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Central government, local government  
and agglomeration

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**Abstract**

This paper analyzes the effect of local and central government on agglomeration. The local and central government do not utilize a regional redistribution policy. In this case, the local government can cause agglomeration by providing for the local public good. When the central government provides the pure public good, would the relationship between the local government and agglomeration be changed or not?

When the effect of the local public good is large relative to that of private goods, the local government causes full agglomeration that may be undesirable. The central government lowers the possibility of the undesirable agglomeration. When the effect of private goods in the utility is large relative to that of the public good, the local government does not cause agglomeration. In this case, the central government causes partial agglomeration.

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# Central government, local government and agglomeration

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# 1 Introduction

This paper analyzes the public sector that relates to the agglomeration of economic activity. Burridge and Myers (1994) examine the local government transfer policy to control the agglomeration. Glazer and Kondo (2007) analyze the local government that voters determine its policy influences the agglomeration. Roos (2004) shows that the local government has a centripetal force and may cause the agglomeration without depending on increasing returns to scale in private production. Mark, McGuire and Papke (2000) estimate that the local government's tax scheme and the public expenditure increase the population size in that region. But these studies do not consider the role of the central government.

The public sector includes not only the local government but also the central government. Riou (2006) analyzes the central government's transfer policy when the local government causes agglomeration. In Riou's model, the central government only redistributes income among regions. But the main object of the central government is not only redistribution but also to provide public services. This paper evaluates the relationship between the public sector and the agglomeration. Here, the public sector contains not only the local government but also the central government that provides the public services.

This paper analyzes that the government that does not utilize a regional redistribution policy. Following Roos (2004), the local government may cause the agglomeration without depending on increasing returns technology. In this paper, when the local public good is valuable and the local government has a more productive technology, all population agglomerates in one region. But this full agglomeration may be undesirable. In this case, the central government is introduced as in Dascher (2002) and Furukawa (2010). The central government produces the public good in one region using that region's labor force. I evaluate whether or not the central government improves the relationship between the local government and the agglomeration without the regional redistribution

policy.

The remainder of this paper is as follows. Section 2 defines the model. Section 3 analyzes the relationship between the local government behavior and agglomeration without the central government. Section 4 introduces the central government. Section 5 summarizes the results.

## 2 The model

The economy consists of regions 1 and 2. In each region, private goods are produced. In this model, there are two types of private goods, which can be traded across regions without cost.

The local government exists in each region. The local government provides a local public good that can not be traded across regions. The local public good in region  $i$  ( $i=1,2$ ) can be consumed by region  $i$ 's individuals as the pure public good. When the central government exists, it locates in one region and provides the pure public good that can be consumed by all individuals.

Individuals consume private goods, the local public good and the public good. Each individual has one unit of labor. In this economy, the total amount of labor is  $\bar{L} = L_1 + L_2$ , where  $L_i$  is region  $i$ 's population. Each individual can migrate across regions without cost.

The individual in region  $i$  has the utility function

$$U^i = (x_1^i x_2^i)^{\frac{1-p}{2}} g_i^p G$$

where  $x_j^i$  is the private good  $j$  ( $j=1,2$ ),  $g_i$  is the local public good which is provided by the local government, and  $G$  is the public good which is provided by the central government. The budget constraint of individuals in region  $i$  is

$$P_1 x_1^i + P_2 x_2^i = (1 - t_i - T)w_i$$

where  $P_j$  is the price of the private good  $j$ ,  $w_i$  is region  $i$ 's labor wage,  $t_i$  is the tax rate that is imposed by the local government, and  $T$  is the central government's tax rate that is equal in both regions.

Individuals maximize the utility subject to the budget constraint. Then, the individual demand of private good  $j$  is

$$x_j^i = \frac{(1 - t_i - T)w_i}{2P_j} \quad (1)$$

From the demand function, the indirect utility function in region  $i$  is given by equation (2) :

$$V^i = \left( \frac{(1 - t_i - T)^2 w_i^2}{4P_1 P_2} \right)^{\frac{1-p}{2}} g_i^p G \quad (2)$$

Because individuals are free to migrate to the region where the utility is higher, the utility is equal across regions in the equilibrium.

