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Estimation of the true almost Ideal demand system model: MCMC Bayesian approach

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Abstract

The almost ideal demand (AID) system model by Deaton and Muellbauer (1980) has desirable properties for consumer demand function. However, since it is difficult to estimate the "true" AID system , linearized almost ideal demand (LAID) system model is estimated. In this paper, we employ the estimation method for the "true" AID system model by Bayesian method. An advantage of our estimation method is to calculate not-linearized demand parameters.

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1 Introduction

When we estimate the demand system, we generally use the piglog-type utility function. The reason why we generally use piglog-type function is to solve an aggregation problem. The aggregation problem is that functional-form of aggregated model demand system at demand function is not equal to the one of not aggregated model. If the piglog-type utility function is used, aggregated demand function is equal to not aggregated demand function. Namely, when we estimate the demand system using macro date, we generally estimate the demand function based on the piglog-type utility function.

The almost ideal demand (AID) system model suggested by Deaton and Muellbauer (1980) is piglog model. Furthermore, this model has desirable properties for consumer demand function. At the first desirable property, the AID system model tests whether the demand function satisfies first-order condition of utility function. Generally, the other models satisfy first-order condition of utility function, and can not to test one. In the second property, the AID system model is linear model for prices and Income. However, at a price index, the AID system model is nonlinear. If the linearly approximated model replaced the nonlinear price index with the linear one was employed, it is simple to estimate the AID system model. It is often called as the linearized almost ideal demand (LAID) system model. However, we found that the use of the linear price index brings the serious problems in econometrics such as the bias in estimates, the errors in variables, and the simultaneity in budget shares (Alston et al., 1994; Buse, 1994; Parshardes, 1993). Above all, the estimation in demand system largely depends on the price index, and then if there is a biased problem in price index itself, their estimates derived from using it must also involve biases. In order to resolve these problems fundamentally, we suggest a estimation method for the "true" AID system model in this paper.

The "true" AID system model is nonlinear for price index parameter. Thus, the "true" AID system model is able to be estimated for maximum likelihood method. However, because the "true" AID system model has complex parameter, it is difficult to estimate one. Therefore we suggest a Bayesian estimation method for the "true" AID system model. The Bayesian method has already applied to the estimation of AID system model, for example, Chalfant et al. (1983), Tiffin and Aguiar (1995), and Lariviere et al. (2000). In these previous studies, LAID system models are estimated. An advantage of our estimation method is to estimate parameters of the AID system model not depending on their price index; therefore we can avoid their imperative problem to biases. In addition, it is also possible to calculate the true value of the AI price index by followed to our posterior estimates. Moreover, our complicated estimation can easily calculate by using the Markov Chain Monte Carlo (MCMC) method, but if not converge in the classical estimation.

This paper is organized as follows. We transform the AID system model by Deaton and Muellbauer (1980) not to depend on their price index in section 2, and express it as the Bayesian framework in section 3. Further we apply to real data in section 4, and compare posterior estimates in the AID system model with the LAID system model. Finally, we describe the summary in this paper.

2 Model

The AID system model by Deaton and Muellbauer (1980) is expressed as

$$w_{it} = a_i + \sum_{j=1}^{n} b_{ij} \ln p_{jt} + c_i \ln\left(\frac{x_t}{P_t}\right) + u_{it}, \qquad i = 1, \cdots, n. \quad t = 1, \cdots, T. \tag{1}$$

With the nonlinear price index

$$\ln P_t = a_0 + \sum_{j=1}^{n} a_j \ln p_{jt} + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} b_{jk} \ln p_{jt} \ln p_{kt}\,, \tag{2}$$

where w_{it} is the i th expenditure share in the period t, p_{jt} is the j th price in the period t, and x_t denotes the total expenditure on all commodities in a system. The AID system model needs to satisfy with linear restrictions as adding-up, homogeneity and symmetry respectively, and the imposition of these properties can test in their parameters. The adding-up condition is given by $\mathbf{\Sigma}_i\, a_i = 1$, $\mathbf{\Sigma}_i\, b_{ij} = 0$ and $\mathbf{\Sigma}_i\, c_i = 0$, the homogeneity condition is $\mathbf{\Sigma}_j\, b_{ij} = 0$, and the symmetry condition is $b_{ij} = b_{ji}$.

Substituting (2) for (1), we can obtain the following as:

$$w_{it} = (a_i - c_i a_0) + \sum_{j=1}^{n} (b_{ij} - c_i a_j) \ln p_{jt} + c_i \ln x_t + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} c_i b_{jk} \ln p_{jt} \ln p_{kt} + u_{it}. \tag{3}$$

Letting the constant parameters $\boldsymbol{a} = (a_1, a_2, \cdots, a_n)'$ in a $n \times 1$ vector, the price parameters $\boldsymbol{b} = (b_1{}',$

b_2',⋯, b_n')′ in a $n^2 \times 1$ vector where $b_i = (b_{1i}, b_{2i},\cdots, b_{ni})'$, and the total expenditure parameters $\boldsymbol{c}$ = $(c_1, c_2,\cdots,c_n)'$ in a $n \times 1$ vector. Then, we define their parameter vectors as $\boldsymbol{\theta} = (\boldsymbol{\theta}_1', \boldsymbol{\theta}_2')'$, where θ_1 = $(\boldsymbol{a}', \boldsymbol{b}')'$ and $\theta_2 = \boldsymbol{c}'$. The vector of explanatory variables in (3) is $Z_t = (1, \boldsymbol{p_t}, \ln x_t, \boldsymbol{p_t}^{**})$ in a $1 \times (2 + n + n^2)$ vector, where the price terms vector is $\boldsymbol{p_t} = (\ln p_{1t}, \ln p_{2t},\cdots, \ln p_{nt})$ in a $1 \times n$ vector. We denote the quadratic price terms as a row vector, $\boldsymbol{p_t}^{**} = (p_{1t}^*, p_{2t}^*,\cdots, p_{nt}^*)$ in a $1 \times n$ vector. Therefore, (3) can be simply rewritten by

