

We now derive the level of p in the equilibrium. It is assumed that if the parent chooses $p = 0$ at the first stage, she necessarily chooses positive transfers to her child at the third stage. In other words, without financial support from her parent, the child invests a small amount in her education and her income is assumed to be low enough to motivate ex-post transfers from her parent. Since reaction functions (7), (10) and (11) imply that ex-post transfers decrease as p increases ($dA^+ / dp < 0$), we define p_0 as p that satisfies FOC with respect to A, (6), with equality when $A = 0$:[6]

$$
(16) \quad -v_p'[(1+r)S] + \delta v_k'[Y_k(k(p_0, S)) - (1+r)D(p_0, S)] = 0.
$$

To examine the parent's choice of p, we draw the graph of (14) in the pU_p-plane, dividing the range of p into (i) $0 \le p \le p_0$ (where the non-negativity constraint on A is non-binding) and (ii) $p_0 < p \le 1$ (where the non-negativity constraint on A is binding).

(i) $0 \le p \le p_0$: Differentiating (14) with respect to p and using the envelope theorem yields

[6] The proof of $dA^+ / dp < 0$ and $p_0 < 1$ is shown in Appendix.

(17) $$\left(\frac{\partial U_p}{\partial p}\right)_{0\le p\le p_0} = (-u'_p + \delta u'_k)(k + pk_p^+) + \delta\{[u'_k - (1+r)v'_k]D_p^+ + (-u'_k + v'_k Y'_k)k_p^+\}.$$

Using (8), (13) and (15), we rewrite (17) as

(18) $$\left(\frac{\partial U_p}{\partial p}\right)_{0\le p\le p_0} = \delta v'_k k_p^+[1-\eta(1-\rho)][Y'_k - (1+r)],$$

where $\rho \equiv u''_k /[u''_k + (1-\eta)^2(1+r)^2 v''_k]$ $(0<\rho<1)$. Since we have $Y'_k = 1+r$ for $p=0$ while $Y'_k < 1+r$ for $p>0$ from Proposition 2, (18) implies

(19) $$\left(\frac{\partial U_p}{\partial p}\right)_{p=0} = 0, \quad \left(\frac{\partial U_p}{\partial p}\right)_{0<p\le p_0} < 0.$$

(ii) $p_0 < p \le 1$: Differentiating (14) with $A=0$ with respect to p and substituting (8) and (15) with $\partial A/\partial D = 0$ into the resulting equation yields[7]

(20) $$\left(\frac{\partial U_p}{\partial p}\right)_{p_0<p\le 1} = (-u'_p + \delta u'_k)(k + pk_p^0) + \delta v'_k k_p^0[Y'_k - (1+r)] < 0.$$

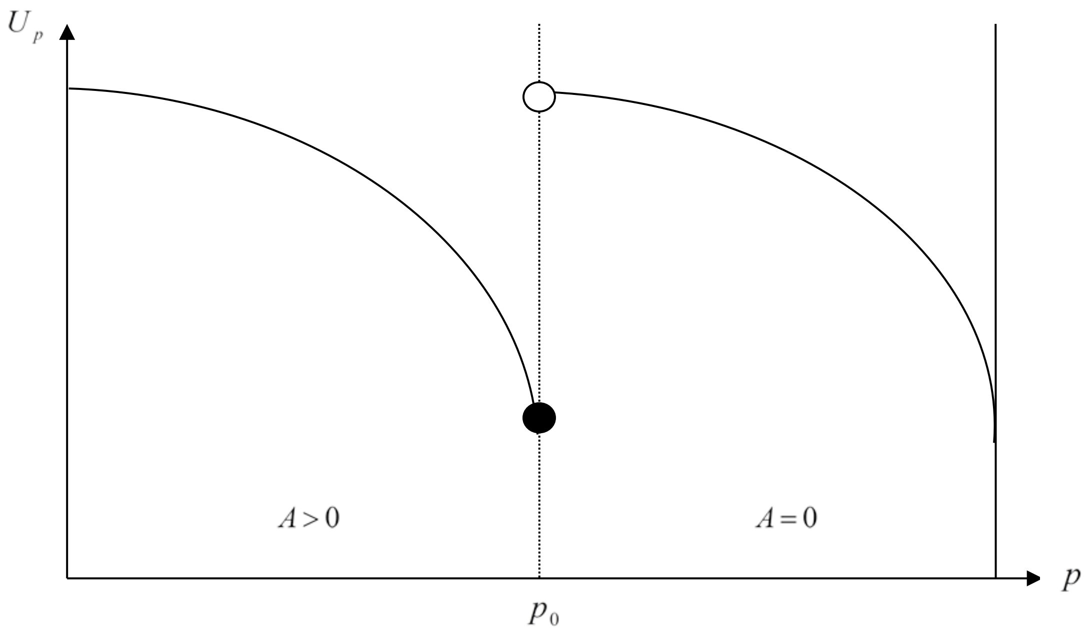

Fig. 2 . Parental utility function for families with *non-binding* liquidity constraints

[7] Note that we have $-u'_p + \delta u'_k < 0$ when the non-negativity constraint on A is binding.

Fig. 2 shows the graph of U_p. The jump at $p = p_0$ should be noted. While the Samaritan's Dilemma arises for $0 \le p \le p_0$ because (6) holds with equality and thus $\partial A / \partial D = \partial A^+ / \partial D > 0$ holds in (8), the dilemma is absent for $p_0 < p \le 1$ because (6) holds with strict inequality and thus $\partial A / \partial D = 0$ holds in (8). Resolution of the Samaritan's Dilemma has a positive effect on the parental utility, and we have the following lemma:

Lemma 1. $\lim_{\varepsilon \to 0} [U_p \big|_{p = p_0 + \varepsilon} - U_p \big|_{p = p_0}] > 0$

Proof. See Appendix.

From (19), (20) and Lemma 1, we derive the equilibrium as characterized in the following proposition:

Proposition 4. *For families with non-binding liquidity constraints, the parental share of educational expenditures in the equilibrium is either* $p* = 0$ *or* $p* = p_0 (+\lim_{\varepsilon \to 0} \varepsilon)$. *If* $p* = 0$, *the child over-consumes in period 1 (giving rise to the Samaritan's Dilemma), while the child chooses the optimal level of educational investment for the family. If* $p* = p_0 (+\lim_{\varepsilon \to 0} \varepsilon)$, *the child chooses the level of educational investment higher than the family's optimal level, while the child's consumption allocation is efficient.*

This result is similar to that of Bruce and Waldman (1990) in that the parent is forced to choose between two types of inefficiency.[8]

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[8] Based on a model in which the parent chooses *inter vivos* transfers as well as bequests, and the child not only chooses savings but also actions that affect the level of family income, Bruce and Waldman (1990) show that while the Samaritan's Dilemma arises when bequests are made, there is no Samaritan's Dilemma but the Rotten Kid Theorem (Becker, 1974) fails when *inter vivos* transfers are made and bequests are not.

5. Families with binding liquidity constraint

In this section, we examine the behavior of families whose borrowing takes a corner solution.

5.1. Ex-post transfers, borrowings and educational investments

The third stage in this case is the same as that for the case of families with non-binding liquidity constraints described in the previous section, except that here FOC with respect to A is assumed to be satisfied with equality.[9]

In the second stage, since we suppose the liquidity constraint to be binding in this section, FOCs for the maximization problem of the child are

(21) $$u_k'(D-(1-p)k) - v_k'(Y_k(k)-(1+r)D+A(k,\ D,\ S))\cdot\left[(1+r)-A_D^+\right] > 0,$$

(22) $$-u_k'(D-(1-p)k)\cdot(1-p) + v_k'(Y_k(k)-(1+r)D+A(k,\ D,\ S))\cdot\left[Y_k'(k)+A_k^+\right] = 0,$$

with $D = \bar{D}(Y_p)$. From (22), we obtain the child's reaction function:

(23) $$k = k^+(p,\ S;\ \bar{D}(Y_p)),$$

with $k_p^+ > 0$, $k_S^+ < 0$ and $k_D^+ \equiv \partial k / \partial \bar{D}(Y_p) > 0$.[10]

Although (21) does not determine the sign of $u_k'(c_k^1) - (1+r)v_k'(c_k^2)$ in this case, (22) can be rewritten as

(24) $$\frac{u_k'(c_k^1)}{v_k'(c_k^2)} = \frac{Y_k'(k)+A_k^+}{1-p} = \frac{(1-\eta)Y_k'(k)}{1-p}.$$

From (24), we obtain the following proposition:

Proposition 5. (i) *The child under-consumes in period 1 if* $\dfrac{(1-\eta)Y_k'(k)}{1-p} > 1+r$.

