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The Struggle to Survive in the R&D Sector:
Implications for Innovation and Growth

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Abstract

We develop a growth model capturing an important aspect of the real world: the struggle to survive in the research and development (R&D) sector in the form of endogenous intertemporal investments by R&D firms to prevent product obsolescence. The core finding is that if legal patent protection is too strong, a higher R&D subsidy rate delivers insufficient investments to survive in the R&D sector, depressing innovation and growth in the long run. Quantitative analysis indicates that, in countries with a high R&D subsidy rate, the current real-world patent protection may be high enough to have a negative effect on R&D subsidies because of the reduced number of R&D firms.

JEL classification: O31, O34, O41
Keywords: Survival, subsidy, patent breadth, R&D-based endogenous growth

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1 Introduction

The essential role of the entry, exit, and survival of firms has been emphasized in growth theory. In Schumpeterian growth models (Segerstrom, Anant, and Dinopoulos 1990, Grossman and Helpman 1991, Aghion and Howitt 1992), the economy grows through survival cycles between the entry of a firm inventing a new high-quality technology and the exit of the firm by destruction of its rents once a newer technology is introduced. Lai (1998) developed a variety-based growth model à la Romer (1990) that captures a dynamic process of firm survival in which rents decrease over time because of gradual product obsolescence. Glass (2001) considers a quality-based model with gradual obsolescence. In these models, survival of firms is exogenous; an important exception is Thoenig and Verdier (2003),[1] who consider, in a quality-based model, that a firm can endogenously survive against obsolescence by using a defensive, more tacit-knowledge-intensive technology. However, although the struggle to survive in the real world typically requires that firms take dynamic decisions,[2] in their model, the survival activity of a firm is static.

We modify a variety-based growth model to capture the struggle for survival by R&D firms as an endogenous dynamic phenomenon and to investigate the effects on the implications of an R&D-based growth model that considers policies, patent protection, and the growth rate. In the model, not only the growth rate but also the probability of survival of an R&D firm are endogenous.

We consider dynamic programming for R&D firms, which engage in intertemporal investments with the aim of increasing their probability of survival against obsolescence. The solution to this governs the evolution of survival investments and probability, which positively or negatively affects the long-run growth rate in market equilibrium. This implies that the struggle to survive plays a significant but ambiguous role in innovation and growth. To further clarify this role, we examine the effects of two standard R&D policy levers: subsidies and legal patent protection. Following Li (2001) and many others, we measure the strength of patent protection by *patent breadth*.

The core finding is that if patent protection is too strong, a higher R&D subsidy

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[1] See Dinopoulos and Syropoulos (2007) and Eicher and García-Peñalosa (2008) for recent growth models with endogenous survival (defense) of firms. See also Akiyama and Furukawa (2009) and Davis and Şener (2012). In these models, the survival activity is essentially not dynamic.

[2] This is the common view in a variety of fields including industrial organization, marketing, and technology management. See, for example, Agarwal and Gort (2002).

rate delivers insufficient investments for the survival of R&D investments, depressing innovation and growth in the long run. Quantitative analysis indicates that, in countries with a high R&D subsidy rate, the current real-world patent protection may be high enough to imply a negative effect of R&D subsidies because of the reduced number of R&D firms.

Our paper is related to studies in the R&D-based growth literature that examine the effects of R&D policies such as subsidies and patent breadth on technological progress and growth; see, for example, Segerstrom (2000), Li (2001), Goh and Olivier (2002), Chu (2009, 2011), Chu, Cozzi, and Galli (2012), Chu, Pan, and Sun (2012), and Iwaisako and Futagami (2012). However, these studies do not consider endogenous or dynamic investments in the struggle for survival, so they do not analyze the policy effects on those investments.[3]

The policy implication of our result is new to this literature in suggesting a substantial interdependence between the two R&D policy instruments: the effect of R&D subsidies depends on patent protection. This is different from the results in the literature. For example, Li (2001) showed that both these policies are growth enhancing. However, we find that these two policy levers interact with each other to be interdependent in equilibrium, which emphasizes the critical role of investment in survival in any consideration of the policy implications in an R&D-based growth model.