$$w_{it} = Z_t \boldsymbol{\beta}_i + u_{it}, \tag{4}$$

where β_i is a parameter vector, and can be defined as

$$\boldsymbol{\beta}_i = \begin{bmatrix} J_i & 0 \\ 0 & J_i^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{\theta}_1 + \boldsymbol{\theta}_2' J_i' \begin{bmatrix} 0 & 0 \\ -I & 0 \\ 0 & \frac{1}{2} I \\ 0 & 0 \end{bmatrix} \boldsymbol{\theta}_1 + \boldsymbol{\theta}_2' J_i' \begin{bmatrix} -a_0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \tag{5}$$

and let

$$J_i = (\overbrace{0,\ldots,0}^{i-1},1,\ldots,0),$$

$$J_i^* = J_i \otimes I_n,$$

where J_i is an operation vector which winkles out the i th scalar from a $1 \times n$ vector, namely $a_i = J_i a_i$ and $c_i = J_i c_i$. And J_i^* is $n \times n^n$ matrix parameter vector which winkles out b_i from $\boldsymbol{b}$, namely $b_i = J_i^* \boldsymbol{b}$.

Further, stacking (4) for n commodities,

$$Y_t = Z_t^* \boldsymbol{\beta} + u_t, \tag{6}$$

where $Y_t = (w_{1t},\cdots,w_{nt})'$, $Z_t^* = Z_t \otimes I_n$, $\boldsymbol{\beta} = (\boldsymbol{\beta}_1,\cdots, \boldsymbol{\beta}_n)'$ and $\boldsymbol{u_t} = (u_{1t},\cdots,u_{nt})'$ is normally distributed as $\boldsymbol{u_t} \sim N(0, \boldsymbol{\Sigma})$. When we estimate the AID system model, the n - 1 equations are used because the adding-up condition is automatically satisfied. The homogeneity and symmetry conditions need to impose further restrictions on parameters. When we consider the demand restrictions in parameters, we denote the restricted parameters as $\boldsymbol{a}^*$, $\boldsymbol{b_i}^*$, $\boldsymbol{c}^*$, $\boldsymbol{\theta}_1^*$ and $\boldsymbol{\theta}_2^*$. These can be rewritten as $\boldsymbol{a} = E\boldsymbol{a}^* + \iota$, $\boldsymbol{b_i} = E\boldsymbol{b_i}^*$, $\boldsymbol{c} = E\boldsymbol{c}^*$,

$$\boldsymbol{\theta}_1 = E^* \boldsymbol{\theta}_1^* + \boldsymbol{\iota}^*,$$

and $\boldsymbol{\theta}_2 = E_0 \boldsymbol{\theta}_2^*$, where $\iota = (0,\cdots,0,1)'$ and $\boldsymbol{\iota}^* = (\iota',0',\cdots,0')'$. And we express as

$$E_0 = \begin{bmatrix} J_1 \\ \vdots \\ J_{n-1} \\ -\sum_{i=1}^{n-1} J_i \end{bmatrix}, \tag{7}$$

and

$$E^* = \begin{bmatrix} E_0 & 0 \\ 0 & E_i \end{bmatrix}, \tag{8}$$

where E_i (i = 1,2,3) is differently set each restriction.

In the case of adding-up, we set $E_i = E_1$ as

$$E_1 = E_0 \otimes I_n. \tag{9}$$

In the homogeneity-constrained model, we set E_2 as

$$E_2 = \begin{bmatrix} E_0 & 0 & \cdots & 0 & 0 \\ 0 & E_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & E_0 & 0 \\ -E_0 & -E_0 & \cdots & -E_0 & 0 \end{bmatrix}, \tag{10}$$

In the symmetry-constrained model, we set E_3 as

$$E_3 = \begin{bmatrix} O_1 & 0 & \cdots & 0 & 0 \\ P_{21} & O_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{n-1,1} & P_{n-1,2} & \cdots & O_{n-1} & 0 \\ Q_1 & Q_2 & \cdots & Q_{n-1} & 0 \end{bmatrix}, \tag{11}$$

where

$$O_i = \begin{bmatrix} J_1 K_i(1) \\ \vdots \\ J_{n-1} K_i(n-1) \\ -\sum_{i=1}^{n-1} J_i K_i(n-1) \end{bmatrix},$$

$$P_{ij} = \begin{bmatrix} J_i H_j(1) \\ \vdots \\ J_i H_j(n-1) \\ -\sum_{s=1}^{n-1} J_i H_j(s) \end{bmatrix},$$

$$Q_i = \begin{bmatrix} \sum_{j=i}^{n-1} J_j H_i(1) + K_{i+1} J_1 \\ \vdots \\ \sum_{j=i}^{n-1} J_j H_i(n-1) + K_{i+1} J_{n-1} \\ -\sum_{s=1}^{n-1}\{\sum_{j=i}^{n-1} J_j H_i(s) + K_{i+1} J_s\} \end{bmatrix},$$

and $K_u(v)$ and $H_u(v)$ are the indicator functions such as

$$K_u(v) = \begin{cases} 1 & if \quad u \geq v \\ 0 & if \quad u < v \end{cases},$$

$$H_u(v) = \begin{cases} 1 & if \quad u = v \\ 0 & if \quad u \neq v \end{cases}.$$