This paper follows Takahashi (1998) in terms of the production of private goods. The private goods are produced with labor as the input. In region 1, the production functions are as follows:

$$X_1^1 = \Gamma \beta L_1^1 \quad X_2^1 = \beta L_2^1 \quad (3)$$

Similarly, in region 2, the production functions are as follows:

$$X_1^2 = L_1^2 \quad X_2^2 = \Gamma L_2^2 \quad (4)$$

where  $X_j^i$  is the amount of good  $j$  produced in region  $i$ , and  $L_j^i$  denotes the amount of labor used to produce the good  $j$  in region  $i$ .  $\Gamma > 1$  shows the degree of comparative advantage. It means that region 1(2) has the comparative advantage in the production of good 1(2). In the model, I assume the regional asymmetry of the production. The parameter  $\beta > 1$  shows that region 1 is superior to region 2 in the production of private goods. Moreover, I assume that  $\Gamma > \beta$ .

Because private goods can be traded across regions without cost, the price of good  $j$   $P_j$  is the same in both regions. Under perfect competition, the zero profit condition implies that

$$P_1 \Gamma \beta = w_1 \quad P_2 \Gamma = w_2 \quad (5)$$

$$P_2 \beta \leq w_1 \quad (6)$$

$$P_1 \leq w_2 \quad (7)$$

(5) means that region 1(2) always produces the good 1(2) because of the comparative advantage. If region 1 produces the good 2, equality holds in (6) . Similarly, if region 2 produces the good 1, equality holds in (7) . When these equations do not hold, region  $i$  does not produce the good  $j$  ( $i \neq j$ ). This specialize case holds when  $1/\Gamma < P_1/P_2 < \Gamma$  . When  $P_1/P_2 = 1/\Gamma$  , region 1 produces both goods while region 2 specializes. When  $P_1/P_2 = \Gamma$  , region 2 produces both goods while region 1 specializes.

The local government produces the local public good with the labor as the inputs. The production function of local public good in region  $i$   $g_i$  is as follows:

$$g_i = L_{g_i}^\gamma$$

where  $L_{g_i}$  is the amount of labor used to produce the local public good in region  $i$  and  $\gamma$  is the local public good's output elasticity for  $L_{g_i}$  . To finance the expenditure of the production, the local government collects income tax from individuals in the region. The local government only uses the labor in the same region. Thus, the budget constraint of the local government is

$$w_i L_{g_i} = t_i w_i L_i \tag{8}$$

### 3 Agglomeration: the effect of local government

In this section, I analyze the equilibrium population when the central government does not exist. I assume that the central government's tax rate  $T$  is zero and the pure public good  $G$  in the utility function is 1. In section 4, I examine the case where the central government exists.

First, suppose that each region specializes the production. Then, the market clearing conditions for private goods using (1) , (3) and (4) may be stated as follows:

$$\frac{(1 - t_1)w_1L_1 + (1 - t_2)w_2L_2}{2P_1} = \Gamma\beta(L_1 - L_{g_1}) \tag{9}$$

$$\frac{(1 - t_1)w_1L_1 + (1 - t_2)w_2L_2}{2P_2} = \Gamma(L_2 - L_{g_2}) \tag{10}$$

From (8) , (9) and (10), I obtain

$$\frac{P_1}{P_2} = \frac{(1-t_2)L_2}{\beta(1-t_1)L_1} \quad (11)$$

From section 2, if  $1/\Gamma < P_1/P_2 < \Gamma$ , each region specializes the production. Using (11), the condition is

$$\frac{1}{\Gamma} < \frac{(1-t_2)L_2}{\beta(1-t_1)L_1} < \Gamma \quad (12)$$

In this case, the indirect utility functions in each region are

$$\begin{aligned} V_1 &= \left( (1-t_1)(1-t_2) \frac{L_2}{L_1} \right)^{\frac{1-p}{2}} (t_1 L_1)^{\gamma p} \left( \frac{\Gamma^2 \beta}{4} \right)^{\frac{1-p}{2}} \\ V_2 &= \left( (1-t_1)(1-t_2) \frac{L_1}{L_2} \right)^{\frac{1-p}{2}} (t_2 L_2)^{\gamma p} \left( \frac{\Gamma^2 \beta}{4} \right)^{\frac{1-p}{2}} \end{aligned}$$