The policy interdependence in our analysis is, in fact, substantially due to the endogenous and intertemporal nature of investment in survival. In short, a higher R&D subsidy rate negatively affects the probability of survival of R&D firms, by making investment in R&D more profitable for firms than investment in survival. The reduced survival probability decreases the expectation of an R&D firm's future value. This reduction in the expected future value is much more serious when future temporary profits, on average, are larger due to a stronger patent protection. Therefore, when patent protection is very strong, the negative effect of a larger R&D subsidy rate tends to dominate the usual growth-enhancing effect of R&D subsidies.

[3] Broadly, our approach applies the market quality theory of Yano (2008, 2009) to survival dynamics.

2 An R&D-based Growth Model with the Struggle for Survival

We consider a variety expansion model of endogenous growth à la Romer (1990) and Grossman and Helpman (1991). There is the infinitely lived representative consumer who inelastically supplies L units of labor in each period. This consumer is endowed with the utility function $U = \sum_{t=0}^{\infty} \beta^t \ln C_t$, where the consumption C_t is defined as a constant elasticity of substitution (CES) function on the continuum of differentiated goods: $C_t = \left(\int_0^{N_t} x_t(j)^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)}$, where $\sigma > 1$ is the elasticity of substitution; $x_t(j)$ is the amount of differentiated good j; and N_t is the number of goods available in period t. It is well known that the corresponding dynamic optimization problem has a solution that yields the Euler equation:

$$\frac{E_{t+1}}{E_t} = \beta(1 + r_t), \tag{1}$$

where r_t is the interest rate and $E_t = \int_0^{N_t} p_t(j)x_t(j)dj$ represents the consumer's spending in period t with the price $p_t(j)$ of final good j. The static demand function for good j is given by $x_t(j) = E_t (p_t(j))^{-\sigma} / (P_t)^{1-\sigma}$, where P_t is the price index defined by $P_t = \left(\int_0^{N_t} p_t(j)^{1-\sigma} dj \right)^{1/(1-\sigma)}$. Assume that a unit of each good j can be manufactured from a unit of labor. If the good j survives up until period t, it is manufactured by the monopolistic firm (patent holder).

To allow for a role for patent policy, we consider an upper-bound $\mu \in (1, \sigma/(\sigma-1)]$ in the markup.[4] Therefore, the equilibrium price becomes $p_t(i) = \mu w_t$, where w_t is the wage rate. As in the existing literature,[5] we interpret μ as patent breadth (i.e., a measure for the strength of patent protection). In this setting, a larger patent breadth μ means a higher markup in accordance with the seminal vision of Gilbert and Shapiro (1990) on "breadth as the ability of the patentee to raise the price." This pricing gives rise to the demand and profit functions, given by:

$$x_t(j) = x_t = \frac{E_t}{\mu w_t N_t} \quad \text{and} \quad \pi_t(j) = \pi_t = \left(\frac{\mu - 1}{\mu} \right) \frac{E_t}{N_t}. \tag{2}$$

[4]To allow for a sufficiently large patent breadth μ, we consider that σ is sufficiently small. To verify that sufficiently large patent breadths are not empirically too restrictive, we show in section 3.1 that our results hold for empirically plausible levels of patent breadth.

[5]See also Li (2001), Goh and Olivier (2002), Iwaisako and Futagami (2012), Chu (2011), and Iwaisako and Futagami (2012) for a similar formulation in the dynamic general equilibrium model.

Equation (2) shows that the profit π_t is reduced by an increase in the number N_t of goods. This property, which is usual in this class of growth models, captures that the struggle to survive becomes more difficult as more goods (more firms) survive in the market.