Then, rewriting the constrained vector $\boldsymbol{\beta}_i^*$ along above definitions,

$$\boldsymbol{\beta}_i^* = \begin{bmatrix} J_i & 0 \\ 0 & J_i^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix} E^* \boldsymbol{\theta}_1^* + \boldsymbol{\theta}_2^{*\prime} E_0{}' J_i{}' \begin{bmatrix} 0 & 0 \\ -I & 0 \\ 0 & \frac{1}{2} I \\ 0 & 0 \end{bmatrix} E^* \boldsymbol{\theta}_1^* + \boldsymbol{\theta}_2^{*\prime} E_0{}' J_i{}' \begin{bmatrix} -a_0 \\ 0 \\ 0 \\ I \end{bmatrix} + \begin{bmatrix} J_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{\iota}^*. \qquad (12)$$

$$= D_{1i}(\boldsymbol{\theta}_2^*)\boldsymbol{\theta}_1^* + D_{2i}(\boldsymbol{\theta}_2^*) = D_{3i}(\boldsymbol{\theta}_1^*)\boldsymbol{\theta}_2^* + D_{4i}(\boldsymbol{\theta}_1^*),$$

where

$$D_{1i}(\boldsymbol{\theta}_1^*) = \begin{bmatrix} J_i & 0 \\ 0 & J_i^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix} E^* + \begin{bmatrix} 0 & 0 \\ -I & 0 \\ 0 & \frac{1}{2} I \\ 0 & 0 \end{bmatrix} J_i E_0 \boldsymbol{\theta}_2^*,$$

$$D_{2i}(\boldsymbol{\theta}_1^*) = \begin{bmatrix} -a_0 \\ 0 \\ 0 \\ 1 \end{bmatrix} J_i \boldsymbol{\theta}_2^* + \begin{bmatrix} J_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{\iota}^*,$$

$$D_{3i}(\boldsymbol{\theta}_1^*) = \begin{bmatrix} 0 & 0 \\ -I & 0 \\ 0 & \frac{1}{2} I \\ 0 & 0 \end{bmatrix} E^* \boldsymbol{\theta}_1^* J_i E_0 + \begin{bmatrix} -a_0 \\ 0 \\ 0 \\ 1 \end{bmatrix} J_i E_0,$$

and

$$D_{4i}(\boldsymbol{\theta}_1^*) = \begin{bmatrix} J_i & 0 \\ 0 & J_i^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix} E^* \boldsymbol{\theta}_1^* + \begin{bmatrix} J_i & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{\iota}^*.$$

Moreover, we stack the parameter vector (12) as the following representation:

$$\boldsymbol{\beta}^* = \begin{bmatrix} D_{11}(\boldsymbol{\theta}_2^*) \\ \vdots \\ D_{1n}(\boldsymbol{\theta}_2^*) \end{bmatrix} \boldsymbol{\theta}_1^* + \begin{bmatrix} D_{21}(\boldsymbol{\theta}_2^*) \\ \vdots \\ D_{2n}(\boldsymbol{\theta}_2^*) \end{bmatrix} = \begin{bmatrix} D_{31}(\boldsymbol{\theta}_1^*) \\ \vdots \\ D_{3n}(\boldsymbol{\theta}_1^*) \end{bmatrix} \boldsymbol{\theta}_2^* + \begin{bmatrix} D_{41}(\boldsymbol{\theta}_1^*) \\ \vdots \\ D_{4n}(\boldsymbol{\theta}_1^*) \end{bmatrix}$$

$$= D_5(\boldsymbol{\theta}_2^*)\boldsymbol{\theta}_1^* + D_6(\boldsymbol{\theta}_2^*) = D_7(\boldsymbol{\theta}_1^*)\boldsymbol{\theta}_2^* + D_8(\boldsymbol{\theta}_1^*) \tag{13}$$

Therefore the parameter vector $\boldsymbol{\beta}^*$ can be obtained by simulating $\boldsymbol{\theta}_1{}^*$ and $\boldsymbol{\theta}_2{}^*$.

3 Bayesian framework

We can easily simulate parameters $\boldsymbol{\theta}^*$ of the AID system model by sampling them dividing into two blocks ($\boldsymbol{\theta}_1{}^*$, $\boldsymbol{\theta}_2{}^*$). The prior distributions of constrained parameters $\boldsymbol{\theta}^*$ and the variance - covariance matrix $\boldsymbol{\Sigma}$ are given by

$$\boldsymbol{\theta}^* \sim N(b_0, B_0), \qquad \Sigma \sim W^{-1}(v_0, R_0)$$

where $W^{-1}(\cdot)$ denotes an inverse Wishart distribution with degrees of freedom v_0 and covariance matrix R_0, $b_0 = (b_0{}^1, b_0{}^2)$ and

$$B_0 = \begin{bmatrix} B_0^{11} & B_0^{12} \\ B_0^{12\prime} & B_0^{22} \end{bmatrix}.$$

Therefore, the parted inverse matrix of B_0 is given by

$$B_0^{-1} = \begin{bmatrix} (B_0^{11} - B_0^{12} B_0^{22-1} B_0^{12\prime})^{-1} & (B_0^{11} - B_0^{12} B_0^{22-1} B_0^{12\prime})^{-1} B_0^{12} B_0^{22-1} \\ B_0^{22-1} B_0^{12\prime} (B_0^{11} - B_0^{12} B_0^{22-1} B_0^{12\prime})^{-1} & (B_0^{22} - B_0^{12\prime} B_0^{11-1} B_0^{12})^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} F_{11} & F_{12} \\ F_{12}' & F_{22} \end{bmatrix}.$$

The log likelihood function of (6) in restricted parameters can be expressed as

$$l(Y_t \mid \boldsymbol{\theta}^*, \Sigma, Z_t^*) \propto |\Sigma|^{-T/2} \exp\left[-\frac{1}{2}(Y_t - Z_t^*\boldsymbol{\beta})'(\Sigma^{-1} \otimes I)(Y_t - Z_t^*\boldsymbol{\beta})\right]. \qquad (14)$$

And prior distribution can be represented by

$$\boldsymbol{\pi}(\boldsymbol{\theta}^*) \propto \exp\left[-\frac{1}{2}(\boldsymbol{\theta}^* - b_0)' B_0^{-1}(\boldsymbol{\theta}^* - b_0)\right], \qquad (15)$$

and

$$\boldsymbol{\pi}(\Sigma) \propto |\Sigma|^{-(v_0+n+1)/2} \exp\left[-\frac{1}{2} tr(R_0 \Sigma^{-1})\right]. \qquad (16)$$

