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Analysis of reforming off-peak fee discount for ETC to reduce highway congestion

Takaji SUZUKI*

Abstract

This study analyses discounts on highway fee for the off-peak commuting with ETC as the application of the traditional bottleneck congestion model in order to consider self-select fee system and post payment related to historical usage. The ETC off-peak commuting discounts improves efficiency of road system due to utilization of off-peak capacity of highway so as to shift traffic demand from open road to highway, by contrast, ETC peak commuting discount which has been in place in Japanese highway. Furthermore, ETC off-peak commuting discount plus that is depend on historical highway usage is an effective measure to give an incentive for usage of ETC device, restraining aggravation of road congestion.

Keywords: Electric Toll Collection, Self-select fee system, Bottleneck congestion.

School of Economics, Chukyo University. Email: takaji@mecl.chukyo-u.ac.jp

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Takaji SUZUKI
School of Economics, Chukyo University
takaji@mecl.chukyo-u.ac.jp

1. Introduction

Some characteristic features have been added to the Japanese highway fee system such as (1) a self-select fee system, (2) non-stop toll collection, and (3) post-payment by introduction of electronic toll collection (ETC). These features enable us to charge a highway fee through a varied payment structure in order to reduce congestion on the highways. First, the self-select fee system, including ETC and non-ETC, achieves a Pareto improvement for the users by appropriate price discrimination. Second, the non-stop toll collection reduces the time and energy consumption due to toll-gate congestion. Third, the post-payment related to the usage history allows us to charge a highway fee that restrains the intensive demand related to repeated commuting during a peak period.

In the case of almost all road-pricing policies related to highway congestion, the focus is on levying an additional charge during the peak period. However, discounts are offered on many public transportation fees during the off-peak period in order to meet the time-varying demand. In fact, the additional charge levied during the peak period and the reduced charge levied during the off-peak period have similar effects on the congested transportation system.

Suzuki (2008) studied the effects of the Japanese highway fee discount for ETC users during the peak period on the highway bottleneck congestion. It was found that the discount obviously worsens the highway congestion during the peak period in many cases; however, it may be workable if the toll gate performance would be improved by the high usage rate of the ETC booths. Furthermore, the mixed use of the roads by both ETC and non-ETC users avoids the increase in highway congestion due to the fee discount for ETC users.

Considering the results of the previous study, I have investigated the off-peak discounts for ETC in order to systematize in the highway fee policies in this study. The analyses relate these discounts to the self-select fee system and the post-payment method.

2. Bottleneck congestion model associated with self-select ETC discounted fee system

The Japanese highway fee system has changed into a self-select fee system in the sense that a highway user can choose between a normal fee system and a discounted fee system even if the application of the discounted fee is limited to the specified possession of an ETC device, time period, distance, and area. This change seems to achieve a Pareto improvement between users because a user can choose the current normal fee system if the ETC discounted fee system is worse than the current system for the user, as discussed by Train (1994). However, the assumption of consumer independence is not satisfied because the choices of the highway users are influenced by the choices of the other users because of the congestion.

The Japanese ETC commuting discount system targets highway commuters during the rush hours in order to rapidly increase the use of ETC. This type of inconsequential fee discounts during a congested time period has not attracted attention in congestion charging studies in the past. Therefore, we need to know the appropriate discounts on highway fees during a congested period if such discounts exist. Suzuki (2008a) applied the bottleneck congestion model to the current Japanese highway fee system as a self-select fee system in order to study the effect of the introduction of an ETC commuting discount on highway congestion. On the basis of the results, the conditions that make an ETC commuting discount system functional in order to ensure an incentive to use the ETC commuting discount without increasing the highway congestion were clarified; furthermore, the aggravated situation of bottleneck congestion and its mechanism were clarified by comparative studies.

Suzuki (2008b) specified three bottleneck points and the relations among them that need to be considered while attempting to improve the performance of a toll gate by increasing the non-stop fee collection at the bottleneck points. As a result, the ETC commuting discount fee system reduces congestion when the utilization rate of the ETC booth increases. On the other hand, the possibility that a high discount may generate bottleneck congestion outside of the toll gate such as on a main track on the highway even if the toll-gate performance is improved is also discussed.

An ETC system can be imposed depending on several factors that affect highway use, such as section, distance, time, and vehicle type. The consideration of these factors helps to develop an efficient fee system. This study focuses on an ETC off-peak commuting discount to reduce the travel costs of the highway users by the intensive use of the off-peak road capacity and to give an incentive for using ETC to the users. For the sake of simplification, the bottleneck points are not specified.

At the point of purchase of the ETC device that postulates the ETC commuting discount, users may choose a highway fee system without considering the variation in travel time due to a change in travel demand but considering the costs of an ETC device and highway fees. For this reason, this study analyses the relationship between the travel choices of highway users and the bottleneck congestion during their morning commute in the short term with respect to their long-term choice of purchasing an ETC device.

The following assumptions are introduced for the simplification. The residential area and the workplace are connected by a highway route A and an open road route B. Highway fees are collected at the toll gate located at the end of the highway.

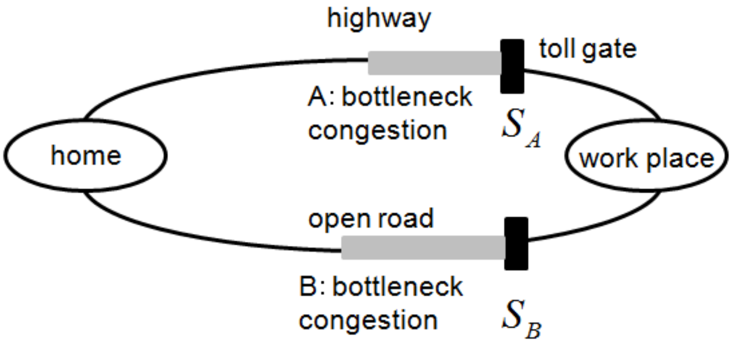

Figure 1 Commuting network

The total number of commuters, N, is constant in the mornings.

$$N = N_A + N_B \tag{1}$$

The commuters choose their departure time and route on the basis of the following three highway fee systems:

(i) Normal fee system: τ for each commute,

(ii) ETC off-peak commuting discount: δ discount from τ for each commute during the off-peak period with ETC, and

(iii) ETC off-peak commuting discount plus: δ discount from τ for one commute during the peak period with ETC if out of P commutes, (P - 1) commutes are during the off-peak hours.