2.1 R&D and Survival

There are a number of perfectly competitive potential R&D firms. A potential R&D firm can innovate one new technology to produce a new intermediate good in period t by investing $1/(\kappa N_{t-1})$ units of labor in period $t-1$, where knowledge spillover is assumed in a standard manner (Romer 1990). Here, $\kappa \in [0, \infty)$ denotes the productivity of R&D. We denote $s \in [0,1)$ as a subsidy rate for innovation, so that the unit cost of R&D is equal to $(1-s)\, w_{t-1}/\kappa$.[6]

A firm that successfully innovates a new product, j, manufactures product j monopolistically, thereby earning a monopolistic rent in period t, π_t. This rent continues through subsequent periods. At an endogenous probability of $1-\iota_t(j)$, where $\iota_t(j) \in [0,1]$ stands for the probability of survival at the end of period t, we assume that an innovated good j is obsoleted and the R&D firm innovating good j has to leave the market. This assumption is based on Lai's (1998) vision of assuming product obsolescence over the endogenously expanding variety of differentiated goods.[7]

We consider that the R&D firm engages in a struggle to survive against obsolescence. To incorporate this, we assume the firm can increase the probability of survival $\iota_t(j)$ by investing $z_t(j)/N_t$ units of labor in period t.[8] Specifically, $\iota_t(j) = (z_t(j))^{\alpha}$, in which $z_t(j) \in [0,1]$ denotes the intensity of survival investment and $\alpha \in (0,1)$ is a technological parameter.[9]

Before proceeding, it is important to consider more specifically the survival investment against product obsolescence. If we took the interpretation of Ethier (1982)

[6] This subsidy is financed by a lump-sum tax.

[7] Whereas his focus is on gradual obsolescence, we consider that product obsolescence is stochastic and discrete. We leave for future research the task of analyzing firm survival against gradual obsolescence.

[8] We also assume the usual external effect of knowledge (Romer 1990) for the survival investment.

[9] For the sake of explanation, we adopt the simplest function for survival probability $\iota_t(j)$, but we obtain the qualitatively same results using a more general form of the survival probability such as $\iota_t(j) = (z_t(j))^{\alpha} + \phi$ or $\left(\gamma\, (z_t(j))^{\alpha} + (1-\gamma)\, (\phi)^{\alpha}\right)^{1/\alpha}$, where $\phi \in (0,1)$ and $\gamma \in (0,1)$ are parameters that capture market or institutional attributes for firm survival.

that the differentiated goods were intermediate goods used for producing the consumption good C_t through the CES production function, then we would suppose that an intermediate product is obsoleted by the introduction of new, more high-tech intermediate goods. The survival investment would be made to update/upgrade the invented intermediate product to catch up with cutting-edge standards. In this paper, we interpret the differentiated goods as consumption goods. The survival investment of a firm is made to keep the consumer interested in its innovated consumption good; this is more akin to the vision of Lai (1998) that a consumption good is obsoleted by the "introduction of more sophisticated goods" for the consumer with a "love of sophistication." For either interpretation, our point is that the incumbent firms invest in their survival against product obsolescence.

An active R&D firm's value is the expectation of the net present discounted value of profits, which is equal to:

$$V_t(j) = \sum_{\tau=t}^{\infty} \left(\left(\prod\nolimits_{s=t+1}^{\tau} \frac{\iota_{s-1}(j)}{1+r_{s-1}} \right) (\pi_\tau(j) - z_\tau(j)) \right). \tag{3}$$

Given that $\pi_t(j) = \pi_t$ in (2), we have $z_t(j) = z_t$ and $\iota_t(j) = \iota_t$ for all j in equilibrium. Thus, we can describe the active R&D firm's behavior as the following dynamic programming problem, following Akiyama, Furukawa, and Yano (2011):

$$V_t^* = \max_{z_t \in [0,1];\ \iota_t = (z_t)^\alpha} \left[\pi_t - \frac{w_t z_t}{N_t} + \iota_t \frac{V_{t+1}^*}{1+r_t} \right]. \tag{4}$$

The solution to (4) gives rise to the following policy function:

$$z_t^* = \min \left\{ \left(\frac{\alpha V_{t+1}^*/(1+r_t)}{w_t/N_t} \right)^{1/(1-\alpha)},\ 1 \right\}.^{10} \tag{5}$$

The evolution of the survival investment and probability, $\{z_t^*\}$ and $\{\iota_t^*\}$, is governed by (5) with the survival function. Over the course of evolution, the larger the discounted future value of innovation ($V_{t+1}^*/(1+r_t)$) or the lower the marginal cost to survive (w_t/N_t), the larger the survival investment and probability (z_t^* and ι_t^*).