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[9] Sufficient conditions for $A > 0$ in the equilibrium are shown in Appendix. See also footnote 15 .

[10] See Appendix.

(ii) *The child's consumption allocation is efficient if* $\frac{(1-\eta)Y_k'(k)}{1-p} = 1+r$.

(iii) *The child over-consumes in period 1 if* $\frac{(1-\eta)Y_k'(k)}{1-p} < 1+r$.

Proposition 5 (iii) shows that the Samaritan's Dilemma may arise even when the liquidity constraint is binding. This is because the child's strategic incentive to obtain more parental transfers may be strong enough to allocate only small share of borrowings for educational investment.

Further, (21) and (22) imply

(25) $$Y_k'(k) > (1-p)(1+r),$$

which derives the following proposition about the educational investment:

Proposition 6. *If* $p = 0$*, the child chooses a level of educational investment below the family optimum.*

On the other hand, if $p > 0$, whether the level of educational investment is too high or too low relative to the family optimum is indeterminate. Proposition 6 is in contrast with Proposition 2 in that, without the parent's financial contributions to the child's education, the child invests insufficient amounts in her education due to the liquidity constraint.

5.2. Parental share of educational expenditure

In the first stage, anticipating the child's reaction (23), the parent chooses savings S and the parental share of educational expenditures p .

Using (6) with equality and (22), FOC with respect to S is reduced to

(26) $$-u_p' + v_p' \cdot (1+r) - (u_p' p + v_p' A_k^+) k_S^+ = 0.$$

Next, we examine the marginal utility of p , which is obtained as

(27) $$\frac{dU_p}{dp} = (-u_p' + \delta u_k')k + \{-pu_p' + \delta[-(1-p)u_k' + v_k' Y_k']\} k_p^+.$$

The first term in (27) is the direct effect of p on parental utility, while the second term is the indirect effect through the reaction of educational investments ($k_p^+ > 0$). From (27), we derive the following lemma:

Lemma 2. If $\sigma \equiv -kY_k''/Y_k' < 1$, then $(dU_p/dp)_{p=0} > 0$.

Proof. See Appendix.

This lemma implies that, in families where the child faces liquidity constraints, the parent supports the child's education financially if $\sigma < 1$. This condition holds when $Y_k(k)$ takes the Cobb-Douglas functional form,[11] and we assume $\sigma < 1$ in the remainder of this paper.

We now examine the equilibrium level of p. Fig. 3 shows the marginal rate of return to educational investments $Y_k'(k)$ and the marginal rate of consumption substitution $(1-\eta)Y_k'(k)/(1-p)$ $(= u_k'(c_k^1)/v_k'(c_k^2))$, which are denoted by MRE and MRS respectively. Since $\partial Y_k'(k(p;\ \bar{D}(Y_p)))/\partial p < 0$ and $\partial[(1-\eta)Y_k'(k(p;\ \bar{D}(Y_p)))/(1-p)]/\partial p > 0$, MRE slopes downward and MRS slopes upward. Both MRE and MRS depend negatively on Y_p because educational investments k depend positively on the limit of borrowings $\bar{D}(Y_p)$ and $Y_k'(k)$ depends negatively on k. Therefore, defining $\tilde{Y}_p$ as Y_p that satisfies MRE= MRS $=1+r$ under a certain level of p, MRE and MRS with $Y_p = \tilde{Y}_p$ locate above those with $Y_p > \tilde{Y}_p$ and below those with $Y_p < \tilde{Y}_p$, as shown in Fig.3.

[11] Given $Y_k = Bk^{\alpha}$ $(B > 0,\ \ 0 < \alpha < 1)$, we have $\sigma = -Y_k''k/Y_k' = 1-\alpha < 1$.

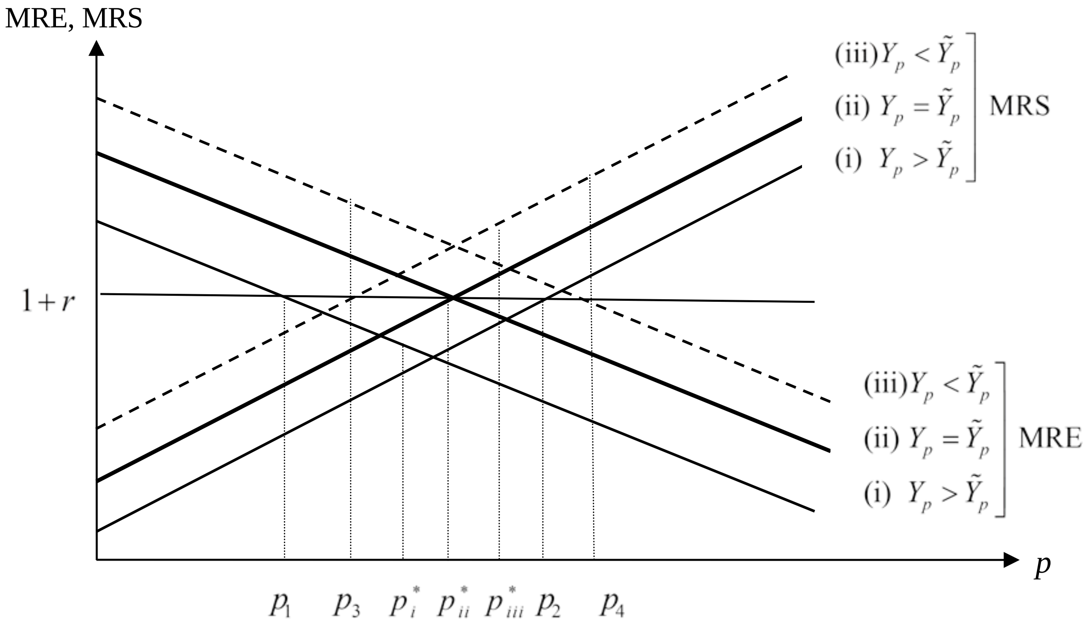

Fig. 3. Parental share of educational expenditure for each income category of families with *binding* liquidity constraints

We present the following lemma to examine the determination of p in each income category of families.

Lemma 3. (i) *If* $\text{MRE} = 1+r$ *and* $\text{MRS} \leq (\geq) 1+r$, *then* $\dfrac{dU_p}{dp} \geq (\leq) 0$ *;*

(ii) *If* $\text{MRS} = 1+r$ *and* $\text{MRE} \geq (\leq) 1+r$, *then* $\dfrac{dU_p}{dp} \geq (\leq) 0$.

Proof. See Appendix.

Lemma 3(i) implies that, under p that induces efficient educational investment, a rise in p increases (decreases) the parent's utility if the child over-consumes (under-consumes) in the first period. Lemma 3(ii) implies that, under p that induces the efficient intertemporal allocation of consumption, a rise in p increases (decreases) the parent's utility if the educational investment is insufficient (excessive).

(i) Families with $Y_p > \tilde{Y}_p$

In Fig. 3, MRE and MRS for families with $Y_p > \tilde{Y}_p$ are shown by the thin lines. When $p = 0$, we have $\partial U_p / \partial p > 0$. Raising p incrementally from zero, we attain p_1, which represents p satisfying $\text{MRE} = 1 + r$. When $p = p_1$, $\text{MRS} < 1 + r$ holds, and Lemma 3(i) implies that $\partial U_p / \partial p > 0$. Raising p further from p_1, we reach to p_2, which represents p satisfying $\text{MRS} = 1 + r$. When $p = p_2$, $\text{MRE} < 1 + r$ holds, and Lemma 3(ii) implies that $\partial U_p / \partial p < 0$. Therefore, under an assumption that $\partial^2 U_p / \partial p^2 < 0$, the equilibrium solution p_i^* must be located between p_1 and p_2, and thus $\text{MRS} < 1 + r$ and $\text{MRE} < 1 + r$ simultaneously hold for p_i^*. [12] From Proposition 5, this implies emergence of the Samaritan's Dilemma (the child's over-consumption in period 1), with excessive amounts invested in education in the equilibrium.

