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The optimal study support interval policy in e-learning

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Abstract:

If a lecture does not provide the facing study support to the student in an e-learning system, the learner has the possibility of dropping out of the target subject to be studied. However, if the lecture indeed provides such support to the learner every time, a problem occurs from the viewpoint of its cost-effectiveness. In this paper, we consider the optimal facing study support interval for the learner and derive analytically the optimal interval study support policy by a stochastic model using access LOG of the e-learning system to contents.

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1 Introduction

High efficiency education and *Efficient education* are responsibilities of every school and society [1]. In the result valuing type society, the object of evaluation is whether it reache a high target level with little effort. Because education is the external economy, the education has been thought to be not related with marginal utility in economics up to now. If the lecturer does not provide the study support to the learner of the e-learning system, the learner often drops out of the study. The problem in study causes study stagnation due to changes in the life environment. Then, when study stagnation is prolonged, student's motivation decrease [3]. Therefore, it is an important role of an appropriate frequency to do the study support so that the lecturer may lead to the finishing completion in the e-learning system. In this paper, when the lecturer uses the e-learning system, we consider how to give the learner efficient study support. In general, the content of study support is different depending on the lecture. Therefore, neither the content, method nor level of study support are considered; instead we consider the frequency of the study support. In addition, we consider that do the learner's understanding leve to the number of access logs of the e-learning system. That is, in the e-learning system of self-study, if the access to a certain learner's e-learning system increases, we consider that learner's master items has risen. Conversely, we assume that the learner masters few items when there is little access in the e-learning system. We use the study support history data in the e-learning systems and it is necessary to decide the study support frequency, and to reduce the work burden of the lecturer.

2 e-learning

Suppose that a learner should master the total number of items $K(>0)$ in the e-learning system for period $[0,S]$ and the number of master items at time t is At in proportion to time. If the number of master items at the final date S is $AS \geq K$ in the e-learning system, the study support is effective. Conversely, if the number of master items at S is $AS < K$, the study support is ineffective. Thus, assume that A is the random variable with mean $E\{A\} = a > 0$ and a probability $G(x) \equiv \Pr\{A \leq x\}$.

The probability that a learner can achieve the number of items K for period S is

$$\Pr\{AS > K\} = \Pr\{A > K/S\} = 1 - G(K/S). \tag{1}$$

The probability that a learner cannot achieve K for S is

$$\Pr\{AS \leq K\} = \Pr\{A \leq K/S\} = G(K/S). \tag{2}$$

3 Optimal Study Support Policy

A learner has to achieve the finishing regulation number K of items for a period S. The lecturer checks the number of the learner's finishing of items at $jT(j = 1, 2, \ldots, N)$ intervals where $NT = S$ and supports a lecture's study. In this study support system, it is assumed that the master item numbers $b(> 0)$ are effective by the lecture's support at every jT.

The probability of achieving the regulations finishing number K of items until time S is

$$\Pr\{AS + Nb > K\} = \Pr\left(A > \frac{K - Nb}{S}\right) = 1 - G\left(\frac{K - Nb}{S}\right), \tag{3}$$

and the probability of no achieving N is

$$\Pr\{AS + Nb \leq K\} = G\left\{\frac{K - Nb}{S}\right\}. \tag{4}$$

Here, we introduce the following costs:

c_1 : Cost when a learner of e-learning system cannot achieve the regulated master item K for period S.

c_2 : Cost for which the lecturer checks a learner in the e-learning system and study support is executed.

The total cost of a learner in this e-learning system is:

$$C(N) = c_1 G\left\{\frac{K - Nb}{S}\right\} + Nc_2 \quad (N = 0, 1, 2, \ldots). \tag{5}$$

We seek an optimal number $N^*(N^* = 1, 2, \ldots)$ that minimize $C(N)$ in (5), *i.e.,* the number N^* by which the lecturer can supports a learner by a minimum number. From $Nb \leq K$, it is clear that $N \leq K/b$. Thus, we may obtain N^* for $N = 0, 1, 2, \ldots, [K/b]$.

Letting $Nb \equiv x$, then $0 \leq x \leq K$, from(5),

$$\begin{aligned}\tilde{C}(x) \equiv \frac{bC(N)}{c_2} &= b\frac{c_1}{c_2}G\left\{\frac{K-x}{S}\right\} + x, \\ \tilde{C}(0) &= \frac{bC(N)}{c_2}G\left\{\frac{K}{S}\right\}, \\ \tilde{C}(K) &= K.\end{aligned} \tag{6}$$

Differenticting $\tilde{C}(x)$ with respect to x and setting it equal to zero,

$$g(\frac{K-x}{S}) = \frac{c_2 S}{c_1 b} \tag{7}$$

4 Numerical Example

4.1 Exponential Case

Suppose that A is a random variable with an exponential distribution $G(x) = 1 - \mathrm{e}^{-x/a}$ and $aS = K = 10$, *i.e.,* A learner has to usually master 10 items until time $10/a$. Then, from (7),

$$\frac{1}{a}\mathrm{e}^{-\frac{K-x}{aS}} = \frac{c_2 S}{c_1 b}. \tag{8}$$

Thus, $\tilde{C}(x)$ is strictly increasing function by x.

(i) If $\frac{1}{a} \leq \frac{c_2 S}{c_1 b}$ then $\tilde{C}(x)$ is strictly increasing in x and $x^* = 0$, *i.e.,* $N^* = 0$.

(ii) If $\frac{1}{a} > \frac{c_2 S}{c_1 b}$ then there exists a finite an unique x^* $(0 < x^* < \infty)$ which satisfies (8), and $N^* = \left[\frac{x}{b}\right]$ or $\left[\frac{x}{b}\right] + 1$ where $[x]$ is the greatest integer contained in x. In case of $N^* > k/b$, set that $N^* = K/b$.