[10] Clearly, $z_t = 0$ is not an equilibrium choice because $d\iota_t/dz_t \to \infty$ as $z_t \to 0$. Noting $\iota_t \leq 1$, the usual Karush–Kuhn–Tucker solution leads to (5). Note that the transversality condition is satisfied, because ι_t^* is uniformly bounded in the present model.

2.2 Market Equilibrium

Free entry into the R&D market ensures that the discounted value of an innovation is equal to the cost, so that we have:

$$\frac{V_{t+1}^{*}}{1+r_t} = \frac{(1-s)\,w_t}{\kappa N_t}. \tag{6}$$

From (5) and (6), in market equilibrium, the intensity of survival investment and the probability of survival, z_t^* and ι_t^*, are independent of time: $z_t^* = z^*$ and $\iota_t^* = \iota^*$ for all t. Specifically:

$$z^* = \begin{cases} (\alpha(1-s)/\kappa)^{\frac{1}{1-\alpha}} & \text{if } \alpha(1-s)/\kappa < 1 \\ 1 & \text{if } \alpha(1-s)/\kappa \geq 1 \end{cases}, \tag{7}$$

$$\iota^* = \begin{cases} (\alpha(1-s)/\kappa)^{\frac{\alpha}{1-\alpha}} & \text{if } \alpha(1-s)/\kappa < 1 \\ 1 & \text{if } \alpha(1-s)/\kappa \geq 1 \end{cases}. \tag{8}$$

There are two kinds of equilibrium: with or without the exit of firms. When the unit cost of an innovation $1/\kappa$ is lower, the exit of firms occurs, in equilibrium, ($\iota^* < 1$), in which some firms leave the market in each period. When $1/\kappa$ is higher, there are no exits, in equilibrium, ($\iota^* = 1$).

Equations (7) and (8) reveal that the survival lifetime of an R&D firm (ι^*) is determined by both technology and policy factors. For the technology factor, the lifetime of a firm is longer as the unit cost of an innovation ($1/\kappa$) is higher. This property comes from a larger value of an innovation that is more difficult (costly) to develop, which gives the firm an incentive to invest more in defense (z^*) of an existing innovation. This results in a longer survival probability/lifetime of the firm. For the policy factor, the lifetime (ι^*) becomes shorter when the R&D subsidy rate s increases. This is because the firm responds to large R&D subsidies by investing more in innovation than in survival.

Now we can close the model by considering the condition for labor market equilibrium. Before proceeding, the number N_t of consumption goods changes over time, which increases with an innovation and decreases with the exit of firms. Then, we have:

$$N_{t+1} = \iota^* N_t + M_t, \tag{9}$$

where M_t denotes the inflow of innovation made in period t and $\iota^* N_t$ is the number of firms that survive at the end of period t. The labor market clearing condition is given by:

$$L = N_t x_t + \left(\frac{1}{\kappa N_t}\right) M_t + \left(\frac{z^*}{N_t}\right) N_t, \tag{10}$$

in which the right-hand side denotes the three labor demands: $N_t x_t$ for production, $\left(\frac{1}{\kappa N_t}\right) M_t$ for innovation, and $\left(\frac{z^*}{N_t}\right) N_t$ for survival.

By (1), (2), (4), (6), (9) and (10), we can characterize the long-run equilibrium of the model with the following theorem.