The intuition behind this result is as follows. Under the efficient level of educational investment that satisfies $\text{MRE} = 1 + r$, since the Samaritan's Dilemma arises, the parent increases p in order to induce her child to pursue a higher education. This is because the educational investment reallocates resources forward and thus reduces distortions caused by the Samaritan's Dilemma.

(ii) Families with $Y_p = \tilde{Y}_p$

In Fig. 3, MRE and MRS for families with $Y_p = \tilde{Y}_p$ are shown by the thick lines. When $p = 0$, we have $\partial U_p / \partial p > 0$. Raising p incrementally from zero, we attain p_{ii}^*, where $\text{MRE} = 1 + r$ and $\text{MRS} = 1 + r$ are simultaneously satisfied. Therefore, both the intertemporal consumption allocation and the educational investment are efficient. In addition, since $p = \eta$ holds, the family optimum is achieved in the

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[12] It should be noted that p_i^* is smaller than p that satisfies $\text{MRE=MRS}(<1+r)$. This is because, for category (i) families, the amount of transfers from the parent to the child in the first period, pk, is excessive under MRE=MRS, namely

$$\left.\frac{dU_p}{dp}\right|_{MRE=MRS} = \frac{\delta v_k'(k + pk_p^+)}{1 + pk_S^+}[MRE - (1+r)] < 0,$$

which is derived from (A27).

equilibrium.[13]

(iii) Families with $Y_p < \tilde{Y}_p$

Families with $Y_p < \tilde{Y}_p$ are further divided into two categories, depending on whether MRS is smaller or greater than $1+r$ when $p=0$. In Fig. 3, MRE and MRS for the former category of families are shown by the dashed lines. When $p=0$, we have $\partial U_p / \partial p > 0$. Raising p incrementally from zero, we attain p_3, which satisfies $\text{MRS} = 1+r$. At p_3, MRE>$1+r$ holds and Lemma 3(ii) implies that $\partial U_p / \partial p > 0$. Raising p further, we reach p_4, which satisfies MRS>$1+r$ and $\text{MRE} = 1+r$. At p_4, Lemma 3(i) implies that $\partial U_p / \partial p < 0$. Under an assumption that $\partial^2 U_p / \partial p^2 < 0$, therefore, MRS>$1+r$ and MRE>$1+r$ simultaneously hold at the equilibrium solution p^*_{iii}.[14] From Proposition 5, this implies that the child under-consumes in period 1 and that educational investment is insufficient in the equilibrium. It is apparent from Lemma 3(i) that this property of equilibrium is also applied to the category of families whose MRS is greater than $1+r$ when $p=0$.

In contrast to category (i), under the efficient level of educational investment, the child of families in category (iii) under-consumes in the first period because she is less able to borrow than the child of category (i) families. The parent, therefore, chooses a lower p in order to induce the child to invest less in education and to consume more in the first period.

The following proposition summarizes the above analysis:

Proposition 7. (i) *The Samaritan's Dilemma and over-investment in education simultaneously arise for families with* $Y_p > \tilde{Y}_p$.
(ii) *Family optimum is achieved for families with* $Y_p = \tilde{Y}_p$.

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[13] From (26), we obtain $-u'_p + (1+r)v'_p = [v'_p / 1 + pk_S^+][-\eta Y'_k + p(1+r)]k_S^+$. Substituting $p = \eta$ and $Y'_k = 1+r$ into the above equation and (24) yields the optimal conditions.

[14] It should be noted that p^*_{iii} is greater than p that satisfies MRE=MRS(>$1+r$). This is because the amount of transfers from the parent to the child in the first period, pk, is insufficient under MRE=MRS, namely $(dU_p / dp)_{MRE=MRS} > 0$, for category (iii) families (see Note 12).

(iii) *Insufficient filial consumption in period 1 and under-investment in education simultaneously arise for families with* $Y_p < \tilde{Y}_p$.

In contrast to the case of non-binding liquidity constraint (Proposition 4), Proposition 7 suggests that, in the case of binding liquidity constraint, the property of equilibrium is different from that in Bruce and Waldman (1990). While either the intertemporal allocation of consumption or the level of filial action is efficient from the family perspective in Bruce and Waldman (1990), neither is efficient in this case (except for families with $Y_p = \tilde{Y}_p$).[15]

6. The effects of income on educational investment

Based on results obtained in sections 4 and 5, this section clarifies the differences in educational investment among families with differing incomes.

Define $\hat{Y}_p$ as Y_p of families whose FOC with respect to D is satisfied with equality for $D = \bar{D}(Y_p)$. In other words, the child's most-preferred level of borrowings in a family with $\hat{Y}_p$, which is denoted by $D^{**}(\hat{Y}_p)$, is just equal to the limit of how much she can borrow, namely $\bar{D}(\hat{Y}_p)$. Assuming that $d\bar{D}(Y_p)/dY_p > dD^{**}(Y_p)/dY_p$,[16] the

[15] If we suppose $A = 0$ in the equilibrium, the child's consumption in period 1 would be insufficient due to liquidity constraints, which is obtained from (21) with $A_D^+ = 0$. Also, educational investment would be insufficient for the following reason. We have $Y_k' > 1 + r$ from (22) when $p = 0$. Raising p from zero increases k and thus overcomes its insufficiency, but it worsens the insufficiency in C_k^1. When p reaches the level consistent with $Y_k' = 1 + r$, a further increase in p induces excessive educational investment. Therefore, the parent would not choose p greater than that level, given $A = 0$. For families with $Y_p = \tilde{Y}_p$, however, $A = 0$ never arises in the equilibrium, because they can attain the family optimum by choosing positive transfers (their FOCs imply $-v_p' + \delta v_k' = 0$). Furthermore, if A is normal goods for the parent, we would have $A > 0$ in the equilibrium also for families with $Y_p > \tilde{Y}_p$. On the other hand, A could be zero in the equilibrium for families with $Y_p < \tilde{Y}_p$. However, even if this is the case, we obtain basically the same result, under-investment in education and insufficient filial consumption in period 1, for these families. Therefore, Proposition 7 would be maintained, even if we explicitly consider the possibility that $A = 0$.

[16] Since $dD^{**}(Y_p)/dY_p$ is positive, we need this assumption. See Appendix for the sign of dD^{**}/dY_p.

liquidity constraint is binding if $Y_p < \hat{Y}_p$, but not if $Y_p \geq \hat{Y}_p$.

From Propositions 4 and 7, we obtain the following proposition.

Proposition 8. *Whether investment in a child's education is too great or too small relative to the family optimum depends on parental income:*

(i) $k^* \geq k^F$, *if* $Y_p \geq \hat{Y}_p$,

(ii) $k^* > k^F$, *if* $\tilde{Y}_p < Y_p < \hat{Y}_p$,

(iii) $k^* = k^F$, *if* $Y_p = \tilde{Y}_p$,

(iv) $k^* < k^F$, *if* $Y_p < \tilde{Y}_p$,

where k^F *is the optimal level of educational investment satisfying* (5).

7. Public policy to achieve the family optimum

The previous sections showed that the level of educational investment is inefficient and the family optimum is not attained for all families except those with $Y_p = \tilde{Y}_p$. This section examines whether a public policy can lead each family into the family optimum. As a public policy, we consider the government intervention on the loan market rather than standard education policies such as government provision of education and a subsidy to educational investments. This is because such education policies are intended for children to receive more education, and would not remedy the excessive private investment in education. In our model, the sources of inefficiency are liquidity constraints and the child's strategic consumption behavior. Since both of them distort the inter-temporal consumption allocation, a policy that directly impinges upon borrowings would be required to achieve optimality. It should be noted, however, that to get rid of the liquidity constraints for all families is useless, because, as shown in section 4, families whose liquidity constraint is not binding fail to attain the family optimum. The policy we consider here is that the government rations credit to each family. Families are not allowed to borrow more than their ration, but are provided additional loans by the government if they cannot borrow as much as their ration from the loan market.