Then, $\boldsymbol{\theta}_1^*$ given $\boldsymbol{\theta}_2^*$ and $\boldsymbol{\theta}_2^*$ given $\boldsymbol{\theta}_1^*$ are normally distributed with

$$\boldsymbol{\theta}_1^* \mid \boldsymbol{\theta}_2^* \sim N(b_0^1 + F_{11}^{-1}F_{12}(\boldsymbol{\theta}_2^* - b_0^2), F_{11}^{-1}),$$

and

$$\boldsymbol{\theta}_2^* \mid \boldsymbol{\theta}_1^* \sim N(b_0^2 + F_{22}^{-1}F_{12}'(\boldsymbol{\theta}_1^* - b_0^1), F_{22}^{-1}).$$

Therefore we can obtain the following posterior distribution as:

$$\boldsymbol{\pi}(\boldsymbol{\theta}^*, \Sigma \mid Y_t, Z_t^*) \propto |\Sigma|^{-T/2} \exp\left[-\frac{1}{2}(Y_t - Z_t^*\boldsymbol{\beta})'(\Sigma^{-1} \otimes I)(Y_t - Z_t^*\boldsymbol{\beta})\right] \qquad (17)$$

$$\times \exp\left[-\frac{1}{2}(\boldsymbol{\theta}^* - b_0)' B_0^{-1}(\boldsymbol{\theta}^* - b_0)\right] |\Sigma|^{-(v_0+n+1)/2} \exp\left[-\frac{1}{2} tr(R_0 \Sigma^{-1})\right]$$

We cannot calculate the posterior distribution directly; however we can easily investigate the properties of estimates by exploiting the Gibbs sampling algorithm. The full conditional distributions of $\boldsymbol{\theta}_1^*$, $\boldsymbol{\theta}_2^*$, and $\boldsymbol{\Sigma}$ are given by

$$\boldsymbol{\theta}_1^* \mid \boldsymbol{\theta}_2^*, \Sigma, Y_t, Z_t^* \sim N(b_1^1, B_1^1),$$

$$\boldsymbol{\theta}_2^* \mid \boldsymbol{\theta}_1^*, \Sigma, Y_t, Z_t^* \sim N(b_1^2, B_1^2),$$

$$\Sigma^{-1} \mid \boldsymbol{\theta}_1^*, \boldsymbol{\theta}_2^*, Y_t, Z_t^* \sim W^{-1}[v_0 + T, R_0 + (Y_t - Z_t^*\boldsymbol{\beta})'(Y_t - Z_t^*\boldsymbol{\beta})],$$

where

$$B_1^1 = [D_5'(\boldsymbol{\theta}_2^*) Z_t'^* (\Sigma^{-1} \otimes I) Z_t^* D_5(\boldsymbol{\theta}_2^*) + F_{11}^{-1}]^{-1},$$

$$B_1^2 = [D_7'(\boldsymbol{\theta}_1^*) Z_t'^* (\Sigma^{-1} \otimes I) Z_t^* D_7(\boldsymbol{\theta}_1^*) + F_{22}^{-1}]^{-1},$$

$$b_1^1 = B_1^1[D_5'(\boldsymbol{\theta}_2^*) Z_t'^* (\Sigma^{-1} \otimes I)(Y_t - Z_t^* D_6(\boldsymbol{\theta}_2^*)) + \{b_0^1 + F_{11}^{-1}F_{12}(\boldsymbol{\theta}_2^* - b_0^2)\} F_{11}^{-1}],$$

$$b_1^2 = B_1^2[D_7'(\boldsymbol{\theta}_1^*) Z_t'^* (\Sigma^{-1} \otimes I)(Y_t - Z_t^* D_8(\boldsymbol{\theta}_1^*)) + \{b_0^2 + F_{22}^{-1}F_{12}'(\boldsymbol{\theta}_1^* - b_0^1)\} F_{22}^{-1}].$$

Since the full conditional distributions have a standard form, we can apply the Gibbs sampling to calculate $\boldsymbol{\theta}^*$ and $\mathbf{\Sigma}$.

4 Application to the real data

The Household Survey data used in this analysis is an annual time series of the Family Income and Expenditure Survey (FIES)[1] by the Statistics Bureau from 1963 to 2001 in Japan. The consumption expenditure is roughly classified into five commodities; 1.Food, 2.Housing, 3.Fuel, 4.Clothing, and 5.Miscellaneous. The price data have been obtained from the Consumer Price Index and calculated using 2000 as the base year. We calculate the Bayesian estimates from the posterior distribution and compare the posterior results in the AID system model with those in the LAID system model. In addition, we measure model complexity and fit by Deviance Information Criterion (DIC) by Spiegelhalter et al. (2002) for two models with three demand restrictions as a model selection criterion.

4.1 Posterior results

We compute posterior results in the AID and the LAID system models.[2] In the LAID system model, we use Stone's index instead of (2) as

$$\ln P_t = \sum_{i=1}^{n} w_{it} \ln p_{it}. \tag{18}$$

The details of a Bayesian approach for the LAID system model are described in Appendix. We calculate their parameters $\boldsymbol{\theta}^*$ or $\boldsymbol{\beta}^*$ of two models by using the Gibbs sampling algorithm. In the AID system model, parameters $\boldsymbol{\beta}^*$ can obtain by simulating the parameter vector $\boldsymbol{\theta}^*$, and it is partitioned into two blocks ($\boldsymbol{\theta}_1^*$, $\boldsymbol{\theta}_2^*$) to generate separately in simulation. In the MCMC estimations, we choose the prior distribution as $\boldsymbol{\theta}^* \sim N(0,\ 10^4 I)$ and $\mathbf{\Sigma} \sim IW(10,\ 0.05I)$.[3] We use Zellner (1963)'s SUR estimates in the non-Bayesian approach as the initial values of $\boldsymbol{\theta}^*_{(0)}$ and $\mathbf{\Sigma}_{(0)}$ in the Gibbs sampling algorithm. The MCMC simulation is generated 15,000 draws from posterior distribution after discarding the first 5,000 draws as burn-in periods. Our results are executed on a personal computer,

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1 The data is a year average of monthly receipts and disbursements per household in all household.
2 The full conditional distribution in the LAID system model is indicated in Appendix.
3 In the LAID system model, we assume the prior distribution as $\boldsymbol{\beta}^* \sim N(0,\ 10^4 I)$ and $\mathbf{\Sigma} \sim IW(10,\ 0.05I)$.

using the OX code.

