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Altruism, Liquidity Constraint, and Education Investment[*]

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Abstract

In Japan and other East Asian societies, the household's education expenditure (especially the private tutoring expenditure) has sharply increased. The purpose of this paper is to provide a rationale for the fact that a number of families very actively invest in education. Introducing altruism and liquidity constraints into a simple parent-child model, we show that the investment in education can be too much or too little, depending on the income level of the family. Our model also has implications for the rotten kid theorem (Becker, 1974). There exist families in which the parental welfare is higher in the equilibrium than in the parent's second best if the liquidity constraint is binding.

JEL classification: I2; D1

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1. Introduction

Many societies are experiencing an aging population due to falling fertility rates as well as increasing longevity. Japan, in particular, provides an extreme case of fertility rates falling dramatically, and it is often pointed out that one of the crucial factors in such a trend is the increasing cost of educating a child. The education expenditure per household in Japan continued to increase until the end of the 1990s, but has stabilized since then. However, noting the fact that the number of children per household has fallen consistently, we can see that the education expenditure per child still continues to increase. In addition, the total expenditure on private supplementary tutoring accounts for quite a large share of the total education expenditure. Such a phenomenon prevails also in other East Asian societies such as South Korea, Taiwan and Hong Kong (Bray and Kwok, 2003). The fact that private tutoring is freely chosen by households suggests that they very actively invest in education.

To evaluate whether or not households expend too much on private education, the rate of return to investment in education should provide useful information. Based on Japanese cross-sectional data from 1986 to 1995, Arai (2001) finds that the average internal rate of return to a university education is 5.93-6.42% for women and 4.81-5.36% for men; likewise for Japan, the Cabinet Office, Government of Japan (2005) estimates that the rate of return to a university education for men born in 1975 is 5.7%. In other countries, a large number of studies have been conducted since the late 1950s. Psacharopoulos and Patrinos (2004), who review the recent empirical results for a wide variety of countries, summarize that the world average rate of return to an additional year of schooling is 10%, which is higher than the average for the high-income countries of the OECD. According to the cross-country analysis by Trostel et al. (2002), the rate of return to schooling is less than 4% for several countries such as Germany (West), Netherlands, Norway, Sweden and Canada. From these results, it is difficult to conclude that the rate of return to investment in education is disproportionately high or low relative to investments in physical capital. On this point, we should note that several factors causing an upward bias in estimations of the rate of return to education investment have been pointed out, such as the correlation between years of schooling and the innate ability to earn income, the effects of liquidity constraints on education decisions, and the direct costs of education (including private tutoring).[1] Furthermore, the downward trend in the rate of return to education (Psacharopoulos and Patrinos, 2004; Cabinet Office, Government of Japan, 2005) implies that children today may face lower rates of return than those estimated in previous studies.

Despite these findings, most of the economic literature argues that private investment in education tends to be insufficient due to the external effect on economic growth, the liquidity constraint (Barham et al., 1995; De Fraja, 2002), the family constitution (Balestrino, 1997; Anderberg and Balestrino, 2003), and the strategic bequest motives (Cremer and Pestieau, 1992). An exception is Cremer and Pestieau (2006), who consider the joy of giving to be the motivation behind parental involvement in their children's education. They show that, if the joy of giving term is not included in the social welfare function, parents may invest more than the social optimum in their children's education.

In order to investigate the education decisions of a family, we consider a simple model of families, each of which consists of one parent and one child. Key features of the model are as follows. First, the parents are purely altruistic toward their child, and this motivates them to be involved in their child's education. Second, the parents differ in income, which is exogenously determined. Third, the children can borrow to finance education investment (and consumption) within limits that differ among children, depending on their parent's income: a child whose parent earns a higher income can borrow a larger amount.

Fourth, while the children choose the level of their education investment,

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[1] In many studies (especially those using the Mincer specification), the cost of education is measured by forgone earnings alone.

its cost is shared between them and their parent, with the share being determined by the parent, who chooses how much to pay for the cost of her child’s education. (The children have to borrow to pay their share of the cost.) Who determines the level of education investment in a family is a modeling issue. In the literature, while Balestrino (1997), De Fraja (2002), Anderberg and Balestrino (2003), and Cremer and Pestieau (2006) suppose that the parents decide, Barham et al. (1995) and Boldrin and Montes (2005) suppose that the children do. In our model, while the children’s education investment is their own choice, the parents can indirectly influence it through their decisions on the share of the cost. This means that the education investment is determined as a result of interactions between parents and children. Such a set-up seems to be in line with the practice in many societies, and also has a significant effect on the results obtained below.

Fifth, while the children’s wage income is determined by their education investment, the parents make transfers to their child after the children’s wage income has been realized. Such ex-post transfers, which are motivated by parental altruism, provide an incentive for the child to over-consume in her youth so as to subsequently receive more transfers, thus engendering the Samaritan’s dilemma (Buchanan, 1975; Lindbeck and Weibull,1988).