7.1. Equilibrium with government intervention on the loan market

In the first stage, the government chooses each family's ration of credit $D_{G,\,i}$ $(i=1,...i_b, i_{b+1},...,n)$ so as to maximize the parental welfare of each family, where $i=1,...i_b$ is a family whose liquidity constraint would be binding without government intervention (namely, a family with $Y_p < \hat{Y}_p$), and $i=i_{b+1},...,n$ is a family whose liquidity constraint would not be binding (namely, a family with $Y_p \geq \hat{Y}_p$).

We first examine the equilibrium level of $D_{G,\,i}$ $(i=1,...i_b)$. (We hereafter omit the superscript i for notational simplicity.) Replacing $\bar{D}(Y_p)$ with D_G in the reaction functions (7), (23), (26) and $(27)=0$, we obtain FOC of the government's maximization problem as

$$\begin{aligned}(28)\qquad &\delta[u_k'(C_k^1)-(1+r)v_k'(C_k^2)]\\ &+\left\{-pu_p'(C_p^1)+\delta[-(1-p)u_k'(C_k^1)+v_k'(C_k^2)Y_k'(k)]\right\}\frac{\partial k}{\partial D_G}=0,\end{aligned}$$

where $\partial k/\partial D_G=\partial k/\partial\bar{D}(Y_p)(\equiv k_D^+)$. These equations characterize the equilibrium with the government intervention on the loan market. In the equilibrium, the family optimum is achieved for all families. [17] This is apparent from the analysis in section 5, where the limit of borrowings $\bar{D}$ depends on Y_p, and only for families with $Y_p=\tilde{Y}_p$ there exists p that induces the optimal inter-temporal allocation of consumption and the optimal educational investment simultaneously. This implies that there should exist such p also for other families if the level of borrowings is given appropriately with no relation to Y_p.

Denoting the equilibrium level of D_G as D_G^*, we have $D_G^* < \bar{D}(Y_p)$ for category (i) families with $Y_p > \tilde{Y}_p$, namely, the government rations credit more tightly than the loan market does. As discussed in Section 5, the child in these families over-invests in education and over-consumes in the first period without government intervention on the loan market. A decrease in borrowings restrains educational investments, and hence raises both MRE $(=Y_k'(k(p,\ D_G)))$ and MRS $(=(1-\eta)Y_k'(k(p,\ D_G))/1-p)$, given p. In Fig. 3, decreasing borrowings from $\bar{D}(Y_p)$ to D_G^* shifts up MRE and MRS of category (i) families to those of category (ii) families. Under D_G^*, therefore, they can achieve the

[17] The formal proof is shown in Appendix.

family optimum by choosing p that satisfies MRE=MRS=$1+r$. On the other hand, we have $D_G^* > \bar{D}(Y_p)$ for category (iii) families with $Y_p < \tilde{Y}_p$, namely, the government provides additional loans to loosen their liquidity constraints. Without government intervention, educational investments are insufficient and the child under-consumes in the first period in these families. An increase in borrowings enhances educational investments, and hence lowers both MRE $(=Y_k'(k(p,\ D_G)))$ and MRS $(=(1-\eta)Y_k'(k(p,\ D_G))/1-p)$, given p. In Fig. 3, increasing borrowings from $\bar{D}(Y_p)$ to D_G^* shifts down MRE and MRS of category (iii) families to those of category (ii) families. Under D_G^*, therefore, they can achieve the family optimum by choosing p that satisfies MRE=MRS=$1+r$.

We next examine the ration of credit for families with non-binding liquidity constraints $D_{G,\ i}\ (i = i_{b+1},...,n)$. If the government chooses D_G higher than the child's most-preferred level of borrowings $D^{**}(Y_p)$, the child chooses $D^{**}(Y_p)$, and the family optimum cannot be attained as shown in Section 4. On the other hand, in the same manner as families with binding liquidity constraints, families with non-binding liquidity constraints can achieve the family optimum under a certain level of D_G lower than $D^{**}(Y_p)$. This implies that $D_G^* < D^{**}(Y_p)$. In contrast to the usual outcome of educational investment that efficient amounts are invested and the government intervention is not called for if credit is not constrained for education, this result suggests that credit needs to be rationed for efficiency and family optimumality. This difference stems from the strategic interaction between the parent and child over the child's educational investment in our model, which causes loan market failure and justifies the government intervention.

From the above analysis, we obtain the following proposition:

Proposition 9. *With government intervention on the loan market, the efficiency in both educational investments and inter-temporal consumption allocation can be restored and the family optimum can be attained for all families.*

Fig.4 shows the ration of credit for each family in the equilibrium as a function of family income. While the ration of credit increases as family income rises, the slope of $D_G^*(Y_p)$ is smaller than that of $\bar{D}(Y_p)$.

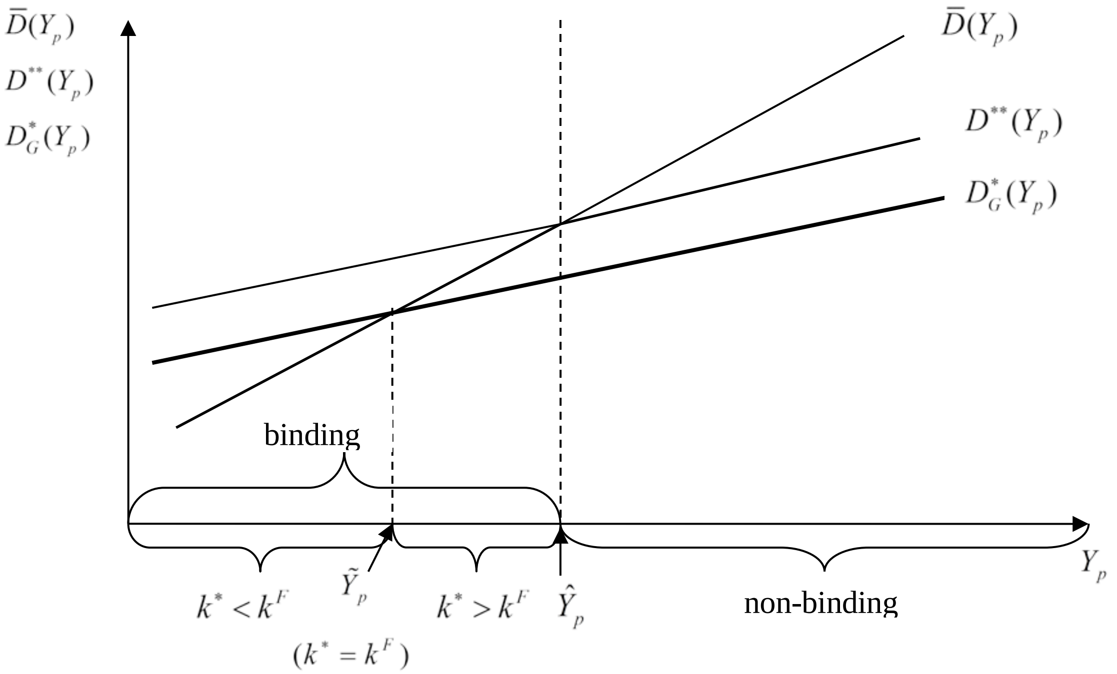

Fig. 4. Limit of borrowings from the loan market ($\bar{D}$), most-preferred level of borrowings (D^{**}), and ration of credit to achieve family optimum (D_G^*)

Behind Proposition 9, the government in our model can observe each family's income. However, the implementation of the optimal policy in the real economy may be hindered by the government's inability to know each individual's lifetime income (or ability to repay the loan). While this sort of asymmetric information is not considered here, our analysis points to a case in which the government intervention on the loan market by means of rationing credit to each family is desirable. The second-best policy under the asymmetric information needs to be investigated as an extension of this study.