Further, (ii) and (iii) require the user to purchase an ETC device.

The commuters are also divided into two categories—ETC users: N^E and non-ETC users:

N^N —depending on the fee type that they choose.

$$N = N^E + N^N \tag{2}$$

Furthermore, users who opt for the ETC off-peak discount plus system are discriminated as N^{EE} from N^E wherever necessary.

The bottleneck congestions arise in one place for each route. S_j represents the capacity of bottleneck j, and $Q_j(t)$ represents the queue that a commuter who alights at t faces at the bottleneck j. By omitting the uncongested part of travel, we can express the travel time of a commuter who departs at t as

$$T_j(t) = \frac{Q_j(t)}{S_j} \tag{3}$$

The departure rate of commuters who alight at t and choose route j and fee type k (k = E for ETC and N for non-ETC) is given by $r_j^k(t)$. The variation of the queue is formulated as follows:

$$\frac{dQ_j(t)}{dt} = \begin{cases} 0, & Q_j(t) = 0 \text{ and } \sum\limits_{k=N,E} r_j^k(t) \le S_j \\ \sum\limits_{k=N,E} r_j^k(t) - S_j \ , & \sum\limits_{k=N,E} r_j^k(t) \ge S_j \end{cases} \tag{4}$$

The travel costs of commuters consist of the following costs: the travel time cost, the schedule cost, and the highway fee.

$$C_j^k(t) = \alpha T_j(t) + SD_j(t) + \left(1 - \delta_j^k(t)\right)\tau \tag{5}$$

where α is the shadow value of the travel time, $SD_j(t)$ is the schedule cost, and $\delta_j^k(t)$ is the discount rate that changes according to the time period, commuting route, and fee type.

All commuters wish to arrive at work at $t*$. Let $t_{n,j}$ be the departure time for which a commuter arrives at work on time using route j, then $t_{n,j} + T_j\left(t_{n,j}\right) = t^*$. If a commuter departs earlier than $t_{n,j}$, he/she is early by $t^* - t - T_j(t)$, while if he/she departs later than $t_{n,j}$, he/she is late by $t + T_j(t) - t^*$. The travel costs of commuters are expressed as

$$C_j^k(t) = \begin{cases} \alpha T_j^k(t) + \beta\left(t^* - t - T_j^k(t)\right) + \left(1 - \delta_j^k(t)\right)\tau & \left(t \le t_{n,j}\right) \\ \alpha T_j^k(t) + \gamma\left(t + T_j^k(t) - t^*\right) + \left(1 - \delta_j^k(t)\right)\tau & \left(t \ge t_{n,j}\right) \end{cases} \tag{6}$$

where β and γ are the shadow values of the time at which the user leaves to reach early and the time at which the user leaves and is late. The order of these values is assumed to be $\beta < \alpha < \gamma$ with respect to α, as given in Small (1982). In addition, $\left(1 - \delta_j^k(t)\right)\tau$ represents the highway fee imposed for departure time t, route choice j, and fee type k. Therefore, commuters can choose an appropriate departure time, route, and fee type to minimize their travel costs.

3. Analyses of reforming highway fee system by using ETC off-peak commuting discounts

3.1 Bottleneck congestions without ETC off-peak commuting discounts

An equilibrium without any discounts, that is the case of $\delta_A^k(t) = 0$ and $\delta_B^k(t) = 1$, in equation (6) is solved as a benchmark for following the highway fee discount policies. Here, only the normal fee is imposed for highway users; therefore, ETC users and non-ETC users are not discriminated. This case is labelled case (I). The equilibrium is defined since commuters are not given an incentive to change their departure times and routes. Let the queue continue from $t_{0,j}$ to $t_{e,j}$. The morning congested period equals the total number of commuters divided by the bottleneck capacity for each route j.

$$t_{e,j} - t_{0,j} = \frac{N_j}{S_j} \tag{7}$$

The first and last commuters do not face a queue. The travel cost for the first and last commuters are simplified from equation (5) by reducing $T_j^k(t)$ as shown below:

$$C_j\left(t_{0,j}\right)=\beta\left(t^*-t_{0,j}\right)+\left(1-\delta_j^k(t)\right)\tau \tag{8}$$

$$C_j\left(t_{e,j}\right)=\gamma\left(t_{e,j}-t^*\right)+\left(1-\delta_j^k(t)\right)\tau \tag{9}$$

The travel costs of all commuters should be equal at this equilibrium; therefore, the departure time and travel cost are derived as

$$t_{0,j}=t^*-\frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j},\quad t_{n,j}=t^*-\frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j},\quad t_{e,j}=t^*+\frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{10}$$

$$C_j=\frac{\beta\gamma}{\beta+\gamma}\frac{N_j}{S_j}+\left(1-\delta_j^k(t)\right)\tau \tag{11}$$

The travel time and the departure rate are calculated as follows:

$$T_j(t)=\begin{cases}\dfrac{\beta}{\alpha-\beta}\left(t-t_{0,j}\right) & for\ t\in\left[t_{0,j},t_{n,j}\right]\\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,j}-t\right) & for\ t\in\left[t_{n,j},t_{e,j}\right]\end{cases} \tag{12}$$

$$r_j^k(t)=\begin{cases}\dfrac{\alpha}{\alpha-\beta}S_j & for\ t\in\left[t_{0,j},t_{n,j}\right]\\ \dfrac{\alpha}{\alpha+\gamma}S_j & for\ t\in\left[t_{n,j},t_{e,j}\right]\end{cases} \tag{13}$$

The numbers of highway commuters and open-road users are calculated by using the following equations:

$$N_A=\frac{S_A}{S_A+S_b}\left(N-S_B\frac{\beta+\gamma}{\beta\gamma}\tau\right)\quad ,N_B=N-N_A \tag{14}$$