The local government maximizes the regional individual utility over the tax rate and takes the populations and the other region's tax rate as given. To solve the utility maximizing problem, I obtain the tax rate in region i as follows

$$t_i = \frac{2\gamma p}{1-p+2\gamma p} \quad (13)$$

Substituting (13) into condition (12), I have

$$\frac{1}{\Gamma\beta} < \frac{L_1}{L_2} < \frac{\Gamma}{\beta} \quad (14)$$

When the equilibrium populations satisfy this condition, the case where each region specializes the production holds. Moreover, substituting (13) into the indirect utility function, I have

$$\begin{aligned} V_1 &= \left[ \left( \frac{1-p}{1-p+2\gamma p} \right)^2 \frac{L_2}{L_1} \right]^{\frac{1-p}{2}} \left[ \frac{2\gamma p}{1-p+2\gamma p} L_1 \right]^{\gamma p} \left( \frac{\Gamma^2 \beta}{4} \right)^{\frac{1-p}{2}} \\ V_2 &= \left[ \left( \frac{1-p}{1-p+2\gamma p} \right)^2 \frac{L_1}{L_2} \right]^{\frac{1-p}{2}} \left[ \frac{2\gamma p}{1-p+2\gamma p} L_2 \right]^{\gamma p} \left( \frac{\Gamma^2 \beta}{4} \right)^{\frac{1-p}{2}} \end{aligned}$$

A ratio of the utility in two regions  $v = V_1/V_2$  is

$$v = \frac{V_1}{V_2} = \left[ \frac{L_1}{L_2} \right]^{(\gamma+1)p-1} \quad (15)$$

Because individuals can migrate across regions without cost, in the equilibrium the ratio of the utility  $v$  equals 1. From this condition and  $\bar{L} = L_1 + L_2$ , the equilibrium populations are

$$L_1 = L_2 = \frac{\bar{L}}{2} \quad (16)$$

If  $(\gamma + 1)p - 1 < 0$ ,  $v$  is the decreasing function of  $L_1$ . Then, the equilibrium populations are stable.

If  $(\gamma + 1)p - 1 > 0$ , the equilibrium populations are unstable.

Second, I explain the case where region 1 produces two private goods. From (14), if  $L_1/L_2 \geq \Gamma/\beta > 1$ , the price ratio  $P_1/P_2$  equals  $1/\Gamma$  and region 1 produces two private goods. In this case, the indirect utility functions in each region are

$$\begin{aligned} V_1 &= (1 - t_1)^{1-p} (t_1 L_1)^{\gamma p} \left( \frac{\Gamma \beta^2}{4} \right)^{\frac{1-p}{2}} \\ V_2 &= (1 - t_2)^{1-p} (t_2 L_2)^{\gamma p} \left( \frac{\Gamma^3}{4} \right)^{\frac{1-p}{2}} \end{aligned}$$

Similar to the first case, the tax rate in region  $i$  is

$$t_i = \frac{\gamma p}{1 - p + \gamma p} \quad (17)$$

Substituting (17) into the indirect utility function, I have

$$\begin{aligned} V_1 &= \left( \frac{1 - p}{1 - p + \gamma p} \right)^{1-p} \left[ \frac{\gamma p}{1 - p + \gamma p} L_1 \right]^{\gamma p} \left( \frac{\Gamma \beta^2}{4} \right)^{\frac{1-p}{2}} \\ V_2 &= \left( \frac{1 - p}{1 - p + \gamma p} \right)^{1-p} \left[ \frac{\gamma p}{1 - p + \gamma p} L_2 \right]^{\gamma p} \left( \frac{\Gamma^3}{4} \right)^{\frac{1-p}{2}} \end{aligned}$$

The ratio of the utility is

$$v = \frac{V_1}{V_2} = \left( \frac{\beta}{\Gamma} \right)^{1-p} \left[ \frac{L_1}{L_2} \right]^{\gamma p} \quad (18)$$



From the condition  $v = 1$ , the ratio of equilibrium populations is

$$\frac{L_1}{L_2} = \left(\frac{\Gamma}{\beta}\right)^{\frac{1-p}{\gamma p}} \quad (19)$$

If  $(\gamma + 1)p - 1 < 0$ ,  $\left(\frac{\Gamma}{\beta}\right)^{\frac{1-p}{\gamma p}} > \frac{\Gamma}{\beta}$ . Hence, the equilibrium populations that are unstable exist when  $L_1/L_2 \geq \Gamma/\beta$ .