8. Conclusion

In many countries, private investment in children's education is increasing and has reached levels that seem excessive relative to levels that maximize family welfare. On the other hand, it is undoubtedly true that some low-income families are reconciled to little or no investment in their children's educations due to liquidity constraints. Modeling the parent-child interaction over the child's education, this study attempted to provide a rationale behind such a phenomenon.

Our main finding is that investment in education can be too great or too little relative to the family optimum, depending on the family's income. In obtaining this result, the child's strategic behavior in consumption allocation plays a key role. We consider two types of transfers from parent to child: the parent's financial contribution to the child's education during the child's youth, and ex-post transfers after the child begins to earn an income. The latter transfers provide an incentive for the child to consume too much when younger, engendering the Samaritan's Dilemma. Whether the parent faces the Samaritan's Dilemma depends on the parent's income. In families with high income, which are not liquidity-constrained, the Samaritan's Dilemma arises if ex-post transfers are made. The parent, therefore, may transfer income only in the form of financial contributions to the child's education, inducing the child to excessive educational investment. In families with middle income, which are liquidity-constrained, the Samaritan's Dilemma still arises if the parent chooses the level of financial contribution consistent with the efficient educational investment. The parent, therefore, behaves so as to induce her child to pursue educational investments exceeding efficiency because educational investments reallocate resources forward, thereby counteracting the Samaritan's Dilemma. This results in inefficient allocation of consumption and educational investment. In families with low income, which are highly liquidity-constrained, the child's consumption in the first period is insufficient if the parent chooses the level of financial contribution consistent with efficient educational investment. The parent, therefore, chooses a lesser financial contribution, leading to too little educational investment.

We also studied the role of public policy in remedying this inefficiency. It has been shown that, rationing credit to each family, the government can induce all families to invest an efficient amount in education, and can replicate the family optimum.

One possible extension of this study is as follows. As shown in Fig. 1, a decrease in number of children and an increase in educational expenditure per child are simultaneously occurring at present. This may imply that parents prefer fewer children and higher educational investments per child over more children and lower educational investments per child. Further investigation by extending our model to incorporate endogenous fertility might suggest a design for public policies that prevent further declines in fertility rates.

Appendix

Derivatives of the reaction functions. For families with non-binding liquidity constraints, from (8) and (9) (or (13)), we have

(A1) $$k_p^+ = k_p^0 = -\frac{1+r}{Y_k''(k)} > 0,$$

(A2) $$k_S^+ = k_S^0 = 0,$$

(A3) $$D_S^+ = 1-\rho > 0,$$

(A4) $$D_S^0 = 0,$$

(A5) $$D_p^+ = -\rho k - \frac{Y_k'(k)}{Y_k''(k)} = -\rho k + (1-p)k_p^+ = \left.\frac{\partial D^+}{\partial p}\right|_{k=const.} + (1-p)k_p^+,$$

(A6) $$D_p^0 = -\rho^0 k - \frac{Y_k'(k)}{Y_k''(k)} = -\rho^0 k + (1-p)k_p^0 = \left.\frac{\partial D^0}{\partial p}\right|_{k=const.} + (1-p)k_p^0,$$

where $\rho \equiv u_k'' / [u_k'' + (1+r)^2(1-\eta)^2 v_k''] \ (0 < \rho < 1)$, $\rho^0 \equiv u_k'' / [u_k'' + (1+r)^2 v_k''] \ (0 < \rho^0 < 1)$. Under the assumption that $\sigma\left(\equiv -kY_k''/Y_k'\right) < 1$, we have

$$D_p^+ = k\left[-\rho + \left(1/\sigma\right)\right] > 0,$$

(A7) $$D_p^0 = k\left[-\rho^0 + \left(1/\sigma\right)\right] > 0.$$

For families with binding liquidity constraint, from (22), we have

(A8) $$k_p^+ = \frac{u_k''k(1-p) - u_k'}{F} > 0,$$

(A9) $$k_S^+ = \frac{-v_k''(1-\eta)^2(1+r)Y_k'}{F} < 0,$$

(A10) $$k_D^+ = \frac{u_k''(1-p) + v_k''(1+r)(1-\eta)^2 Y_k'}{F} > 0,$$

where $F = u_k''(1-p)^2 + v_k''(1-\eta)^2(Y_k')^2 + v_k'(1-\eta)Y_k'' < 0$.

Proof of $dA^+/dp < 0$ and $p_0 < 1$. If $A > 0$, from (7), (10) and (11), we have $A = A^+(k^+(p,\ S),\ D^+(p,\ S),\ S)$. Differentiating this equation with respect to p yields

$$\frac{dA^+}{dp} = \frac{\partial A^+}{\partial k}\frac{\partial k^+}{\partial p} + \frac{\partial A^+}{\partial D}\frac{\partial D^+}{\partial p} + \left(\frac{\partial A^+}{\partial k}\frac{\partial k^+}{\partial S} + \frac{\partial A^+}{\partial D}\frac{\partial D^+}{\partial S} + \frac{\partial A^+}{\partial S}\right)\frac{\partial S}{\partial p}$$

(A11)
$$= \eta Y_k'(k)\frac{(1+r)}{Y_k''(k)} + (1+r)\eta\left(-k\rho - \frac{Y_k'(k)}{Y_k''(k)}\right) + [(1+r)\eta(1-\rho) + (1-\eta)(1+r)]\frac{\partial S}{\partial p}$$
$$= -(1+r)\eta\rho k + (1+r)(1-\eta\rho)\frac{\partial S}{\partial p}.$$

Since

$$\frac{\partial S}{\partial p} = -k - \frac{u_p'' pk_p}{u_p'' + v_p''(1+r)^2\eta\rho(1-\eta(1-\rho))} < 0,$$

we have $dA^+/dp < 0$.

Next, we prove $p_0 < 1$. Equation (13) implies that $k \to \infty$ as $p \to 1$. However, since pk cannot exceed Y_p, we have $S \to 0$ as $p \to 1$. Hence, (6) holds with strict inequality when p exceeds a certain value smaller than 1. This implies $p_0 < 1$.

Proof of Lemma 1. For $p = p_0$, we define the following function with dummy variable θ:

(A12)
$$u_k'(D-(1-p_0)k) - v_k'(Y_k(k)-(1+r)D)\left[(1+r) - \theta A_D^+\right] = 0,$$

(A13)
$$-(1-p_0)u_k'(D-(1-p_0)k) + v_k'(Y_k(k)-(1+r)D)\left[Y_k'(k) + \theta A_k^+\right] = 0,$$

(A14)
$$u_p'(Y_p - S - p_0k) - v_p'((1+r)S)\left[(1+r) - \theta A_D^+ D_S^+\right] = 0.$$

D, k and S satisfy (A12)–(A14) with $\theta = 0$ when (6) holds with strict inequality, whereas they satisfy (A12)–(A14) with $\theta = 1$ when (6) holds with equality. Equations (A12) and (A13) imply

(A15)
$$Y_k'(k) - (1+r)(1-p_0) = 0.$$

From (A12), (A14) and (A15), we obtain $(D(\theta),\ k(\theta),\ S(\theta))$. Differentiating (A12), (A14) and (A15) yields $dk/d\theta (\equiv k'(\theta)) = 0$,

(A16)
$$\frac{dD}{d\theta}(\equiv D'(\theta)) = \frac{-(1+r)\eta v_k'}{u_k'' + (1+r)^2(1-\theta\eta)v_k''} > 0,$$

(A17) $$\frac{dS}{d\theta}(\equiv S'(\theta)) = \frac{(1+r)(1-\rho)\eta v_p'}{u_p'' + (1+r)^2[1-\theta\eta(1-\rho)]v_p''} < 0 .$$

Noting that $A=0$ when $p = p_0$, the parent's utility function is given by

(A18) $$\begin{aligned} U_p\Big|_{p=p_0} &= u_p[Y_p - S(\theta) - p_0 k(\theta)] + v_p[(1+r)S(\theta)] \\ &+ \delta\{u_k[D(\theta) - (1-p_0)k(\theta)] + v_k[Y_k(k(\theta) - (1+r)D(\theta)]\}. \end{aligned}$$

Differentiating (A18) with respect to θ yields

(A19) $$\left.\frac{\partial U_p}{\partial\theta}\right|_{p=p_0} = [-u_p' + (1+r)v_p']S'(\theta) + \delta\{\left[u_k' - v_k' \cdot (1+r)\right]D'(\theta) .$$

From (A12)–(A14), (A16), (A17) and (6), we have

(A20) $$\left.\frac{\partial U_p}{\partial\theta}\right|_{p=p_0} = [D_S^+ S'(\theta)v_p' - D'(\theta)v_k']\theta A_D^+ < 0 ,$$

which implies that, when θ moves from $\theta = 1$ to $\theta = 0$, the parent's utility increases.