Tables 1 to 3 show the posterior means and standard deviations of parameters in the AID and LAID system models. In the AID system model, we set a_0 = 0. In adding-up condition of Table 1, the posterior means of parameters in both models are relatively similar values each other. In particular, the income parameters are remarkable. However, the estimates of the AID model have the smaller standard deviations than the LAID model. In homogeneity of Table 2 and symmetry conditions of Table 3, their parameters between two models occasionally depart from each other. For instance, intercept a_2 of the AID model has still smaller value than the LAID model in both conditions. Generally, the differences of posterior results in both models come to be large with demand restrictions.

In (1), the income elasticity is given by

$$\eta_i = 1 + \frac{c_i}{w_i}. \tag{19}$$

And the uncompensated (Marshallian) price elasticity is given by

$$e_{ij} = -\delta_{ij} + \frac{b_{ij}}{w_i} + \frac{c_i}{w_i}\left[a_j - \sum_{k=1}^{n} b_{kj} \ln p_k \right], \tag{20}$$

where δ_{ij} is the Kronecker delta (when $i = j$, $\delta_{ij} = 1$, and otherwise 0). Tables 4 and 5 report the income and own-price elasticities from the posterior results. In both models of Table 4, the income elasticities in adding-up show that 2.Housing, 4.Clothing and 5.Miscellaneous are elastic, and 1.Food and 3.Fuel are inelastic. Then, in the imposition of homogeneity restriction, the income elasticity for 3.Fuel changes to be elastic and that for 4.Clothing changes to be inelastic. In symmetry condition, their difference of estimate between the AID and LAID models appears for the income elasticity of 3.Fuel. In the LAID model, the income elasticity for 3.Fuel largely changes by the imposition of demand restrictions, that is, estimates in the LAID model is sensitive to parameter restriction. For the own price elasticities of Table 5, these values in both models largely change by each condition in almost commodities. For example, the own price elasticity for 4.Clothing in the LAID model changes from -0.78 in the adding-up condition to -1.13 in the homogeneity condition. Like the posterior results in Table 2, the standard deviations of elasticities in the AID model show smaller than those in the LAID model. Thus, the AID model in our estimation can simulate the parameters having the smaller variance than the LAID system model.

In addition, we can calculate the true AI price index using the posterior results. These are indicated on the lower side of Tables 1 to 3. With the imposition of demand restrictions, the AI price index comes to be smaller, at the same time, the divergences between the AI and LAI price indices are also larger. That is, as showed in Table 3, the more we impose the demand restrictions, the larger both estimates between the AID and LAID system models diverge. In the Bayesian approach, we can compute the standard deviation and the 95% credible interval of AI price index, and judge its statistical significance.

Further we compute the correlation between the LAI price index and their disturbances. The correlation between the price index and disturbances will violate the standard orthogonality condition and yield the errors in variables problem. The calculated correlation $\boldsymbol{r}$ = (0.997, -0.996, -0.962, 0.646) where $\boldsymbol{r}$ denotes the correlation between the LAI price index and the i th equation disturbance ($i = 1,\cdots, n - 1$). This shows the positive or negative high collinearity between them, and the use of the LAI price index brings the serious problems in econometrics.

4.2 Model selection by Deviance Information Criterion

We introduce the Deviance Information Criterion (DIC) by Spiegelhalter et al. (2002), which measures model complexity and fit as the model selection criterion. The DIC is an appropriate measurement for model selection because it accounts for model fit and model size well. Xiao et al. (2007) has applied this criterion to the selection of best functional forms in U.S. electricity demand. Model with smaller DIC values is preferred to one with larger DIC. We use the DIC for the model selection between the AID and LAID system models. The DIC can be calculated by

$$DIC = \overline{D(\boldsymbol{\theta}^*)} - D(\bar{\boldsymbol{\theta}}^*), \tag{21}$$

where

$$D(\boldsymbol{\theta}^*) = -2\log\{l(Y_t \mid \boldsymbol{\theta}^*, \Sigma)\} + 2\log\{f(Y_t)\}.$$

$\overline{D(\boldsymbol{\theta}^*)}$ is the average of $D(\boldsymbol{\theta}^*)$ and $D(\bar{\boldsymbol{\theta}}^*)$ computes at the average values of $\boldsymbol{\theta}^*$. $l(\cdot)$ is the evaluated likelihood at the $\boldsymbol{\theta}^*$. We set $f(Y_t) = 1$ following to Spiegelhalter et al. (2002). The DIC computed from the posterior results are reported in Table 6.

We calculate each three DIC values for the AID and LAID system models. Table 6 shows the DIC values in the AID system are smaller than the LAID system in all demand restrictions. Therefore we find the AID system is preferred to the LAID system model. Especially, the homogeneity constrained model in the AID system outperforms the other model specifications. These support our posterior results in model selection.