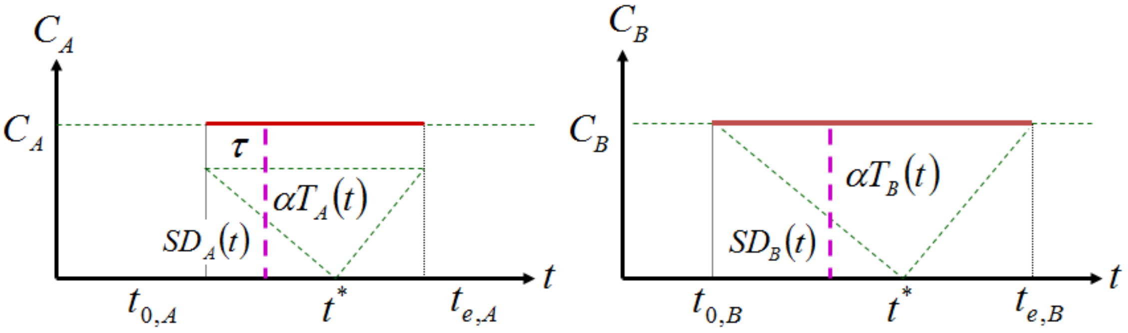

Figure 2 Transition of travel cost components in case (I)

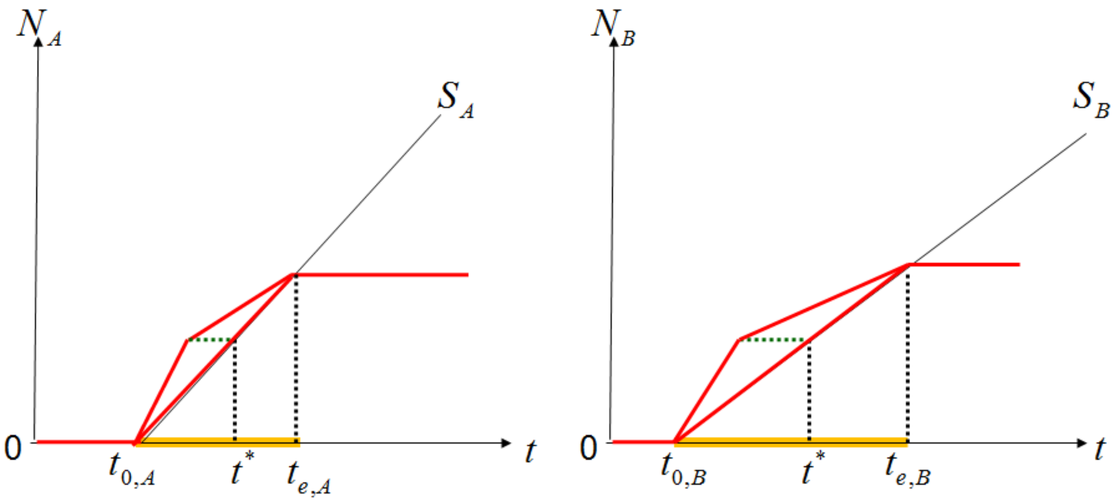

Figure 3 Transitions of cumulative number of departures and arrivals in case (I)

Figure 2 depicts the transitions of travel cost components when the benchmark equilibrium and the implied equilibrium conditions are satisfied because the travel time costs $\alpha T_j(t)$, schedule cost $SD_j(t)$, are highway fee $\tau(t)$ are different corresponding to the arrival time; however, the travel costs are constant for all commuters. Vickrey (1969) insisted that a social optimal be achieved by charging a time-varying fee that corresponds to the travel time cost $\alpha T_j(t)$.

Figure 3 illustrates the transitions of highway and open-road usage during the morning commuting period at a benchmark equilibrium. The horizontal axes denote the timescales, and the vertical axes denote the cumulative number of automobiles. The vertical gaps between the cumulative departures and cumulative arrivals functions are equal to the queue length. The horizontal gaps correspond to the travel times. Hence, the total social losses of bottleneck congestions, i.e. the total travel times, are calculated by using the area between the two functions.

3.2 Bottleneck congestions with ETC off-peak commuting discount

In this subsection, the combined highway fee systems (i) and (ii) are analyzed. This case is called case (II). The commuters are divided into two categories—ETC users and non-ETC users—according to the fee type that they choose. As mentioned in the previous section, a commuter who arrives at t* faces the longest queue. Therefore, let the off-peak period be the time period that does not include the arrival time t*; this period is denoted by $[t_{0,A}, t_A^+], [t_A^-, t_{e,A}]$. The travel costs are formulated on the basis of the fee type, route choice, and departure time, as expressed in equation (15).

$$C_j^k(t)=\begin{cases}\alpha T(t)+\beta\left(t^*-t-T_j(t)\right)+\left(1-\delta_j^k(t)\right)\tau & for\ k\in(N,E), t\in\left[t_{0,j},t_{n,j}\right]\\ \alpha T(t)+\gamma\left(t+T_j(t)-t^*\right)+\left(1-\delta_j^k(t)\right)\tau & for\ k\in(N,E), t\in\left[t_{n,j},t_{e,j}\right]\end{cases} \tag{15}$$

$$\text{where}\quad \delta_j^k(t)\begin{cases}=0 & for\ j=A, k=N, t\in\left[t_{0,A},t_{e,A}\right]\\ =0 & for\ j=A, k=E, t\in\left[t_A^+,t_A^-\right]\\ =\delta & for\ j=A, k=E, t\in\left[t_{0,A},t_A^+\right],\left[t_A^-,t_{e,A}\right]\\ =1 & for\ j=B, k=N,E, t\in\left[t_{0,j},t_{e,j}\right]\end{cases}$$