Third, consider that region 2 produces two private goods. From (14), if  $L_1/L_2 \leq 1/(\Gamma\beta) < 1$ , the price ratio  $P_1/P_2$  equals  $\Gamma$  and the case realizes. Similar to the second case, the indirect utility functions when the local government maximizes the utility are

$$\begin{aligned} V_1 &= \left(\frac{1-p}{1-p+\gamma p}\right)^{1-p} \left[\frac{\gamma p}{1-p+\gamma p} L_1\right]^{\gamma p} \left(\frac{\Gamma^3 \beta^2}{4}\right)^{\frac{1-p}{2}} \\ V_2 &= \left(\frac{1-p}{1-p+\gamma p}\right)^{1-p} \left[\frac{\gamma p}{1-p+\gamma p} L_2\right]^{\gamma p} \left(\frac{\Gamma}{4}\right)^{\frac{1-p}{2}} \end{aligned}$$

and the ratio of the utility is

$$v = \frac{V_1}{V_2} = (\Gamma\beta)^{1-p} \left[\frac{L_1}{L_2}\right]^{\gamma p} \quad (20)$$

From the condition  $v = 1$ , the ratio of equilibrium populations is

$$\frac{L_1}{L_2} = \left(\frac{1}{\Gamma\beta}\right)^{\frac{1-p}{\gamma p}} \quad (21)$$

If  $(\gamma + 1)p - 1 < 0$ ,  $\left(\frac{1}{\Gamma\beta}\right)^{\frac{1-p}{\gamma p}} < \frac{1}{\Gamma\beta}$ . Then, the equilibrium populations that are unstable exist when  $L_1/L_2 \leq 1/(\Gamma\beta)$ .

Combining the above three conditions, I explain herewith the equilibrium populations. Concerning the initial distribution of the population, suppose that the difference between each population is not so large. If  $(\gamma + 1)p - 1 > 0$ , that is the weight of the local public good in the utility is large relative to that of private goods,  $L_1 = L_2 = \bar{L}/2$  are equilibriums but unstable. When in the initial distribution  $L_1 > L_2$ , all individuals concentrate in region 1 and  $L_1 = \bar{L}$ ,  $L_2 = 0$  in the equilibrium. Conversely, when  $L_1 < L_2$ , all individuals concentrate in region 2 and  $L_1 = 0$ ,  $L_2 = \bar{L}$  in the

equilibrium. If  $(\gamma + 1)p - 1 < 0$ , that is the weight of the private goods in the utility is large relative to that of the public good,  $L_1 = L_2 = \bar{L}/2$  are stable equilibriums. These results are summarized as Proposition 1.

**Proposition 1** Suppose that the difference between initial populations in each region is not so large. If  $(\gamma + 1)p - 1 > 0$ , all individuals concentrate in the region where the initial population is larger.

Conversely, if  $(\gamma + 1)p - 1 < 0$ , the full agglomeration does not arise. The equilibrium populations are equal in each region and stable.

This proposition resembles the proposition 1 of Roos (2004). When the local public good's output elasticity is larger, the unstable interior equilibrium is  $L_1 = L_2 = \bar{L}/2$ . In addition to that of Roos (2004), when the weight of the local public good is larger, the unstable interior equilibrium is  $L_1 = L_2 = \bar{L}/2$ . If the population in one region is larger than the other, individuals concentrate in that region. The reason is that the centripetal effect which arises from the local public good is stronger than the centrifugal effect which arises from the private goods. If the population in region 2 is larger, all individuals concentrate in region 2. This means that region 1 that has the most advanced production technology does not produce the private goods. Then, the welfare is lower than the case where all individuals concentrate in region 1.

When the weight of the private goods is larger ( $(\gamma + 1)p - 1 < 0$ ), the equilibrium distribution of the population is symmetric among regions and stable because the centrifugal effect is more robust than the centripetal effect. If the difference between initial populations in each region is not so large, the most advanced technology is utilized in the equilibrium.

## 4 Existence of central government

The previous section shows that undesirable agglomeration may arise when the local public good plays an important part. But in the previous section, I assumed that the central government does not exist. In this section, I analyze the role of the central government that prevents undesirable agglomeration.

I assume that the central government locates in region 1. The central government produces the pure public good with the labor in region 1 as the input. The production function of the public good is as follows:

$$G = L_G$$

where  $L_G$  is the amount of labor used to produce the public good. The central government collects the income tax from all individuals to cover the cost of the public good production. Then, the central government's budget constraint is

$$w_1 L_G = T(w_1 L_1 + w_2 L_2) \quad (22)$$

where  $T$  is the income tax rate.