Theorem 1 *In the initial period* 0*, the economy jumps into a unique balanced growth path that is characterized by the following long-run rate of economic growth:*

$$1 + g^* = \frac{\beta}{1 - s + \beta(\mu - 1)} \left(\kappa L(\mu - 1) + (\mu - s)\iota^* - \mu\kappa z^*\right), \tag{11}$$

where $g^* = (N_{t+1} - N_t)/N_t$ *for all* $t \geq 0$*. The equilibrium investment and probability of survival,* z^* *and* ι^**, are given by (7) and (8).*

Proof. See Appendix A. ■

Theorem 1 shows that the struggle for survival, represented by the investment z^* and probability ι^*, plays an important but ambiguous role in long-run growth g^*. From (11), the survival probability ι^* contributes to growth g^* by slowing obsolescence while the investment in survival z^* *itself* depresses growth g^* by tightening the resource (labor) market (i.e., fewer resources are left for new innovation).

3 Effects of R&D Subsidies and Patent Protection

3.1 Qualitative Analysis

To further identify the role of the struggle to survive in the R&D sector, we examine the effects of two R&D policy levers: subsidies and patent protection (i.e., patent breadth in our analysis). We present two propositions. The first proposition is as follows.

Proposition 1 *The higher the R&D subsidy rate* s*, the lower the probability of survival of firms* ι^**.*

Proposition 1, by inspection of (7) and (8), shows that the probability of survival ι^* is a decreasing function of the R&D subsidy rate s. This effect is quite intuitive: a higher rate s of R&D subsidies makes the R&D activity more profitable for firms than the survival activity, encouraging firms to invest more resources in R&D and fewer resources in survival. To express it simply, the firms respond to an increased R&D subsidy rate by engaging more in R&D than in survival.

Next, we examine the effects of the R&D subsidy s on growth g^*. First, using (11) together with (7) and (8), we can verify that when no firm exit is to take place in equilibrium ($\iota^* = 1$ as $\alpha(1-s)/\kappa \geq 1$), the R&D subsidy only has the usual growth-enhancing effect: the higher the subsidy rate s, the higher the long-run growth rate g^*.[11]

However, the effect may be different for a more realistic case where the probability of survival is less than 1 and some firms leave the market in each period ($\iota^* < 1$ as $\alpha(1-s)/\kappa < 1$). This is because a higher R&D subsidy rate s affects the survival activity and environment, such as by lowering the probability of survival ι^* (Proposition 1). Differentiating (11) with respect to s, we have the second proposition, as follows.[12]

Proposition 2 *In the presence of firm exit (when $\alpha(1-s)/\kappa < 1$), the effect of an increase in the R&D subsidy rate s is negative on growth g^* if the patent breadth μ is sufficiently large.*

Proposition 2 shows an interdependence between these two policies— subsidies and patent breadth—suggesting that whether the R&D subsidy enhances growth depends on patent breadth. The intuition for this is as follows. In short, a higher R&D subsidy rate s results in a decrease in the expectation of the future value of R&D firms, by reducing the probability of survival ι^* (Proposition 1). This effect of reducing R&D survival ι^* on the expected future value is much more serious when the profits of firms are larger because of a larger patent breadth μ. Therefore, as μ is large, the negative effect of a larger R&D subsidy rate tends to dominate the usual growth-enhancing effect of R&D subsidies.

We elaborate this by means of the Bellman equation (4) with (2) and (6). We can show that the present value of an innovation, denoted as v^*, is balanced with

[11] See Appendix B for the formal proof.

[12] See Appendix B for the formal proof.

the temporary profit plus the expectation of the discounted future value:

$$\underbrace{v^*}_{\textit{Present value}} = \underbrace{(\mu - 1)/\mu}_{\textit{Profit}} + \underbrace{(\overset{\text{Effect 1.}}{\iota^*} - z^* \cdot \overset{\text{Effect 2.}}{\kappa/(1-s)})\beta v^*/g^*}_{\textit{Future value}}. \tag{12}$$

Equation (12) shows that a higher R&D subsidy rate s leads to a lower present value v^* of innovation by decreasing the *future value* through the following two effects. As the R&D subsidy rate s increases,

1. the probability of R&D survival ι^* decreases (Proposition 1), which reduces the expected lifetime of the R&D firm, and