5 Concluding remarks

We calculate the true parameters of the AID system model by the Bayesian method, not depending on their price index. Heretofore, the estimation of the AID system model largely depends on the price index. Therefore the calculated estimates must have a bias problem when we use a problematic price index such as the linearized price index of (18). We equally estimate the LAID system model with Stone's index in order to compare to our estimation in the AID system model. In our application to real data, we report a Japanese household demand with five commodities. Our empirical results show that posterior means of the AID system model derived from our Bayesian estimation have the smaller variance than those from the LAID model. Further, we find that their estimates between the AID and LAID system diverge with the imposition of demand restrictions. This evidence can make a clear from calculating the value itself of the AI price index in (2) from the posterior results. The values of AI price index come to be so smaller with the imposition of demand restrictions that the divergence between the AI and LAI price indices would be large. According to our estimation, we can also investigate the true biases of the linearized price index from the AI price index. Additionally, we compare the validity of both AID and LAID system models by the DIC of model selection criterion. The DIC shows the AID system model is superior to the alternative. Above all, the homogeneity or symmetry constrained model is the best functional form in our application.

Appendix

In the AID system model, $\boldsymbol{\theta}^*$ is constructed by E^* matrix. For n = 5, E^* matrix in (8) can be written as follows:

Adding-up:

$$E_0 = \begin{bmatrix} J_1 \\ \vdots \\ J_4 \\ -J_1 - \cdots - J_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & -1 & 0 \end{bmatrix},$$

and $E_1 = E_0 \otimes I_n$. E_0 matrix is common to all cases.

Homogeneity:

$$E_2 = \begin{bmatrix} E_0 & 0 & 0 & 0 & 0 \\ 0 & E_0 & 0 & 0 & 0 \\ 0 & 0 & E_0 & 0 & 0 \\ 0 & 0 & 0 & E_0 & 0 \\ -E_0 & -E_0 & -E_0 & -E_0 & 0 \end{bmatrix}$$

Symmetry:

$$E_3 = \begin{bmatrix} O_1 & 0 & 0 & 0 & 0 \\ P_{21} & O_2 & 0 & 0 & 0 \\ P_{31} & P_{32} & O_3 & 0 & 0 \\ P_{41} & P_{42} & P_{43} & O_4 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 & 0 \end{bmatrix}$$

where

$$O_1 = \begin{bmatrix} J_1 K_1(1) \\ J_2 K_1(2) \\ J_3 K_1(3) \\ J_4 K_1(4) \\ -J_1 K_1(1) - \cdots - J_4 K_1(4) \end{bmatrix} = \begin{bmatrix} J_1 \\ 0 \\ 0 \\ 0 \\ -J_1 \end{bmatrix},$$

$$O_2 = \begin{bmatrix} J_1 \\ J_2 \\ 0 \\ 0 \\ -J_1 - J_2 \end{bmatrix}, \quad O_3 = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ 0 \\ -J_1 - J_2 - J_3 \end{bmatrix}, \quad O_4 = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ -J_1 - J_2 - J_3 - J_4 \end{bmatrix}.$$

$$P_{21}=\begin{bmatrix} J_2H_1(1)\\ J_2H_1(2)\\ J_2H_1(3)\\ J_2H_1(4)\\ -J_2H_1(1)-\cdots-J_2H_1(4)\end{bmatrix}=\begin{bmatrix} J_2\\ 0\\ 0\\ 0\\ -J_2\end{bmatrix},\quad P_{32}=\begin{bmatrix} 0\\ J_3\\ 0\\ 0\\ -J_3\end{bmatrix},\quad P_{43}=\begin{bmatrix} 0\\ 0\\ J_4\\ 0\\ -J_4\end{bmatrix}.$$

$$Q_1=\begin{bmatrix} J_1H_1(1)+\cdots+J_4H_1(1)+K_2(1)J_1\\ \vdots\\ \vdots\\ J_1H_1(4)+\cdots+J_4H_1(4)+K_2(4)J_4\\ -\sum_{s=1}^{4}\{\sum_{j=1}^{4}J_jH_1(s)+K_2J_s\}\end{bmatrix}=\begin{bmatrix} J_1+J_2+J_3+J_4+J_1\\ J_2\\ 0\\ 0\\ -2J_1-J_2-J_3-J_4\end{bmatrix},$$

$$Q_2=\begin{bmatrix} J_1\\ J_1+J_2+J_3+J_4+J_2\\ J_3\\ 0\\ -2J_1-2J_2-2J_3-J_4\end{bmatrix},\quad Q_3=\begin{bmatrix} J_1\\ J_2\\ J_1+J_2+J_3+J_4+J_3\\ J_4\\ -2J_1-2J_2-2J_3-2J_4\end{bmatrix},\quad Q_3=\begin{bmatrix} J_1\\ J_2\\ J_3\\ J_1+J_2+J_3+J_4+J_4\\ -2J_1-2J_2-2J_3-2J_4\end{bmatrix}.$$

In the LAID system model, Z_t^* of (6) can be written as follows:

Adding-up:

$$Z_t^*=I_{n-1}\otimes[1,\ln p_{1t},\ldots,\ln p_{nt},\ln(x_t/P_t)]$$

Homogeneity:

$$Z_t^*=I_{n-1}\otimes[1,z_{1t},\ldots,z_{n-1,t},\ln(x_t/P_t)],$$

where $z_{jt}=\ln p_{jt}-\ln p_{nt}$ $(j=1,\ldots,n.)$.