3.2.1 Case (IIa): $\left(t_A^+-t_{0,A}+t_{e,A}-t_A^-\right)S_A>N^E$

First, the maximum number of commuters that can be handled at the toll gate during a discount period is defined as the bottleneck capacity of the discount period: $\left(t_A^+-t_{0,A}+t_{e,A}-t_A^-\right)S_A$. Let us begin with the case in which the number of ETC users is less than the bottleneck capacity of the discount time period under the fee system (i) + (ii). This case (case (IIa)) corresponds to a relatively low discount rate, expensive ETC device, large bottleneck capacity, long discount period, early spread period of the ETC device, and so on. The equilibrium holds under the conditions that no commuter has an incentive to change his/her departure time and route in order to reduce his/her travel cost.

$$t_{0,j}=t^*-\frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j},\quad t_{n,j}=t^*-\frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j},\quad t_{e,j}=t^*+\frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{16}$$

$$t_{n,A}^+=\frac{\alpha-\beta}{\alpha}t_A^++\frac{\beta}{\alpha}t_{0,A},\quad t_{n,A}^-=\frac{\alpha+\gamma}{\alpha}t_A^--\frac{\gamma}{\alpha}t_{e,A} \tag{17}$$

$$T_j(t)=\begin{cases}\dfrac{\beta}{\alpha-\beta}\left(t-t_{0,j}\right) & for\ t\in\left[t_{0,j},t_{n,j}\right]\\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,j}-t\right) & for\ t\in\left[t_{n,j},t_{e,j}\right]\end{cases} \tag{18}$$

$$C_A^N=C_B^N \tag{19}$$

$$C_A^N=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\tau \qquad for\ t\in\left[t_{0,A},t_{e,A}\right] \tag{20}$$

$$C_A^E=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\left(1-\delta_A^E(t)\right)\tau \qquad for\ t\in\left[t_{0,A},t_{e,A}\right] \tag{21}$$

$$C_B^N, C_B^E = \frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad for\ t \in [t_{0,B}, t_{e,B}] \tag{22}$$

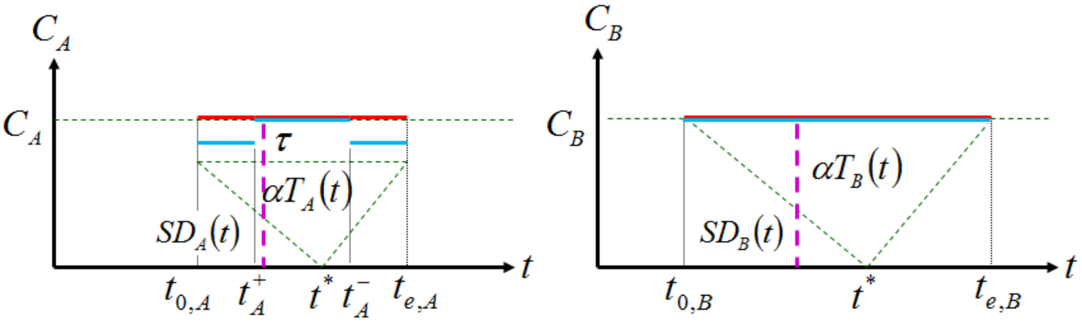

Figure 4 Transitions of travel cost components in case (IIa)

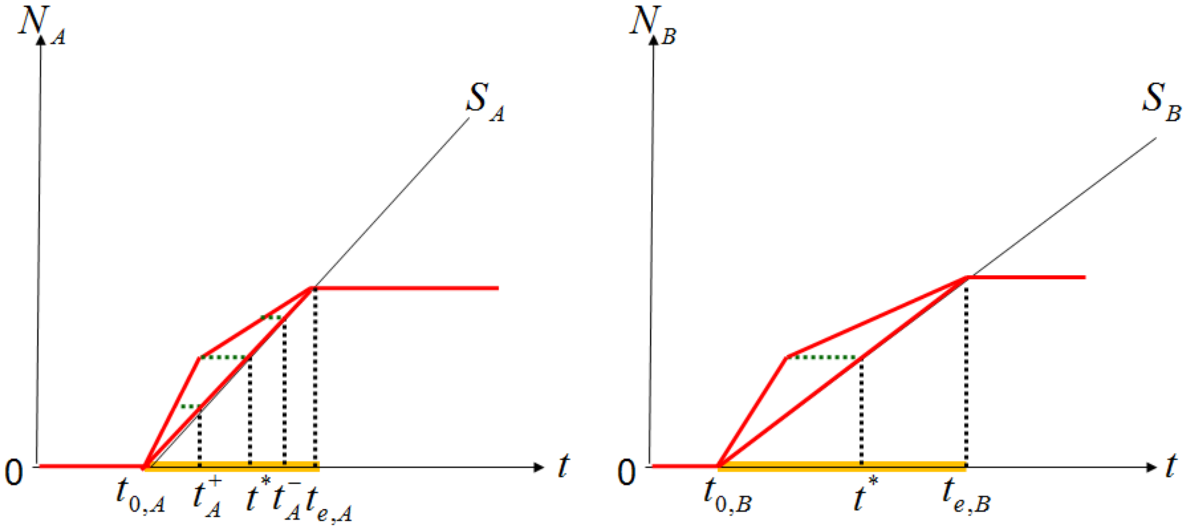

Figure 5 Transitions of cumulative number of departures and arrivals in case (IIa)

In the equilibrium illustrated in Figs. 4 and 5, only the travel cost of ETC users, represented by a light colour, decreases during the off-peak period. The difference between the travel costs of the ETC users and those of the non-ETC users is due to the following reason: ETC users will try to pass the toll gate during an off-peak period, while non-ETC users are not strongly interested in the off-peak period because they cannot enjoy the ETC off-peak commuting discount. On the basis of the given condition, i.e. $(t_A^+ - t_{0,A} + t_{e,A} - t_A^-)S_A > N^E$, it can be concluded that all ETC users can use the highway during the off-peak period, including some non-ETC users. ETC users change their departure time during the off-peak period, and non-ETC users change their departure time during both the off-peak and the peak periods. Further, they change their route choices between the highway and the open road in order to reduce their travel costs. As a result, all ETC users pass the toll gate during the discount period. The travel times for ETC users and non-ETC users are exactly the same on the highway route corresponding to each departure time at the equilibrium; therefore, a commuter cannot distinguish ETC users from other commuters except at the toll gate. Even if the highway fee is discounted, the congestions will not change on both the highway and the open road, as shown in Fig. 5. On the other hand, with respect to the travel costs, only ETC users enjoy reduced travel costs because of the off-peak discount. As there is an incentive for ETC, the number of ETC users will increase in the long term.