2. the marginal cost of R&D survival $\kappa/(1-s)$ increases, which reduces the profitability of the R&D firm.[13]

The key mechanism in (12) is that these two negative effects of the subsidy s on the innovation value v^* are strengthened by a larger patent breadth μ (whereby with a larger *profit*, $(\mu-1)/\mu$). This simply reflects that when the profits are large (due to a large patent breadth μ), the decrease in the value v^* caused by (1) the decreased survival probability ι^* and (2) the increased survival cost $\kappa/(1-s)$ is also large. Finally, given the standard positive relationship between the innovation value v^* and the growth rate g^*,[14] we may complete the description of the intuition for Proposition 2 by summarizing those arguments as follows.

Remark 1 (Policy Interdependence) *The larger the patent breadth μ, the stronger the two negative effects of a higher R&D subsidy rate s on innovation v^* and growth g^* through an impact on the survival activity and environment for R&D firms, i.e., reducing the survival probability ι^* and increasing the survival cost $\kappa/(1-s)$. Proposition 2 demonstrates that if patent breadth μ is large enough, these two effects are strong enough to dominate the standard growth-enhancing effect of R&D subsidies.*

We conclude the qualitative analysis as follows. The struggle for survival in R&D (with endogenous and intertemporal investments in survival) creates an R&D policy interdependence; whether the R&D subsidy increases technological progress

[13] This effect is due to the free-entry condition (6).

[14] This natural relationship is usual in R&D-based growth models, and we can easily verify it by using (10).

and growth crucially depends on the strength of the patent protection (captured by patent breadth). Specifically, if the patent protection is too strong, a higher R&D subsidy rate delivers insufficient investments for survival and, instead, decreases innovation and growth.

3.2 Quantitative Analysis

An important question is whether real-world patent protection results in a positive or negative effect on R&D subsidies. To address this question, we calibrate the model containing a square-root survival function by normalizing $\alpha = 0.5$. Consider the set of variables, $\{\beta, \kappa, \mu, s, g^*\}$. We set the time preference rate β to a standard value of 0.97. As for patent breadth (i.e., the measure for patent protection), we consider two polar levels of the markup from the realistic range, $\mu \in \{1.6,\ 2.5\}$.[15] We work on the entire range of the subsidy rate $s \in (0,1)$. Using a plausible rate of survival, 0.925,[16] we calibrate the R&D productivity κ. Finally, we take a realistic growth rate $g^* = 0.016$ as the benchmark.[17]

Numerical calculations show that, for the large patent breadth case ($\mu = 2.5$), the growth effect of R&D subsidies s is negative above a very low threshold, $s \simeq 0.08$ (about 8 percent). Even for the small patent breadth case ($\mu = 1.6$), the threshold level goes up to $s \simeq 0.18$ (about 18 percent). Given the real-world average rates of R&D subsidies (approximately 10 percent for the US, 20 percent for the UK, 30 percent for Canada, and 40 percent for France),[18] our calculations suggest that, in countries with a high R&D subsidy rate such as Canada and France, the current level of patent breadth may have a negative effect of R&D subsidies on innovation and economic growth because of the decreased survival of R&D firms.

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[15] See the estimates in Hall (1986) and a calibration analysis based on these estimates in Kwan and Lai (2003).

[16] Note that, on average, 90–95 percent of firms survive in a single year (Agarwal and Gort 2002). We take the average of 90 and 95 to obtain 0.925.

[17] See Kwan and Lai (2003).

[18] See Parsons (2011).