Proof of Lemma 2. We define k_0 as k when $p = 0$. Since $Y_k'(k_0) > 1+r$ (Proposition 6), we have the following two cases:

(A21) $$(1-\eta)Y_k'(k_0) < 1+r < Y_k'(k_0),$$

(A22) $$1+r < (1-\eta)Y_k'(k_0) < Y_k'(k_0).$$

Note that $(1-\eta)Y_k'(k_0) = u_k'(c_k^1)/v_k'(c_k^2)$ when $p = 0$.

First, we consider families with (A21) satisfied. Substituting (6) with equality, (24) and (26) into (27) with $p = 0$ yields

(A23) $$\left.\frac{dU_p}{dp}\right|_{p=0} = \delta v_k' \cdot \{[-(1-\eta)Y_k'(k_0) + (1+r)](k_p^+ - k_0) + [Y_k'(k_0) - (1+r)]k_p^+ + A_k^+ k_S^+ k_0\}.$$

From (22) and (A8), we have

(A24) $$k_p^+ - k = -\frac{1}{F\big|_{p=0}}[v_k'(1-\eta)Y_k'(1-\sigma) + v_k'' k(1-\eta)^2 (Y_k')^2] ,$$

where $F\big|_{p=0} = u_k'' + v_k''(1-\eta)^2(Y_k')^2 + v_k'(1-\eta)Y_k'' < 0$. Using (A9), (A24) can be rewritten as

(A25) $$k_p^+ - k = -\frac{v_k'(1-\eta)Y_k'(1-\sigma)}{F\big|_{p=0}} + \frac{kY_k'k_S^+}{1+r}.$$

Noting $A_k^+ = -\eta Y_k'(k)$, we substitute (A25) into (A23) to yield

$$\left.\frac{dU_p}{dp}\right|_{p=0} = \delta v_k'\{[(1-\eta)Y_k' - (1+r)]\frac{v_k'(1-\eta)Y_k'(1-\sigma)}{F\big|_{p=0}} + [Y_k' - (1+r)][k_p^+ - \frac{kY_k'k_S^+}{1+r}(1-\eta)]\}.$$

From (A21), we have that, if $\sigma < 1$, then $(dU_p/dp)_{p=0} > 0$.

Next, we consider families with (A22) satisfied. We rewrite (A23) as $(dU_p/dp)_{p=0} = \delta v_k'\{\eta Y_k'(k)k_p^+ + [(1-\eta)Y_k' - (1+r) + A_k^+ k_S^+]k\}$, which is positive under (A22).

Proof of Lemma 3. We rewrite (27) as

(A26) $$\frac{dU_p}{dp} = (-u_p' + \delta u_k')(k + pk_p^+) + \delta(-u_k' + v_k'Y_k')k_p^+.$$

Substituting (6) with equality and (26) into (A26) yields

(A27) $$\begin{aligned}\frac{dU_p}{dp} = \frac{\delta v_k'}{1+pk_S^+}\{[-\frac{u_k'}{v_k'} + (1+r)][(1+pk_S^+)k_p^+ - (k+pk_p^+)] + [Y_k' - (1+r)]k_p^+(1+pk_S^+) \\ + (k+pk_p^+)k_S^+(p\frac{u_k'}{v_k'} - \eta Y_k')\}.\end{aligned}$$

When $Y_k' = 1+r$ holds, substituting (24) into (A27) yields

(A28) $$\left(\frac{dU_p}{dp}\right)_{Y_k'=1+r} = \frac{\delta v_k'}{1+pk_S^+}[-\frac{(1-\eta)Y_k'(k)}{1-p} + (1+r)][(1-p)k_p^+ - (1+k_S^+)k].$$

Substituting $(1-p)k_p^+ - (1+k_S^+)k = -v_k' \cdot (1-\eta)(1-\sigma)Y'/F$, which is obtained from (A8) and (A9), into (A28) yields

(A29) $$\left(\frac{dU_p}{dp}\right)_{Y_k'=1+r} = \frac{\delta v_k'}{1+pk_S^+}\cdot\frac{v_k'\cdot(1-\eta)(1-\sigma)Y_k'}{F}[\frac{(1-\eta)Y_k'(k)}{1-p} - (1+r)].$$

From (26), (A9) and $A_k^+ = -\eta Y_k'(k)$, we have $1+pk_S^+ > 0$. Since $\sigma < 1$ is assumed, (A29) implies that, if $(1-\eta)Y_k'(k)/(1-p) \geq (\leq) 1+r$,

$$\left(\frac{dU_p}{dp}\right)_{Y_k'=1+r} \leq (\geq) 0.$$

When $(1-\eta)Y_k'/(1-p)(= u_k'/v_k') = 1+r$ holds, (A27) is rewritten as

(A30) $$\left(\frac{dU_p}{dp}\right)_{\frac{(1-\eta)Y_k'}{1-p}=1+r} = \frac{\delta v_k'}{1+pk_S^+}(k_p^+ - kk_S^+)[Y_k' - (1+r)].$$

Hence we have that, if $Y_k'(k) \geq (\leq) 1+r$,

$$\left(\frac{dU_p}{dp}\right)_{\frac{(1-\eta)Y_k'}{1-p}=1+r} \geq (\leq) 0 . \qquad \square$$

Sufficient conditions for $A > 0$ in the case of binding liquidity constraints.

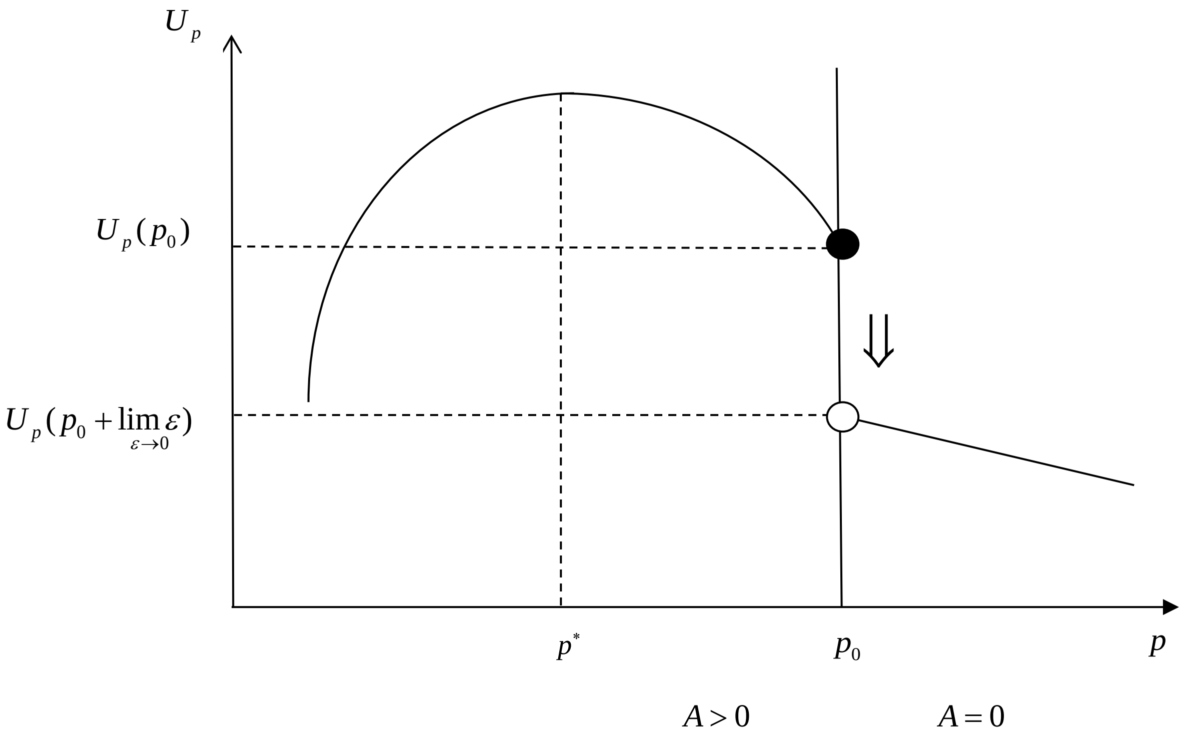

Fig. 5. The parental utility in the case of binding liquidity constraints

Sufficient conditions for $A > 0$ in the case of binding liquidity constraints are as follows (see Fig.5).