Similar to section 3, I analyze the equilibrium populations. In the following, I consider three cases about the private good's production.

First, I consider the case where each region specializes the production. Using (1) , (3) and (4) , market clearing conditions for private goods lead to

$$\frac{(1 - t_1 - T)w_1 L_1 + (1 - t_2 - T)w_2 L_2}{2P_1} = \Gamma\beta(L_1 - L_{g_1} - L_G) \quad (23)$$

$$\frac{(1 - t_1 - T)w_1 L_1 + (1 - t_2 - T)w_2 L_2}{2P_2} = \Gamma(L_2 - L_{g_2}) \quad (24)$$

From (8) , (22) , (23) and (24) , I obtain

$$\frac{P_1}{P_2} = \frac{(1 - t_2 + T)L_2}{\beta(1 - t_1 - T)L_1} \quad (25)$$

If  $\frac{1}{\Gamma} < \frac{(1-t_2+T)L_2}{\beta(1-t_1-T)L_1} < \Gamma$ , each region specializes the production. In this case, the indirect utility functions in each region are

$$\begin{aligned} V_1 &= \left( (1-t_1-T)(1-t_2+T) \frac{L_2}{L_1} \right)^{\frac{1-p}{2}} (t_1 L_1)^{\gamma p} \left( \frac{\Gamma^2 \beta}{4} \right)^{\frac{1-p}{2}} G \\ V_2 &= \left( (1-t_1-T) \frac{(1-t_2-T)^2 L_1}{(1-t_2+T) L_2} \right)^{\frac{1-p}{2}} (t_2 L_2)^{\gamma p} \left( \frac{\Gamma^2 \beta}{4} \right)^{\frac{1-p}{2}} G \end{aligned}$$

The local government maximizes the regional individual utility over the tax rate and takes the populations, the other authorities' tax rates and public goods as given. From the utility maximizing problem, the tax rate in region 1 is

$$t_1^* = \frac{2\gamma p(1-T)}{1-p+2\gamma p} \quad (26)$$

Similarly, the tax rate in region 2 is

$$t_2^* = \frac{1-p+4\gamma p+3(1-p)T - \sqrt{(1-p+4\gamma p+3(1-p)T)^2 - 8\gamma p(1-T^2)(1-p+2\gamma p)}}{2(1-p+2\gamma p)} \quad (27)$$

Comparing  $t_1^*$  and  $t_2^*$ ,  $t_1^* > t_2^*$  holds. Substituting these tax rates into the condition that each region specializes the production, I have:

$$\frac{1}{\Gamma} \frac{1-t_2^*+T}{1-t_1^*-T} < \frac{L_1}{L_2} < \frac{\Gamma}{\beta} \frac{1-t_2^*+T}{1-t_1^*-T} \quad (28)$$

Because  $t_1^* > t_2^*$ ,  $(1-t_2^*+T)/(1-t_1^*-T) > 1$ . This means that the range of the population ratio in which each region specializes the production shifts upward compared with section 3. The ratio of the utility in two regions is

$$v = \frac{V_1}{V_2} = \left[ \frac{1-t_2^*+T}{1-t_2^*-T} \right]^{1-p} \left[ \frac{t_1^*}{t_2^*} \right]^{\gamma p} \left[ \frac{L_1}{L_2} \right]^{(\gamma+1)p-1} \quad (29)$$

In the equilibrium, the utility is equal across regions. Then, the ratio of the equilibrium populations is

$$\frac{L_1^*}{L_2^*} = \left[ \left( \frac{1-t_2^*-T}{1-t_2^*+T} \right)^{1-p} \left( \frac{t_2^*}{t_1^*} \right)^{\gamma p} \right]^{\frac{1}{(\gamma+1)p-1}} \quad (30)$$

When  $(\gamma + 1)p - 1 > 0$ ,  $L_1^*/L_2^* < 1$  and the equilibrium populations are unstable. Moreover, if  $\beta$  is sufficiently large, the equilibrium populations satisfy the condition (28). When  $(\gamma + 1)p - 1 < 0$ ,  $L_1^*/L_2^* > 1$  and the equilibrium populations are stable. If the difference between  $\Gamma$  and  $\beta$  is sufficiently large, the equilibrium populations satisfy the condition (28). I assume that these conditions hold.