3.2.2 Case (IIb): $\left(t_A^{+} - t_{0,A} + t_{e,A} - t_A^{-}\right)S_A < N^E$

Herein, the number of ETC users is more than the bottleneck capacity of the discount period. This case may be caused by a relatively high discount rate, cheap ETC device, small bottleneck capacity, short discount period, late spread period of ETC device, and so on. The equilibrium also holds under the condition that no commuter has an incentive to change his/her departure time and route in order to decrease his/her travel cost.

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{23}$$

$$t_{n,A}^{+} = \frac{\alpha-\beta}{\alpha}t_A^{+} + \frac{\beta}{\alpha}t_{0,A}, \quad t_{n,A}^{-} = \frac{\alpha+\gamma}{\alpha}t_A^{-} - \frac{\gamma}{\alpha}t_{e,A} \tag{24}$$

where both $t_{n,A}^{+}$ and $t_{n,A}^{-}$ are applied off-peak discounts.

$$T_A(t) = \begin{cases} \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,A}\right) + \dfrac{\delta\tau}{\alpha-\beta} & \mathit{for}\ t \in \left[t_{0,A}, t_{n,A}^{+}\right] \\ \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,A}\right) & \mathit{for}\ t \in \left[t_{n,A}^{+}, t_{n,A}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,A} - t\right) & \mathit{for}\ t \in \left[t_{n,A}, t_{n,A}^{-}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_e - t\right) + \dfrac{\delta\tau}{\alpha+\gamma} & \mathit{for}\ t \in \left[t_n^{-}, t_e\right] \end{cases} \tag{25}$$

$$T_B(t) = \begin{cases} \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,B}\right) & \mathit{for}\ t \in \left[t_{0,B}, t_{n,B}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,B} - t\right) & \mathit{for}\ t \in \left[t_{n,B}, t_{e,B}\right] \end{cases} \tag{26}$$

$$C_A^N = C_B^N \tag{27}$$

$$C_A^N = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \left(1+\delta_A^E(t)\right)\tau \qquad for\ t \in \left[t_{0,A}, t_{e,A}\right] \tag{28}$$

$$C_A^E = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \tau \qquad for\ t \in \left[t_{0,A}, t_{e,A}\right] \tag{29}$$

$$C_B^E, C_B^N = \frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad for\ t \in \left[t_{0,B}, t_{e,B}\right] \tag{30}$$

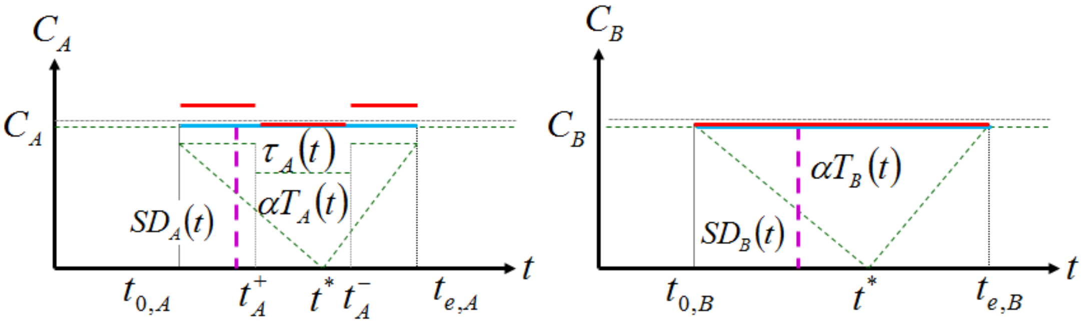

Figure 6 Transitions of travel cost components in case (IIb)

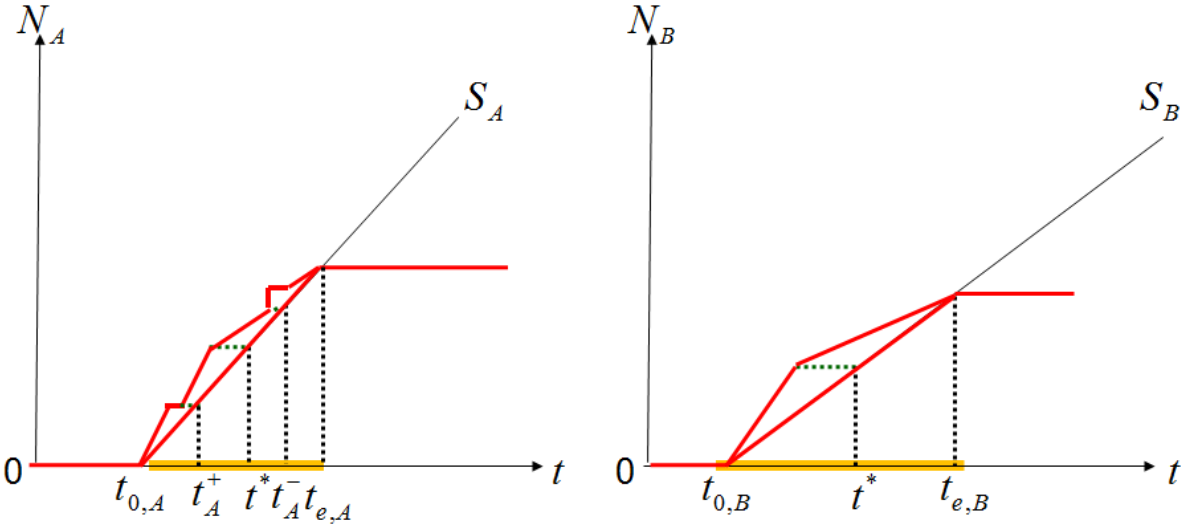

Figure 7 Transitions of cumulative number of departures and arrivals in case (IIb)