(i) $p^* < p_0$, where p^* is defined as p that maximizes U_p for $0 \leq p \leq p_0$. (Put it differently, $-v_p' + \delta v_k' > 0$ holds when $p = p^*$.)

(ii) U_p jumps downward at $p=p_0$ ($\lim_{\varepsilon\to 0}\left[U_p\big|_{p=p_0}-U_p\big|_{p=p_0+\varepsilon}\right]>0$).

Given that condition (i) holds, condition (ii) is satisfied if the child's first-period consumption is insufficient and educational investment is excessive at $p=p_0$. To prove this, we define the following function with dummy variable θ:

(A31) $$-(1-p_0)u_k'(\bar{D}-(1-p_0)k)+v_k'(Y_k(k)-(1+r)\bar{D})\left[Y_k'(k)+\theta A_k^+\right]=0,,$$

(A32) $$\begin{aligned}&-u_p'(Y_p-S-p_0k)-(1+r)v_p'((1+r)S)\\&-\left[p_0u_p'(Y_p-S-p_0k)+\theta v_p'((1+r)S\cdot A_k^+)\right]k_s=0\end{aligned}.$$

k and S satisfy (A31) and (A32) with $\theta=0$ when (6) holds with strict inequality, whereas they satisfy (A31) and (A32) with $\theta=1$ when (6) holds with equality. From (A31) and (A32) , we obtain $(k(\theta),\ S(\theta))$. Differentiating (A31) and (A32) with respect to θ yields

(A33) $$\frac{dk}{d\theta}(\equiv k'(\theta))=\frac{\eta Y_k'v_k'}{u_k''\cdot(1-p_0)^2+v''\cdot(Y_k')^2(1-\theta\eta)+v_k'\cdot(1-\theta\eta)Y_k''}<0.$$

(A34) $$\frac{dS}{d\theta}(\equiv S'(\theta))=\frac{-u_p''p_0k'(\theta)}{u_p''+(1+r)^2v_p''}>0.$$

Noting that $A=0$ when $p=p_0$, the parent's utility function is given by

(A35) $$\begin{aligned}U_p\big|_{p=p_0}&=u_p[Y_p-S(\theta)-p_0k(\theta)]+v_p[(1+r)S(\theta)]\\&+\delta\{u_k[\bar{D}-(1-p_0)k(\theta)]+v_k[Y_k(k(\theta)-(1+r)\bar{D}]\}.\end{aligned}$$

Differentiating (A35) with respect to θ and using the envelop theorem yield

(A36) $$\begin{aligned}\frac{dU_p}{d\theta}\bigg|_{p=p_0}&=\left[-p_0u_p'+\delta(-(1-p_0)u_k'+v_k'Y_k')\right]k'(\theta)\\&=\left\{p_0(-u_p'+\delta u_k')+\delta v_k'\left[\left(-\frac{u_k'}{v_k'}+(1+r)\right)+\left(Y_k'-(1+r)\right)\right]\right\}k'(\theta)\end{aligned}$$

From (27) and condition (i), we have

(A37) $$\frac{dU_p}{dp}\bigg|_{p=p_0}=(-u_p'+\delta u_k')k+\{-pu_p'+\delta[-(1-p)u_k'+v_k'Y_k']\}k_p^+<0.$$

From (A37), if $-u_p'+\delta u_k'>0$, we have $-pu_p'+\delta[-(1-p)u_k'+v_k'Y_k']<0$ and thus (A36)>0, which implies that, when θ increases from $\theta=1$ to $\theta=0$, the parent's utility decreases. On the other hand, if $-u_p' + \delta u_k' \leq 0$, the sign of $-pu_p' + \delta[-(1-p)u_k' + v_k' Y_k']$ is indeterminate in general. However, we have that, if $-u_k' / v_k' + (1+r) < 0$ (the child's first-period consumption is insufficient) and $Y_k' - (1+r) < 0$ (educational investment is excessive), then (A36) is positive, implying that the parent's utility decreases as θ increases.

One may suppose that the condition $dU_p / dp < 0$ *for* $p > p_0$ is also required for $A > 0$**.** However, we have $\left(dU_p / dp\right)_{p=p_0} < 0$ (condition (i)) $\Rightarrow$ $\left(dU_p / dp\right)_{p>p_0} < 0$, which is proved as follows. From (22) and (27), we have

(A38)
$$\left.\frac{dU_p}{dp}\right|_{p=p_0} = (-u_p' + \delta u_k')k + \{-pu_p' + \delta[-(1-p)u_k' + v_k' Y_k']\}k_p^+$$
$$= (-u_p' + \delta u_k')k - u_p' p k_p^+ + v_k' \eta Y_k' k_p^+,$$

whereas from (22) with $A = 0$ we have

(A39)
$$\left.\frac{dU_p}{dp}\right|_{p=p_0+\lim_{\varepsilon\to 0}\varepsilon} = (-u_p' + \delta u_k')k + \{-pu_p' + \delta[-(1-p)u_k' + v_k' Y_k']\}k_p^0$$
$$= (-u_p' + \delta u_k')k - u_p' p k_p^0.$$

Noting that $k_p^+ = k_p^0 = -(1+r)/Y_k''(k)$ and $v_k' \eta Y_k' k_p^+ > 0$, we have that (A39) is negative if (A38) is negative. Assuming the concavity of U_p, we obtain $\left(dU_p / dp\right)_{p>p_0} < 0$.

The sign of $dD^{**}(Y_p)/dY_p$**.** As shown in Section 4, we have two equilibria, (i) $p^* = 0$ and (ii) $p^* = p_0(Y_p) + \lim_{\varepsilon\to 0}\varepsilon$, in the case of non-binding liquidity constraints. We examine the sign of $dD^{**}(Y_p)/dY_p$ in each case.

(i) This type of equilibrium is characterized by $p = 0,\ Y_k'(k) = 1+r,\ A = A^+(k,D,S)$, (9) and (15). Substituting $A = A^+(k,\ D,\ S)$ into (9) and (15) yields

(A40) $$u_k'(D-k) - v_k'(Y_k(k) - (1+r)D + A^+(k,D,S))(1+r)(1-\eta) = 0,$$

(A41) $$-u_p'(Y_p - S) + v_p'((1+r)S - A^+(k,D,S))(1+r)(1-\eta(1-\rho)) = 0.$$

Differentiating (A40) and (A41) with respect to D, S and Y_p, we have

(A42)
$$\begin{bmatrix} U^k_{DD} & U^k_{DS} \\ U^p_{SD} & U^p_{SS} \end{bmatrix}\begin{bmatrix} dD \\ dS \end{bmatrix} = \begin{bmatrix} 0 \\ U^p_{SYp} \end{bmatrix} dY_p ,$$

where

$U^k_{DD} = u''_k + v''_k(1+r)^2(1-\eta)^2 < 0$,

$U^k_{DS} = -v''_k(1+r)^2(1-\eta)^2 > 0$,

$U^p_{SD} = -v''_p\eta(1+r)^2(1-\eta(1-\rho)) > 0$,

$U^p_{SS} = u''_p + v''_p\eta(1+r)^2(1-\eta(1-\rho)) < 0$,

$U^p_{SYp} = u''_p < 0$.