Second, I explain the case where region 1 produces two private goods. From (28), if  $\frac{L_1}{L_2} \geq \frac{\Gamma}{\beta} \frac{1-t_2^*+T}{1-t_1^*-T}$ , the price ratio  $P_1/P_2$  equals  $1/\Gamma$  and region 1 produces two private goods. In this case, the indirect utility functions in each region are

$$\begin{aligned} V_1 &= (1 - t_1 - T)^{1-p} (t_1 L_1)^{\gamma p} \left( \frac{\Gamma \beta^2}{4} \right)^{\frac{1-p}{2}} G \\ V_2 &= (1 - t_2 - T)^{1-p} (t_2 L_2)^{\gamma p} \left( \frac{\Gamma^3}{4} \right)^{\frac{1-p}{2}} G \end{aligned}$$

Utilizing the local government's utility maximizing behavior, the tax rate in region  $i$  is

$$t_i = \frac{\gamma p(1 - T)}{1 - p + \gamma p}$$

Using the indirect utility function and the tax rate, the ratio of the utility is

$$v = \frac{V_1}{V_2} = \left( \frac{\beta}{\Gamma} \right)^{1-p} \left[ \frac{L_1}{L_2} \right]^{\gamma p}$$

This is the same function as that in section 3. Therefore, the analysis in the second case does not change except for the condition that the populations must be satisfied.

Third, consider that region 2 produces two private goods. From (28), if  $\frac{L_1}{L_2} \leq \frac{1}{\Gamma \beta} \frac{1-t_2^*+T}{1-t_1^*-T}$ , the price ratio  $P_1/P_2$  equals  $\Gamma$  and region 2 produces two private goods. In this case, the indirect utility functions in each region are

$$\begin{aligned} V_1 &= (1 - t_1 - T)^{1-p} (t_1 L_1)^{\gamma p} \left( \frac{\Gamma^3 \beta^2}{4} \right)^{\frac{1-p}{2}} G \\ V_2 &= (1 - t_2 - T)^{1-p} (t_2 L_2)^{\gamma p} \left( \frac{\Gamma}{4} \right)^{\frac{1-p}{2}} G \end{aligned}$$

Like the second case, the tax rate in region i is

$$t_i = \frac{\gamma p(1-T)}{1-p+\gamma p}$$

Then, the ratio of the utility is

$$v = \frac{V_1}{V_2} = (\Gamma\beta)^{1-p} \left[ \frac{L_1}{L_2} \right]^{\gamma p}$$

This is the same function as that in section 3. Therefore, the analysis in the third case does not change except for the condition that the populations must be satisfied.

Combining three cases, I analyze the equilibrium populations. Like the analysis in section 3, I suppose that the difference between initial populations in each region is not so large. If  $(\gamma+1)p-1 > 0$ ,  $L_1^*/L_2^* < 1$  is the equilibrium but unstable. When  $L_1 > L_1^* (< \bar{L}/2)$ , all individuals concentrate in region 1 in the equilibrium. Comparing to section 3, the possibility that full agglomeration in region 1 increases. If  $(\gamma+1)p-1 < 0$ ,  $L_1^*/L_2^* > 1$  is the stable equilibrium. Comparing to section 3, these results are summarized as Proposition 2.

**Proposition 2** Suppose that the difference between initial populations in each region is not so large and the central government locates in region 1. If the effect of the local public good is larger than that of private goods ( $(\gamma+1)p-1 > 0$ ), the possibility that all individuals concentrate in region 1 is stronger than the case where the central government does not exist.

Conversely, if  $(\gamma+1)p-1 < 0$ , the full agglomeration does not emerge. In the stable equilibrium, the population in region 1 is larger than that in region 2.

When the effect of the local public good is larger ( $(\gamma+1)p-1 > 0$ ), the local government causes full agglomeration. The central government can not stop the agglomeration. But, the existence of the central government promotes utilization of region 1's production technology that is superior to region 2. In region 1, the wage is higher than the case where the central government does not locate

because the central government increases the demand for labor in region 1. Therefore, the central government increases the possibility of full agglomeration in region 1.

When the effect of private goods is larger ( $(\gamma + 1)p - 1 < 0$ ), the local government does not cause agglomeration. Moreover, the central government does not cause full agglomeration. The equilibrium population in region 1 is larger than that in region 2 because of the central government's labor demand.