The probability that the learner can master the regulated number of master items by the e-learning system is

$$\Pr\{A > K/S\} = \mathrm{e}^{-K/aS} = \mathrm{e}^{-1} = 0.3678. \tag{9}$$

From (6), we seek N^* $(0 \leq N^* \leq K/a)$ which minimizes

$$\frac{C(N)}{c_2} = \frac{c_1}{c_2}\left(1 - \mathrm{e}^{-\frac{10-Nb}{10}}\right) + N(N = 0, 1, 2, [K/b]) \tag{10}$$

For example, when $aS = K = 10$ and $b = 1$, from (10),

$$\begin{aligned} \frac{C(N)}{c_2} &= \frac{c_1}{c_2}\left(1 - \mathrm{e}^{-\frac{10-N}{10}}\right) + N, \\ \frac{C(0)}{c_2} &= \frac{c_1}{c_2}\left(1 - \mathrm{e}^{-1}\right), \\ \frac{C(1)}{c_2} &= \frac{c_1}{c_2}\left(1 - \mathrm{e}^{-\frac{9}{10}}\right) + 1. \end{aligned}$$

That is,

$$\frac{C(0)}{c_2} \geq \frac{C(1)}{c_2} \Longrightarrow \frac{c_1}{c_2} \geq \frac{1}{\mathrm{e}^{-0.9} - \mathrm{e}^{-1}} = 25.846405.$$

Then, if $c_1/c_2 \geq 25.8$ then $N = 1$ is beter than $N = 0$, that is better than the lecturer doesn't suport. In addition,

$$\begin{aligned} \frac{C(10)}{c_2} &= 10, \\ \frac{C(9)}{c_2} &= \frac{c_1}{c_2}\left(1 - \mathrm{e}^{-1/10}\right) + 9, \end{aligned}$$

then,

$$\frac{C(10)}{c_2} \geq \frac{C(9)}{c_2} \Longrightarrow \frac{c_1}{c_2} \leq 10.51.$$

We obtain, if $c_1/c_2 < 10.51$ then $N^* \leq 9$.

4.2 Normal Distribution Case

Assume that a random variable A has a normal distribution $N(a, \sigma^2/S)$ and $aS = K = 10$. Then,

$$G\left(\frac{K - Nb}{S}\right) = \Phi\left(\frac{(K - Nb)/S - a}{\sigma/\sqrt{S}}\right) = \Phi\left(\frac{-Nb}{\sigma\sqrt{S}}\right), \tag{11}$$

where a standard normal distribution $\Phi(x)$ is $N(0, 1)$. From (5), we seek an $N^*(0 \leq N^* \leq K/b)$ which minimizes

$$\frac{C(N)}{c_2} = \frac{c_1}{c_2}\Phi\left(\frac{-Nb}{\sigma\sqrt{S}}\right) + N \tag{12}$$

Letting $Nb = x$,

$$\tilde{C}(x) = \frac{bC(x/b)}{c_2} = \frac{bc_1}{c_2}\Phi(\frac{-x}{\sigma\sqrt{S}}) + x \quad (0 \leq x \leq K). \tag{13}$$

Differentiating $\tilde{C}(x)$ with respect to x and setting it equal to zero,

$$\frac{1}{\delta\sqrt{S}}\phi(\frac{-x}{\delta\sqrt{S}}) = \frac{c_2}{bc_1}, \tag{14}$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{x^2}{2}}$.

(i) If $\dfrac{1}{\delta\sqrt{2\pi S}} \le \dfrac{c_2}{bc_1}$ then $x^* = 0$, *i.e.*, $N^* = 0$.

(ii) If $\dfrac{1}{\delta\sqrt{2\pi S}} > \dfrac{c_2}{bc_1}$ then there exists a finite and unique x^* $(0 < x^* < \infty)$ which satisfies (14), and $N^* = \left[\dfrac{x}{b}\right]$ or $N^* = \left[\dfrac{x}{b}\right] + 1$, In case of $N^* > K/b$, set that $N^* = K/b$.

From case (ii), if $c_2/c_1 > aS/b$, then $N^* \ge 1$. For example, in particular, when $N = 0$, we have

$$C(0) = \frac{c_1}{2}. \tag{15}$$

Table 1: Optimal study support number $b = 1$, $\sigma = 1$, $K = 10$, $S = 15$, $a = 0.5$

c_1/c_2	N^*
10	1
20	4
30	5
50	7

Table 1 shows that, if c_1/c_2 becomes large, then the lecturer should increase the frequency of study support. If the cost becames large when the learner cannot achieve the regulated master item for a period, it is shown that we should frequently provide study support. For example, when $c_1/c_2 = 20$, the lecturer should provide four times the study support. Let cost c_1 when the learner cannot achieve the regulated master item for the period and the ratio to c_2 be 20 times the cost of the check of learner's master situation and the study support. Then, we should provide study support once almost every four weeks (15/4=3.75).

5 Conclusion

In this paper, we assume that the number of accumulation accesses to the e-learning system is linear. It is necessary to verify the linear relation between the number of accumulation accesses and master items in the e-learning system. In the proposed model, the optimal study support interval can be determined analytically by the linear of accumulation access to the e-learning system. However, we do not consider the form and the content of the study support, and there is a support method according to the master progress in the content of the learner's study support. In addition, it is a problem the necessity of every time the same cost though the content of the study support is different.

As a future problems, we should analyze the characteristics of a distributed learner's master situation and cost of the learner's study support and compare them.

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