Symmetry:

$$Z_t^*=I_{n-1}\otimes[1,Q_t,\ln(x_t/P_t)],$$

where

$$Q_t=\left[\begin{array}{ccccc|cccc|ccc|c|c} z_{1t} & z_{2t} & z_{3t} & \cdots & z_{n-1,t} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0\\ 0 & z_{1t} & 0 & \cdots & 0 & z_{2t} & z_{3t} & \cdots & z_{n-1,t} & 0 & \cdots & 0 & \cdots & 0\\ 0 & 0 & z_{1t} & \cdots & 0 & 0 & z_{2t} & \cdots & 0 & z_{3t} & \cdots & z_{n-1,t} & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots\\ 0 & 0 & 0 & \cdots & z_{1t} & 0 & 0 & \cdots & z_{2t} & 0 & \cdots & z_{3t} & \cdots & 0\end{array}\right].$$

The prior distributions of $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ are given by

$$\boldsymbol{\beta} \sim N(\boldsymbol{b}_0, \boldsymbol{B}_0), \qquad \Sigma \sim W^{-1}(v_0, R_0).$$

The full conditional distributions of $\boldsymbol{\beta}$ and $\boldsymbol{\Sigma}$ are given by

$$\boldsymbol{\beta} \mid \Sigma, Y_t, Z_t^* \sim N(b_1^1, B_1^1), \qquad \Sigma \mid \boldsymbol{\beta}, Y_t, Z_t^* \sim W^{-1}(v_1, R_1),$$

where $B_1 = [(Z_t'^*(\Sigma^{-1} \otimes I)Z_t^*)^{-1} + B_0]^{-1}$, $b_1 = B_1[B_0 b_0 + Z_t'^*(\Sigma^{-1} \otimes I)Y_t]$, $v_1 = v_0 + T$, and $R_1 = R_0 + (Y_t - Z_t^*\boldsymbol{\beta})'\,(Y_t - Z_t^*\boldsymbol{\beta})$.

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Table 1 Posterior mean results in adding-up condition

i	a_i	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	c_i
(I) AI							
1	0.9793	0.3943	-0.0326	-0.0832	-0.2049	-0.3302	-0.0500
	(0.0075)	(0.0095)	(0.0096)	(0.0094)	(0.0100)	(0.0094)	(0.0099)
2	0.1689	-0.0446	0.0680	-0.0181	0.1555	-0.0404	-0.0201
	(0.0090)	(0.0095)	(0.0094)	(0.0098)	(0.0090)	(0.0086)	(0.0101)
3	0.1899	-0.0404	-0.0061	0.0354	0.0747	-0.0199	-0.0201
	(0.0087)	(0.0081)	(0.0099)	(0.0078)	(0.0084)	(0.0087)	(0.0097)
4	-0.2684	-0.0622	0.0840	-0.0120	0.0363	-0.0702	0.0601
	(0.0089)	(0.0099)	(0.0086)	(0.0083)	(0.0079)	(0.0085)	(0.0099)
5	-0.0697	-0.2472	-0.1133	0.0779	-0.0616	0.4607	0.0297
	(0.0100)	(0.0099)	(0.0100)	(0.0101)	(0.0100)	(0.0098)	(0.0099)
$\ln P_t$	2.404	[1.8697,	2.6010]				
	(0.0717)						
(II) LAI							
1	0.9098	0.4595	0.0395	0.0095	-0.1706	-0.3405	-0.0518
	(0.2589)	(0.0684)	(0.0654)	(0.0243)	(0.0444)	(0.0645)	(0.0402)
2	0.2703	-0.0390	0.0809	-0.0389	0.0509	-0.0489	-0.0169
	(0.5896)	(0.1558)	(0.1489)	(0.0554)	(0.1010)	(0.1469)	(0.0916)
3	0.1896	-0.0506	-0.0406	0.0891	0.0293	0.0093	-0.0221
	(0.5093)	(0.1346)	(0.1286)	(0.0479)	(0.0873)	(0.1268)	(0.0791)
4	-0.3201	-0.0301	0.0799	0.0080	0.0199	-0.1602	0.0599
	(0.1432)	(0.0378)	(0.0362)	(0.0135)	(0.0245)	(0.0357)	(0.0222)
5	-0.0496	-0.3397	-0.1597	-0.0676	0.0705	0.5404	0.0309
	(0.0202)	(0.0187)	(0.0194)	(0.0193)	(0.0198)	(0.0196)	(0.0145)
$\ln P_t$	4.145						

Notes: The number in commodity denotes 1.Food, 2.Housing, 3.Fuel, 4.Clothing, and 5.Miscellaneous. P_t denotes the value of price index calculated from the posterior results. The values in parentheses are the posterior standard deviations. [] denotes the 95% credible interval of price index.

Table 2 Posterior mean results in homogeneity condition

i	a_i	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	c_i
(I) AI							
1	0.8339	0.4249	-0.0174	-0.0106	-0.1721	-0.3397	-0.0499
	(0.0031)	(0.0011)	(0.0031)	(0.0030)	(0.0032)	(0.0031)	(0.0031)
2	0.0061	-0.0131	0.0372	-0.0193	0.1257	-0.0597	-0.0101
	(0.0030)	(0.0027)	(0.0029)	(0.0030)	(0.0032)	(0.0032)	(0.0031)
3	-0.0236	-0.0477	-0.0867	0.0828	0.0849	-0.0297	0.0059
	(0.0013)	(0.0012)	(0.0028)	(0.0028)	(0.0031)	(0.0031)	(0.0031)
4	0.0617	-0.0256	0.1107	0.0104	0.0392	-0.0977	-0.0010
	(0.0028)	(0.0028)	(0.0029)	(0.0030)	(0.0031)	(0.0032)	(0.0032)
5	-0.1500	-0.3469	-0.1702	-0.0808	0.0706	0.5305	0.0549
	(0.0029)	(0.0027)	(0.0026)	(0.0028)	(0.0031)	(0.0031)	(0.0031)
$\ln P_t$	0.851	[0.3743,	1.8319]				
	(0.2457)						
(II) LAI							
1	0.8699	0.4599	0.0299	0.0199	-0.1700	-0.3397	-0.0501
	(0.0533)	(0.0365)	(0.0311)	(0.0098)	(0.0219)	(0.0201)	(0.0042)
2	0.2298	-0.0500	0.0798	-0.0301	0.0601	-0.0598	-0.0102
	(0.0485)	(0.0331)	(0.0283)	(0.0089)	(0.0198)	(0.0198)	(0.0038)
3	-0.0198	-0.0599	-0.0701	0.1102	0.0499	-0.0299	0.0062
	(0.0532)	(0.0364)	(0.0311)	(0.0097)	(0.0218)	(0.0199)	(0.0041)
4	0.0699	-0.0019	0.1305	-0.0201	-0.0098	-0.0982	-0.0010
	(0.0533)	(0.0365)	(0.0312)	(0.0098)	(0.0218)	(0.0197)	(0.0041)
5	-0.1498	-0.3480	-0.1697	-0.0799	0.0699	0.5277	0.0551
	(0.0199)	(0.0201)	(0.0199)	(0.0201)	(0.0198)	(0.0398)	(0.0116)
$\ln P_t$	4.145						