Figure 1 and Figure 2 depict the ratio of utility where the difference between the populations is not so large. In Figure 1,  $p = 0.6$ ,  $\Gamma = 12$ ,  $\beta = 5$ ,  $\gamma = 1$  and  $T = 0.04$  when the central government exists. In Figure 2,  $p = 0.4$  and the other parameters do not change. The solid line represents the case where the central government exists and the dotted line represents the case when it does not. Figure 1 shows the case  $(\gamma + 1)p - 1 > 0$ . Introducing the central government, the graph shifts upward and the ratio of the unstable equilibrium population decreases. Then, the range within which all individuals concentrate in region 1 becomes larger. Figure 2 shows the case  $(\gamma + 1)p - 1 < 0$ . When the central government is introduced, the graph shifts upward and the interior equilibrium population in region 1 increases.

## 5 Conclusion

This paper analyzes the effect of local and central government on agglomeration. The local government can cause agglomeration by providing the local public good. When the central government provides the pure public good without the regional redistribution policy, whether or not it improves the relationship between the local government and the agglomeration?

When the weight of the local public good in the utility is large relative to that of private goods and the local government's productivity is larger, the local government causes full agglomeration so that all individuals concentrate in one region. In terms of the production of private goods, this

agglomeration may be undesirable. The central government can not stop the agglomeration. But it lowers the possibility of such undesirable agglomeration. When the weight of private goods in the utility is large relative to that of the public good, the local government does not cause agglomeration. In this case, the central government causes partial agglomeration so that the population of one region is larger than that of the other region. But the central government does not cause full agglomeration.



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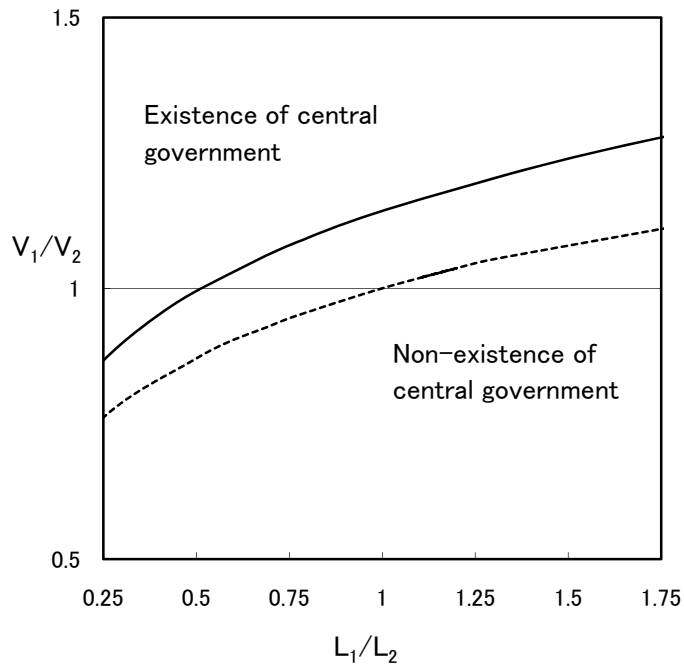


Figure 1 Ratio of utility ( $2p-1 > 0$ )

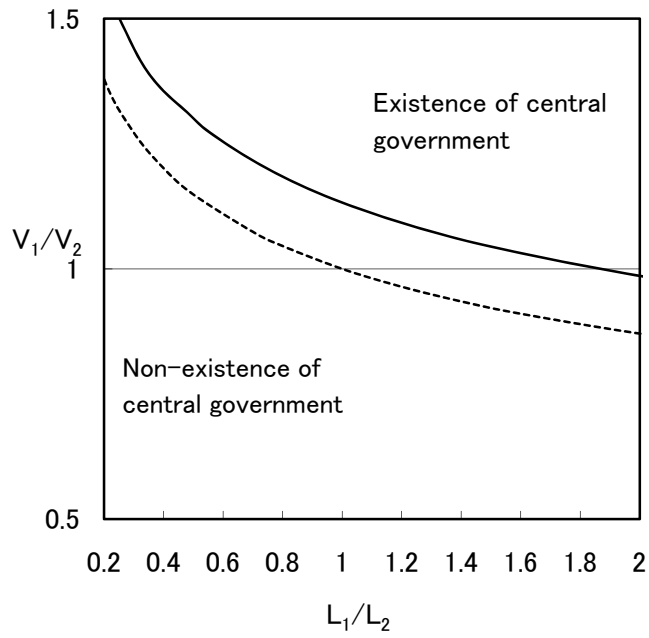


Figure 2 Ratio of utility ( $2p-1 < 0$ )