As shown at the equilibrium in Fig. 6, travel costs decreased slightly except for non-ETC users during the off-peak period. The difference between the travel costs of ETC users and those of non-ETC users is due to the following reason: ETC users try to pass the toll gate during the off-peak period; however, a section of ETC users cannot do so because the number of ETC users is more than the bottleneck capacity in the discount period. Then, ETC users extend the queue during the off-peak period until their travel costs equal the travel costs during the peak period on the highway or the open road. On the other hand, non-ETC users do not need to pass the gate within the off-peak period. Therefore, non-ETC users pass the toll gate during the peak period or use the open road. At the equilibrium, all commuters who pass the toll gate within the off-peak period are ETC users, and commuters who do so within the peak period or use the open road are both ETC users and non-ETC users. Hence, a Pareto improvement is achieved by commuters because the equilibrium travel costs are decreased for all users as compared to case (I). This decrease is caused by the shift in demand from the open road to the highway use despite repressing the increase in congestion during the peak period, as shown in Fig. 6. As there is no incentive for ETC use, the number of ETC users may not increase in the long term.

3.3 Bottleneck congestion with ETC off-peak commuting discount plus

In this subsection, the ETC off-peak commuting discount, i.e. (iii), is combined with (i) and (ii) as an additional incentive to promote off-peak highway commuting. The commuters are divided into two categories—ETC users: N^E and non-ETC users: N^N. Further, the ETC users who travel frequently during the off-peak period are discriminated from the ETC users as N^{EE}.

$$C_j^k(t)=\begin{cases}\alpha T(t)+\beta\left(t^*-t-T_j(t)\right)+\left(1-\delta_j^k(t)\right)\tau & for\ k\in(N,E), t\in\left[t_{0,j},t_{n,j}\right]\\ \alpha T(t)+\gamma\left(t+T_j(t)-t^*\right)+\left(1-\delta_j^k(t)\right)\tau & for\ k\in(N,E), t\in\left[t_{n,j},t_{e,j}\right]\end{cases} \quad (31)$$

$$ただし、\delta_j^k(t)\begin{cases}=0 & for\ j=A, k=N, t\in\left[t_{0,A},t_{e,A}\right]\\ =0 & for\ j=A, k=E, t\in\left[t_A^+,t_A^-\right]\\ =\delta & for\ j=A, k=EE, t\in\left[t_A^+,t_A^-\right]\\ =\delta & for\ j=A, k=E,EE, t\in\left[t_{0,A},t_A^+\right],\left[t_A^-,t_{e,A}\right]\\ =1 & for\ j=B, k=N,E,EE, t\in\left[t_{0,j},t_{e,j}\right]\end{cases}$$

3.3.1 Case (IIIa): $\left(t_A^+-t_{0,A}+t_{e,A}-t_A^-\right)s_A>N^E-N^{EE}$

Herein, the number of ETC users who do not commute frequently during the off-peak period N^E-N^{EE} is less than the bottleneck capacity of the discount period, i.e. $\left(t_A^+-t_{0,A}+t_{e,A}-t_A^-\right)s_A$. This case is labelled case (IIIa). The equilibrium is solved in the same manner as in the previous section.

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{32}$$

$$t^+_{n,A} = \frac{\alpha-\beta}{\alpha} t^+_A + \frac{\beta}{\alpha} t_{0,A}, \quad t^-_{n,A} = \frac{\alpha+\gamma}{\alpha} t^-_A - \frac{\gamma}{\alpha} t_{e,A} \tag{33}$$

$$T_j(t) = \begin{cases} \frac{\beta}{\alpha-\beta}\left(t - t_{0,j}\right) & for\; t \in \left[t_{0,j}, t_{n,j}\right] \\ \frac{\gamma}{\alpha+\gamma}\left(t_{e,j} - t\right) & for\; t \in \left[t_{n,j}, t_{e,j}\right] \end{cases} \tag{34}$$

$$C^N_A = C^N_B \tag{35}$$

$$C^N_A = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \tau \qquad for\; t \in \left[t_{0,A}, t_{e,A}\right] \tag{36}$$

$$C^E_A = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \left(1 - \delta^E_A(t)\right)\tau \qquad for\; t \in \left[t_{0,A}, t_{e,A}\right] \tag{37}$$

$$C^{EE}_A = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \left(1 - \delta^{EE}_A(t)\right)\tau \qquad for\; t \in \left[t_{0,A}, t_{e,A}\right] \tag{38}$$

$$C^E_B, C^N_B, C^{EE}_B = \frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad for\; t \in \left[t_{0,B}, t_{e,B}\right] \tag{39}$$

At the equilibrium illustrated in Figs. 8 and 9, the travel cost of ETC users N^E decreases during the off-peak period, and the travel cost of ETC users who commute frequently during the off-peak period N^{EE} decreases outside of the off-peak period as well. The differences in the travel costs of the users are accrued by the following process: As the ETC users who do not frequently travel during the off-peak period cannot get a discount during the peak period, they try to commute during the off-peak period. On the basis of the given condition, i.e. $\left(t^+_A - t_{0,A} + t_{e,A} - t^-_A\right)S_A > N^E - N^{EE}$, it can be concluded that all users can commute during the off-peak period, including some non-ETC users and ETC users who frequently commute during the off-peak period. ETC users who do not commute frequently during the off-peak period change their departure time to fall within the off-peak period, ETC users who commute frequently during the off-peak period change their departure time within both the off-peak and the peak periods, and non-ETC users change their departure time and route within both the off-peak and the peak periods in order to reduce their travel costs. The equilibrium is valid when no commuters can find an alternative departure time and route to reduce his/her travel costs. As a result, the travel times for all the users are exactly the same with respect to each departure time and route choice at this equilibrium. Even if the users' departure time and route choice are discriminated on the basis of the self-select modes, the congestions do not change on both the highway and the open road in cases (I) and (IIa). On the other hand, with respect to travel costs, ETC users, including those who commute frequently during the off-peak period, enjoy relatively low travel costs because of the off-peak discount, as shown in Figure 8. Furthermore, ETC users who commute frequently within the off-peak period benefit from the ETC off-peak commuting discount plus a discount within the peak period. Therefore, as there is an incentive for ETC and for commuting during the off-peak period, the number of ETC users will increase in the long term.