Notes: 1.Food, 2.Housing, 3.Fuel, 4.Clothing, 5.Miscellaneous. The values in parentheses are the posterior standard deviations. [] denotes the 95% credible interval of price index.

Table 3 Posterior mean results in symmetry condition

i	a_i	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	c_i
(I) AI							
1	0.9281	0.3747	-0.0002	0.0181	-0.1068	-0.3293	-0.0599
	(0.0031)	(0.0030)	(0.0023)	(0.0029)	(0.0030)	(0.0032)	(0.0031)
2	-0.0005		0.0163	-0.0415	0.0812	-0.0993	-0.0130
	(0.0008)		(0.0013)	(0.0028)	(0.0028)	(0.0032)	(0.0032)
3	-0.1152			0.0881	0.0069	-0.0793	0.0099
	(0.0031)			(0.0030)	(0.0027)	(0.0031)	(0.0031)
4	0.1507				0.1003	-0.0193	-0.0079
	(0.0030)				(0.0028)	(0.0031)	(0.0032)
5	-0.3009					0.5348	0.0709
	(0.0032)					(0.0031)	(0.0031)
$\ln P_t$	1.372	[1.2837,	2.0020]				
	(0.2457)						
(II) LAI							
1	0.9731	0.3982	-0.0126	0.0206	-0.0870	-0.3192	-0.0623
	(0.0105)	(0.0099)	(0.0101)	(0.0099)	(0.0102)	(0.0199)	(0.0098)
2	0.2494		0.0630	-0.0316	0.0789	-0.0976	-0.0123
	(0.0120)		(0.0102)	(0.0100)	(0.0104)	(0.0100)	(0.0202)
3	-0.1105			0.1102	-0.0201	-0.0792	-0.0441
	(0.0099)			(0.0101)	(0.0099)	(0.0200)	(0.0199)
4	0.0293				0.0205	0.0076	-0.0012
	(0.0155)				(0.0194)	(0.0096)	(0.0268)
5	-0.1413					0.4884	0.1199
	(0.0209)					(0.0566)	(0.0239)
$\ln P_t$	4.145						

Notes: 1.Food, 2.Housing, 3.Fuel, 4.Clothing, 5.Miscellaneous. The values in parentheses are the posterior standard deviations. [] denotes the 95% credible interval of price index.

Table 4 Income elasticities from the posterior mean results

	1	2	3	4	5
(I) AID					
Adding-up	0.8287	0.8096	0.6716	1.8261	1.0638
	(0.0344)	(0.0963)	(0.1632)	(0.1381)	(0.0225)
Homogeneity	0.8316	0.8949	1.1092	0.9878	1.1163
	(0.0109)	(0.0304)	(0.0516)	(0.0436)	(0.0071)
Symmetry	0.7979	0.8635	1.1825	0.9009	1.1502
	(0.0106)	(0.0332)	(0.0572)	(0.0394)	(0.0066)
(II) LAID					
Adding-up	0.8223	0.8394	0.6389	1.8231	1.0660
	(0.0233)	(0.0687)	(0.1210)	(0.1037)	(0.0309)
Homogeneity	0.8286	0.9028	1.1022	0.9859	1.1175
	(0.0168)	(0.0539)	(0.1013)	(0.0877)	(0.0248)
Symmetry	0.7864	0.8834	0.2783	0.9834	1.2558
	(0.0338)	(0.0945)	(0.3266)	(0.1316)	(0.0512)

Notes: 1.Food, 2.Housing, 3.Fuel, 4.Clothing, 5.Miscellaneous. The values in parentheses are the posterior standard deviations.

Table 5 Own price elasticities from the posterior mean results

	1	2	3	4	5
(I) AID					
Adding-up	0.4009	-0.3345	-0.4005	-0.5611	-1.4100
	(0.0342)	(0.0838)	(0.1445)	(0.1172)	(0.0236)
Homogeneity	0.4821	-0.5998	0.5114	-0.5119	0.0672
	(0.0092)	(0.0154)	(0.0321)	(0.0252)	(0.0076)
Symmetry	0.3229	-0.8159	0.6038	0.2534	0.0603
	(0.0104)	(0.0143)	(0.0541)	(0.0351)	(0.0074)
(II) LAID					
Adding-up	0.6259	-0.2151	0.4809	-0.7867	0.1216
	(0.0355)	(0.0937)	(0.1606)	(0.1361)	(0.0466)
Homogeneity	0.6258	-0.2319	0.7966	-1.1347	0.0705
	(0.0343)	(0.0952)	(0.1643)	(0.1378)	(0.0857)
Symmetry	0.4267	-0.3900	0.8479	-0.7167	-0.0782
	(0.0356)	(0.0972)	(0.1657)	(0.2685)	(0.1285)

Notes: 1.Food, 2.Housing, 3.Fuel, 4.Clothing, 5.Miscellaneous. The values in parentheses are the posterior standard deviations.

Table 6 The DIC for the AID and LAID models

DIC	AI	LAI

Adding-up	1572.6	1723.5
Homogeneity	968.2	1723.5
Symmetry	1245.8	1772.1

Notes: DIC shows the Deviance Information Criterion by Spiegelhalter et al. (2002).