From (A42), we have

$$\frac{dD^{**}}{dY_p} = \frac{u''_p v''_k(1+r)^2(1-\eta)^2}{u''_p v''_k(1+r)^2(1-\eta)^2 + u''_p v''_k + u''_k v''_p\eta(1+r)^2(1-\eta(1-\rho))} > 0 .$$

Since $dD^{**}(Y_p)/dY_p$ is likely to be very small, the assumption that $d\bar{D}/dY_p > dD^{**}/dY_p$ may be acceptable.

(ii) This type of equilibrium is characterized by $A = 0$, (10), (11), (15) and (16). Substituting (10) and (11) with $A = 0$ into (15) with $A = 0$ and (16) yields

(A43)
$$-u'_p(Y_p - S - pk^0(p)) + v'_p((1+r)S)(1+r) = 0$$

(A44)
$$-v'_p\left((1+r)S\right) + \delta v'_k\left(Y_k(k^0(p) - (1+r)D^0(p,S)\right) = 0 .$$

Differentiating (A43) and (A44) with respect to S, p and Y_p, we have

(A45)
$$\begin{bmatrix} U^p_{SS} & U^k_{Sp} \\ U^p_{pS} & U^p_{pp} \end{bmatrix}\begin{bmatrix} dS \\ dp \end{bmatrix} = \begin{bmatrix} u''_p \\ 0 \end{bmatrix} dY_p ,$$

where

$U^p_{SS} = u''_p + v''_p(1+r)^2 < 0$,

$U^k_{Sp} = u''_p(k + pk^0_p) < 0$,

$U^p_{pS} = -v''_p(1+r) > 0$,

$U^p_{pp} = \delta v''_k(Y'_k k^0_p - (1+r)D^0_p) = \delta v''_k u''_k(1+r)k/(u''_k + (1+r)^2 v''_k) < 0$.

From (A45), we have

(A46)
$$\frac{dp}{dY_p} = \frac{u''_p v''_p(1+r)}{\left[\delta k(1+r)u''_k v''_k(u''_p + v''_p(1+r)^2)/(u''_k + (1+r)^2 v''_k)\right] + u''_p v''_p(1+r)(k + pk^0_p)}$$
$$= \frac{w}{kX + w(k + pk^0_p)} > 0,$$

where $w = u_p'' v_p'' (1+r) > 0$ and $X = \delta\rho^0 (1+r) v_k'' (u_p'' + v_p''(1+\mathrm{r})^2) > 0$. This means that the parental share of educational expenditure p increases as the parental income Y_p rises.

From (A7), we have $D_p^0 > 0$ if $\sigma < 1$. As discussed in 4.1, the effect of p on D is divided into the negative direct effect and the positive indirect effect via the change in k. If $\sigma < 1$, the latter effect dominates the former effect. $D_p^0 > 0$ and (A46) imply

(A47)
$$\begin{aligned}\frac{dD^{**}}{dY_p} &= D_p^0 \frac{dp}{dY_p} \\ &= \frac{wD_p^0}{kX + w(k + pk_p^0)} > 0.\end{aligned}$$

Using (A6), we rewrite (A47) as

(A48)
$$\frac{dD^{**}}{dY_p} = \frac{w\left[-k\rho^0 + (1-p)k_p^0\right]}{kX + w(k + pk_p^0)}.$$

Differentiating (A48) with respect to k_p^0 yields

(A49)
$$\frac{\partial\left(dD^{**} / dY_p\right)}{\partial k_p^0} = \frac{kw\left[(1-p)(\mathrm{w}+X) + pw\rho^0\right]}{\left[kX + w(k + pk_p^0)\right]^2} > 0.$$

We can interpret (A49) as follows. If the increase in educational investments by a rise in p is small, the corresponding increase in borrowings is also small. In this case, although an increase in Y_p raises p, its effect on borrowings is limited.

Therefore, if k_p^0 is small enough, then dD^{**} / dY_p is also small enough, and the assumption that $d\bar{D} / dY_p > dD^{**} / dY_p$ is likely to be satisfied.

Proof of optimality of the equilibrium with government loan policy in the case of binding liquidity constraint. Define $(\tilde{A}, \tilde{k}, \tilde{S}, \tilde{p}, \tilde{D}_G)$ as the solution of the following system:

(A50)
$$-v_p'(C_p^2) + \delta v_k'(C_k^2) = 0,$$

(A51)
$$Y_k'(k) = 1 + r,$$

(A52)
$$-u_p'(C_p^1) + (1+r)v_p'(C_p^2) = 0,$$

(A53)
$$p = \eta,$$

(A54)
$$u_k'(C_k^1) - (1+r)v_k'(C_k^2) = 0,$$

where $C_p^1 = Y_p - S - pk,$ $C_p^2 = (1+r)S - A,$ $C_k^1 = D_G - (1-p)k,$ and $C_k^2 = Y_k(k) - (1+r)D_G + A$.

In the following, we show that $(\tilde{A}, \tilde{k}, \tilde{S}, \tilde{p}, \tilde{D}_G)$ coincides with both the equilibrium solution and the family optimum. The equilibrium solution $(A^*, k^*, S^*, p^*, D_G^*)$ satisfies

(A55) $$-v_p'(C_p^2) + \delta v_k'(C_k^2) = 0$$

(A56) $$-(1-p)u_k'(C_k^1) + v_k'(C_k^2)Y_k'(k)(1-\eta) = 0$$

(A57) $$-u_p'(C_p^1) + (1+r)v_p'(C_p^2) - [pu_p'(C_p^1) - v_p'(C_p^2)\eta Y_k'(k)]k_S^+ = 0$$

(A58) $$\begin{aligned}&[-u_p'(C_p^1) + \delta u_k'(C_k^1)]k \\ &+ \left\{-pu_p'(C_p^1) + \delta[-(1-p)u_k'(C_k^1) + v_k'(C_k^2)Y_k'(k)\right\} k_p^+ = 0\end{aligned}$$

and (28).

From (A51), (A53) and (A54), we obtain (A56). From (A52), we have $-u_p'(C_p^1) + (1+r)v_p'(C_p^2) - \eta[u_p'(C_p^1) - (1+r)v_p'(C_p^2)]k_S^+ = 0$. Substituting (A51) and (A53) into this equation yields (A57). From (A50)–(A52) and (A54), we have $[-u_p'(C_p^1) + \delta u_k'(C_k^1)](k + pk_p^+) + \delta[-u_k'(C_k^1) + v_k'(C_k^2)Y_k'(k)]k_p^+ = 0$. Rearranging this equation yields (A58). From (A50)–(A52) and (A54), we have

$$\delta[u_k'(C_k^1) - (1+r)v_k'(C_k^2)] + \left\{p[-u_p'(C_p^1) + \delta u_k'(C_k^1)] + \delta[-u_k'(C_k^1) + v_k'(C_k^2)Y_k'(k)]\right\}(\partial k / \partial D_G) = 0 .$$

Rearranging this equation yields (28). Therefore, $(\tilde{A}, \tilde{k}, \tilde{S}, \tilde{p}, \tilde{D}_G)$ satisfies (A55)–(A58) and (28), and hence equals the equilibrium solution $(A^*, k^*, S^*, p^*, D_G^*)$.

The family optimum satisfies (2)–(5). From (A50)–(A52) and (A54), we have (2)–(5). This implies that $(\tilde{A}, \tilde{k}, \tilde{S}, \tilde{p}, \tilde{D}_G)$ satisfies the family optimality condition. Since $(\tilde{A}, \tilde{k}, \tilde{S}, \tilde{p}, \tilde{D}_G)$ coincides with both the equilibrium solution and the family optimum, the equilibrium solution $(A^*, k^*, S^*, p^*, D_G^*)$ coincides with the family optimum. □

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