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Appendix A:

By incorporating (1), (2), and (9) into (10), we have:

$$\frac{N_{t+1}}{N_t} = \frac{\kappa L + \iota^* - \kappa z^*}{\frac{1-s}{\beta\mu} + \frac{V_{t+1}^* N_{t+1}}{E_{t+1}}} \frac{V_{t+1}^* N_{t+1}}{E_{t+1}}, \tag{A1}$$

in which use has been made of $w_t = \frac{V_{t+1}^*}{1+r_t}\frac{\kappa N_t}{1-s}$ by (6). By incorporating (2) and (A1) into (4),[19]

$$\begin{aligned} \frac{V_{t+1}^* N_{t+1}}{E_{t+1}} &= \frac{\kappa L + \iota^* - \kappa z^*}{\beta\left(\iota^* - \frac{\kappa}{1-s} z^*\right)} \left(\frac{V_t^* N_t}{E_t}\right) \\ &\quad - \frac{1}{\beta}\left(\frac{\kappa L\left(1-\mu^{-1}\right) + \left(1 - s/\mu\right)\iota^* - \kappa z^*}{\iota^* - \frac{\kappa z^*}{1-s}}\right) \end{aligned} \tag{A2}$$

is obtained. By (A2), the usual arguments on the transversality condition imply $\frac{V_{t+1}^* N_{t+1}}{E_{t+1}} = \frac{V_t^* N_t}{E_t} = v^*$ for all $t \geq 0$ (saddle-path stability). Then, we have

$$v^* = \frac{\kappa\left(1 - (1/\mu)\right)L + \left(1 - s/\mu\right)\iota^* - \kappa z^*}{\kappa L + (1-\beta)\iota^* + \frac{\beta - (1-s)}{1-s}\kappa z^*}. \tag{A3}$$

With the definition of v^*, substituting (A3) into (A1) implies (11). To ensure $g^* > 0$, we assume the labor force is sufficiently large to meet:

$$(\mu - 1)L > \mu z^* + \frac{(1-\beta)(1-s)}{\kappa\beta} + (1 - \iota^*)\frac{\mu - s}{\kappa}, \tag{A4}$$

which implies $(\mu - 1)L > \mu z^*$.

Appendix B:

When $\phi \geq 1 - \left(\alpha(1-s)/\kappa\right)^{\frac{\alpha}{1-\alpha}}$, by (7), (8), and (11), we have

$$\frac{d}{ds}(1 + g^*) = \beta\frac{1 - \beta + \frac{\kappa}{\mu-1}\left((\mu-1)L - \mu z^*\right)}{\left((1-s) + \beta(\mu-1)\right)^2}, \tag{B1}$$

[19] Note that:

$$\frac{N_{t+1}V_{t+1}^*}{E_{t+1}} = \frac{N_{t+1}}{N_t}\frac{\frac{V_t^* N_t}{E_t} - \left(\frac{\mu-1}{\mu}\right)}{\beta\left(\iota^* - \frac{\kappa z^*}{1-s}\right)}.$$

which is strictly positive as $(\mu - 1)L > \mu z^*$ must hold for a positive growth rate, $g^* > 0$; see Appendix A.

When $\phi < 1 - (\alpha (1 - s) / \kappa)^{\frac{\alpha}{1-\alpha}}$, by differentiating (11) with respect to s, with (7), we obtain

$$\begin{aligned} & \frac{d}{ds}(1 + g^*) \\ = \; & \frac{\beta}{\mu - 1} \; \frac{\kappa L + (1-\beta)\phi + \left(\frac{\alpha(1-s)}{\kappa}\right)^{\frac{\alpha}{1-\alpha}} \left(s + (1-\alpha)(1-s)\frac{\mu}{\mu-1} - (1-s+\beta(\mu-1))\left(1 + \frac{\alpha s(\mu-1)}{(1-\alpha)(1-s)}\right)\right)}{((\sigma-1)(1-s)+\beta)^2} \; . \end{aligned} \tag{B2}$$

As μ goes to ∞, the first two terms in the right-hand side go to 0 while the third term goes to $-\infty$. By differentiating (11) with respect to μ, we obtain

$$\frac{d}{d\mu}(1 + g^*) = \beta \frac{\kappa L (1 - s) + (1 - s - \beta + s\beta)\, \iota^* - (1 - s - \beta)\, \kappa z^*}{(1 - s + \beta (\mu - 1))^2}, \tag{B3}$$

which is always positive by (7).