3.3.2 Case (IIIb); $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A < N^E - N^{EE}$

Next, the number of ETC users who do not travel frequently during the off-peak period $N^E - N^{EE}$ is more than the bottleneck capacity of the discount time period, i.e. $\left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A$. This case is labelled case (IIIb). The equilibrium is solved in the same manner as in the previous section.

$$t_{0,j} = t^* - \frac{\gamma}{\beta+\gamma}\frac{N_j}{S_j}, \quad t_{n,j} = t^* - \frac{\beta\gamma}{\alpha(\beta+\gamma)}\frac{N_j}{S_j}, \quad t_{e,j} = t^* + \frac{\beta}{\beta+\gamma}\frac{N_j}{S_j} \tag{40}$$

$$t_{n,A}^+ = \frac{\alpha-\beta}{\alpha}t_A^+ + \frac{\beta}{\alpha}t_{0,A}, \quad t_{n,A}^- = \frac{\alpha+\gamma}{\alpha}t_A^- - \frac{\gamma}{\alpha}t_{e,A} \tag{41}$$

$$T_A(t) = \begin{cases} \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,A}\right) + \dfrac{(1+1/P)\delta\tau}{\alpha-\beta} & for\ t \in \left[t_{0,A}, t_{n,A}^+\right] \\ \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,A}\right) & for\ t \in \left[t_{n,A}^+, t_{n,A}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,A} - t\right) & for\ t \in \left[t_{n,A}, t_{n,A}^-\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_e - t\right) + \dfrac{(1+1/P)\delta\tau}{\alpha+\gamma} & for\ t \in \left[t_n^-, t_e\right] \end{cases} \tag{42}$$

$$T_B(t) = \begin{cases} \dfrac{\beta}{\alpha-\beta}\left(t - t_{0,B}\right) & for\ t \in \left[t_{0,B}, t_{n,B}\right] \\ \dfrac{\gamma}{\alpha+\gamma}\left(t_{e,B} - t\right) & for\ t \in \left[t_{n,B}, t_{e,B}\right] \end{cases} \tag{43}$$

$$C_A^N = C_B^N \tag{44}$$

$$C_A^N = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \left(1 + (1+1/P)\delta_A^E(t)\right)\tau \qquad for\ t \in \left[t_{0,A}, t_{e,A}\right] \tag{45}$$

$$C_A^E = \frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A} + \tau \qquad for\ t \in \left[t_{0,A}, t_{e,A}\right] \tag{46}$$

$$C_A^{EE}=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\left(1-\left(1+1/P\right)\delta_A^{EE}(t)\right)\tau \qquad for\ t\in\left[t_{n,A}^{+},t_{n,A}^{-}\right] \tag{47}$$

$$C_A^{EE}=\frac{\beta\gamma}{\beta+\gamma}\frac{N_A}{S_A}+\tau \qquad for\ t\in\left[t_{0,A},t_{n,A}^{+}\right]\left[t_{n,A}^{-},t_{e,A}\right] \tag{48}$$

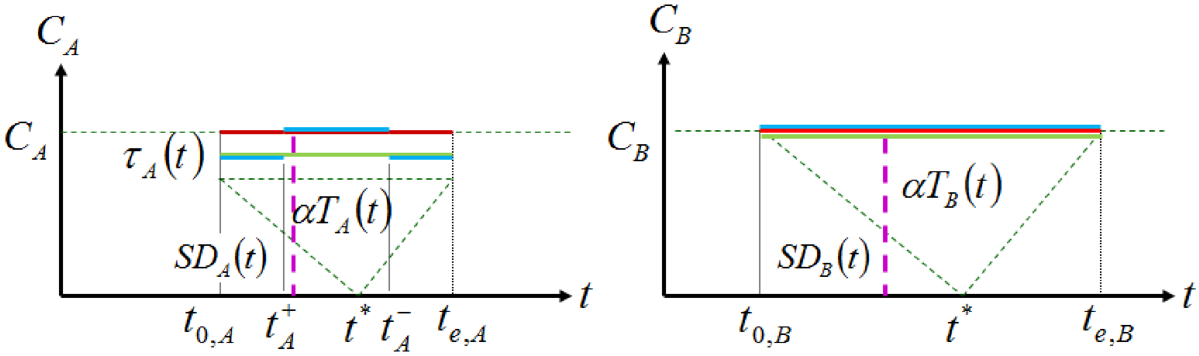

Figure 8 Transitions of travel cost components in case (IIIa)

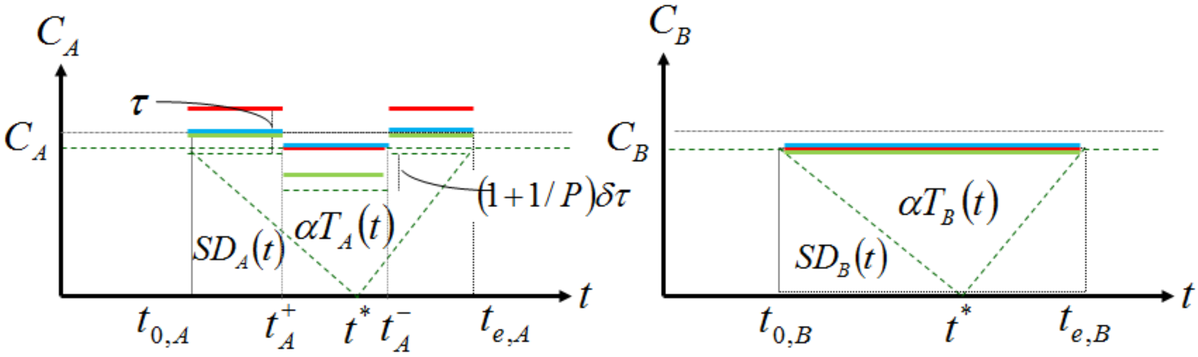

Figure 9 Transitions of travel cost components in case (IIIb)

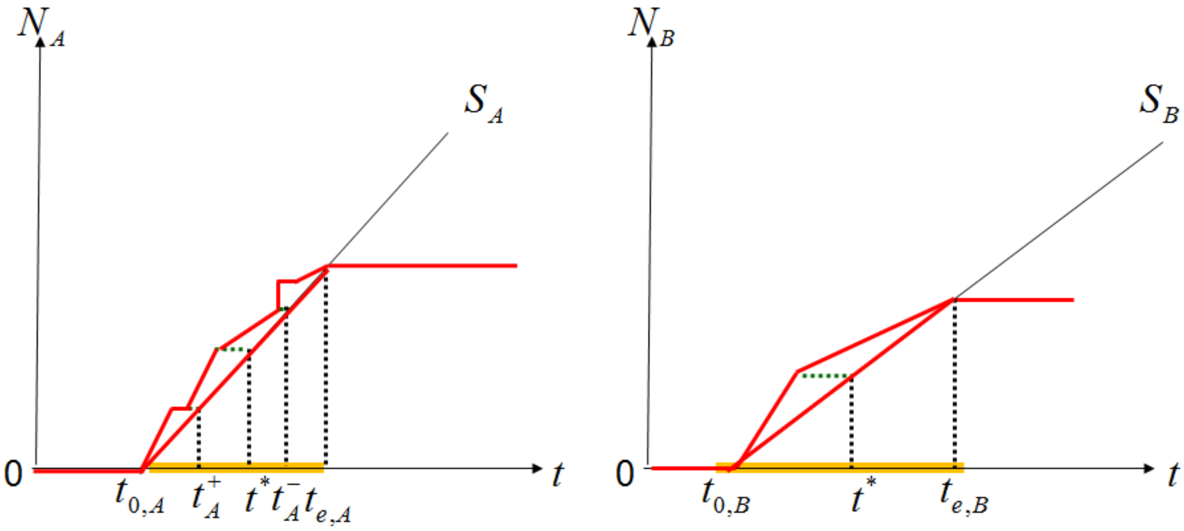

Figure 10 Transitions of cumulative number of departures and arrivals in case (IIIb)

$$C_B^E, C_B^N, C_B^{EE} = \frac{\beta\gamma}{\beta+\gamma}\frac{N_B}{S_B} \qquad for\ t \in \left[t_{0,B}, t_{e,B}\right] \tag{49}$$

At this equilibrium, the travel costs of N^N users increase by $\delta\tau$ as compared to those of N^E within the off-peak period and the travel costs of N^{EE} decrease by $\delta\tau$ as compared to those of the others within the peak period, as shown in Fig. 9.

ETC users who have not chosen ETC off-peak commuting discount plus try to pass the toll gate within the off-peak period; however, some of them cannot do so because $N^E - N^{EE}$ is more than the bottleneck capacity during the discount period. Then, the queue is extended within the off-peak period. ETC users who have selected ETC off-peak commuting discount plus would like to pass the toll gate with the peak period because their travel costs during the peak period are relatively lower than during the off-peak period. Even if the queue is prolonged until the travel costs of the other ETC users within the off-peak period are equal to their travel costs within the peak period, the ETC users who have selected ETC off-peak discount plus can still enjoy their benefits within the peak period at every P^th^ commute. Therefore, the queue is extended until the gaps of travel times are equivalent to $(1+1/P)\delta\tau$ between the off-peak and the peak periods. The travel costs between ETC users are indifferent in the expected value. Hence, all ETC users who go through the toll gate within the off-peak period are potential N^{EE} who will be eligible for the discount under the ETC off-peak commuting discount plus scheme within the peak period for every P^th^ commute; therefore, $N^{EE} = \left(t_A^+ - t_{0,A} + t_{e,A} - t_A^-\right)S_A / (P-1)$.

Meanwhile, non-ETC users have no incentive to join the extended queue during the off-peak period. Therefore, N^N go through the toll gate during the peak period if they use the highway route. Therefore, the three different groups, i.e. N^{EE}, N^E, and N^N, commute mixing within the peak period on the highway route. A short peak period or a small value of P that leads to $\left(t_A^- - t_A^+\right)S_A < N^{EE}$ may disturb the onset of the mitigation of bottleneck congestions. The open road may be used by both N^N and N^E. As a result, the commuting time period on the highway route is expanded, and the travel demand is shifted from the open road to the highway route depicted in Fig. 10. Then, the travel costs of using both the open road and the highway are decreased, and the efficiency of the road system is improved even if the problem of bottleneck congestion on the highway route is aggravated within the off-peak period.

4. Conclusions

This study analyses the discounts on highway fee for the off-peak commuting with ETC by using the traditional bottleneck congestion model in order to consider the self-select fee system and the post-payment method related to historical usage.

The ETC off-peak commuting discount improves the efficiency of the road system because of the utilization of the off-peak capacity of the highway so as to shift the traffic demand from the open road to the highway as compared to the ETC peak commuting discount that was in place in Japanese highway. However, if the number of ETC users who use the discounts is less than the bottleneck capacity during the discount period, then the introduction of the discount does not reduce the bottleneck congestion even if they save on their highway fees. Furthermore, the scale of the effect of the mitigation of road congestion depends on the reduction in the travel costs caused by the shift in demand from the open road to the highway.

It is confirmed that the additional discount, i.e. ETC off-peak commuting discount plus, is an effective incentive for using an ETC device, thereby restraining the aggravation of the road congestion. That is, if the policy maker provides more ETC off-peak commuting discounts as an incentive for using an ETC device, the discount period must be expanded and then an increase in the road congestion during the peak period is inevitable. Meanwhile, the ETC off-peak commuting discount plus provides an incentive for the use of ETC without increasing road congestion. Considering the near-ubiquity of ETC, this discount system must be effective as part of the Japanese highway fee policy.

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