Our main results are as follows. First, the investment in education can be too much or too little, depending on the family income.[2] We can distinguish three categories of families, according to their income level. In families belonging to the first category, who are the wealthy and thus not liquidity constrained in the equilibrium, the level of education investment is either equal to or higher than the parental first-best level. For families belonging to the second category, who are the middle class and are liquidity constrained, the level of education investment is higher than the parental first-best level. In families belonging to the third category, who are the poor and highly liquidity

[2] Barham et al. (1995) and De Fraja (2002) also consider a variation in income among families, and show the effect of parental income on education investments in relation to liquidity constraints. However, excessive education investments never arise in these studies.

constrained, the level of education investment is lower than the parental first-best level.

While families in the first and second categories (namely, the high and middle income classes) may invest too much in a child's education, the reason differs between the two categories. For a family in the first category, if the ex-post transfers are operative, the child chooses the efficient level of education investment because her liquidity constraint is not binding, but the Samaritan's dilemma then arises. On the other hand, if the parent makes sufficiently large transfers in the form of education expenditures and the ex-post transfers are made inoperative, the efficient intertemporal allocation of consumption is achieved, but the child chooses too much education investment. As for a family in the second category, since the liquidity constraint is binding, an efficient consumption allocation is never achieved when ex-post transfers are inoperative. Moreover, the child must marginally adjust her consumption allocation through the education investment. Therefore, the level of education investment that attains an efficient consumption allocation generally does not coincide with its first-best level (namely, the level where the marginal return to education equals the market interest rate). Under the first-best level of education investment, since the Samaritan's dilemma arises in a family in the second category, the parent behaves in a way to induce her child to pursue higher education. This is because the education investment reallocates resources forwards, and counteracts the Samaritan's dilemma.

The results obtained in this paper also have implications for the rotten kid theorem (Becker, 1974). In some families with binding liquidity constraint, the parental welfare in the equilibrium is higher than that in the parent's second best subject to the liquidity constraint.

2. Model

Consider an economy that consists of two generations: the parents' generation and that of the children. A parent lives for three periods of equal length: youth (period 0), middle-age (period 1) and old-age (period 2), and a child also lives for three periods: youth (period 1), middle-age (period 2) and old-age (period 3), with an overlapping of periods 1 and 2. Each member of the parents' generation is heterogeneous with respect to their income level. The population of the parents' generation is N, and each parent produces one child exogenously.

We focus on the periods in which two generations overlap, i.e., periods 1 and 2. In period 1, the parent in family i allocates her income $Y_{p,i}$, which is determined by the education investment made in period 0 (and thus exogenous in period 1), among consumption $C^1_{p,i}$, savings $S_{p,i}$ and financial contributions to the cost of the child's education. We assume that the investment in the child's education k_i is partly financed by the parent, and that the child finances the rest. In period 2, the parent observes the child's income, and allocates her savings carried over from period 1 between her own consumption $C^2_{p,i}$ and ex-post transfers toward her child $A_i (\geq 0)$. Thus, the parent's budget constraints in periods 1 and 2 are

(1) $$Y_{p,i} = C^1_{p,i} + S_i + p_i k_i, \qquad (i = 1, \ldots N)$$

(2) $$(1+r)S_i = C^2_{p,i} + A_i, \qquad (i = 1, \ldots N)$$

where p_i $(0 \leq p_i \leq 1)$ is the parental share of education expenditure, and r is the interest rate, which is determined exogenously.

The child has no income in period 1, and thus must borrow to finance consumption $C^1_{k,i}$ and education expenditure $(1-p_i)k_i$. In period 2, the child receives her income $Y_{k,i}$, which is a function of k_i satisfying $Y'_{k,i}(k_i) > 0,\ Y''_{k,i}(k_i) < 0$ and $\lim_{k_i \to 0} Y'_{k,i}(k_i) = \infty$. She repays the borrowings out of the sum of her income and the ex-post transfers from her parent, and allocates the rest between consumption $C^1_{k,i}$ and savings $S_{k,i}$. Thus the child's budget constraints in periods 1 and 2 are

(3) $$D_i = C^1_{k,i} + (1-p_i)k_i, \qquad (i = 1, \ldots N)$$

(4) $$Y_{k,i}(k_i) - (1+r)D_i + A_i = C^2_{k,i} + S_{k,i}, \qquad (i = 1, \ldots N)$$

where D_i is the child's borrowings. Without a loss in the generality of the model, we can neglect the children's old-age (period 3). Namely, $S_{k,i} = 0$ is assumed hereafter. We assume that the amount the child can borrow has an upper limit $\bar{D}_i$, which depends positively on her parent's income:

(5) $$D_i \le \bar{D}(Y_{p,i}), \quad \bar{D}'(Y_{p,i}) > 0, \quad (\, i = 1,...N\,) \,.$$

The parent is altruistic and her utility function is given by

(6) $$U_{p,i} = u_p(C^1_{p,i}) + v_p(C^2_{p,i}) + \delta U_{k,i},$$

where δ is the weight attached to her child's utility $U_{k,i}$. We assume that $u'_p > 0$, $u''_p < 0$, $\lim\limits_{C^1_{p,i}\to 0} u'_p(C^1_{p,i}) = \infty$, $v'_p > 0$, $v''_p < 0$ and $\lim\limits_{C^2_{p,i}\to 0} v'_p(C^2_{p,i}) = \infty$.

The child is selfish and cares only about her own consumption, and her utility function is given by

(7) $$U_{k,i} = u_k(C^1_{k,i}) + v_k(C^2_{k,i})\,.$$

We assume that $u'_k > 0$, $u''_k < 0$, $\lim\limits_{C^1_{k,i}\to 0} u'_k(C^1_{k,i}) = \infty$, $v'_k > 0$, $v''_k < 0$ and $\lim\limits_{C^2_{k,i}\to 0} v'_k(C^2_{k,i}) = \infty$. We hereafter omit the subscript i as long as that does not cause a misunderstanding.

The timing of the game is as follows: (i) the parent chooses C^1_p, S and p; (ii) the child chooses C^1_k, D and k; (iii) the child's income $Y_{k,i}$ is realized, and the parent chooses C^2_p and A. (As a result, C^2_k is determined.)

3. First best for the parent

As a benchmark, we start by deriving the first-best allocation for the parent in a family. The parent, who implements the optimal allocation with respect to $\left\{C^1_p,\, C^2_p,\, C^1_k,\, C^2_k,\, k\right\}$, maximizes her utility subject to the overall feasibility constraint of her family:

(8) $$\max_{C^1_p, C^2_p, C^1_k, C^2_k, k} \; u_p(C^1_p) + v_p(C^2_p) + \delta[u_k(C^1_k) + v_k(C^2_k)]$$

(9) $$s.t. \;\; C^1_p + \frac{C^2_p}{1+r} + C^1_k + \frac{C^2_k}{1+r} + k = Y_p + \frac{Y_k(k)}{1+r}\,.$$

The first-order conditions (FOCs) for this problem are given by

(10) $$u_p'(C_p^1) = \delta u_k'(C_k^1),$$

(11) $$v_p'(C_p^2) = \delta v_k'(C_k^2),$$

(12) $$\frac{u_p'(C_p^1)}{v_p'(C_p^2)} = \frac{u_k'(C_k^1)}{v_k'(C_k^2)} = 1 + r,$$

(13) $$Y_k'(k) = 1 + r.$$

The optimality conditions (10)-(13) and the feasibility condition (9) determine the first-best allocation for the parent. In the rest of the paper, the first best is denoted by the superscript F.

4. Families with non-binding liquidity constraint

From now on, we examine the behavior of families in the competitive equilibrium. Since the parental income Y_p differs with each family, we can consider two types of families: one with non-binding liquidity constraint and the other with binding liquidity constraint. (We will show in section 6 that families with relatively lower Y_p face the binding liquidity constraint, while those with relatively higher Y_p do not.) In this section, we deal with families whose liquidity constraint is not binding.

4.1. Second and Third Stages: Ex-post transfers, borrowings and education investment

We first examine the optimizing behavior of the parent in a family with non-binding liquidity constraint at the third stage of the game. In period 2, given k, D and S, the parent transfers A toward her child so as to maximize $v_p((1+r)S - A) + \delta v_k(Y_k(k) - (1+r)D + A)$ subject to the non-negativity constraint on A. The FOC is

(14) $$-v_p'((1+r)S - A) + \delta v_k'(Y_k(k) - (1+r)D + A) \le 0 \quad \text{(with equality if } A > 0\text{)}.$$

From (14), we obtain the parent's reaction function:

(15) $$A = A(k,\ D,\ S) = \begin{cases} A^+(k,\ D,\ S), & \text{if (14) holds with equality,} \\ 0, & \text{if (14) holds with strict inequality.} \end{cases}$$

The properties of $A(k,\ D,\ S)$ are summarized as follows:

(16) $$\frac{\partial A^{+}}{\partial k} = -\eta Y_k'(k) < 0\,,$$

(17) $$\frac{\partial A^{+}}{\partial D} = (1+r)\eta > 0\,,$$

(18) $$\frac{\partial A^{+}}{\partial S} = (1-\eta)(1+r) > 0\,,$$

where

$$\eta \equiv \frac{\delta v_k''}{v_p'' + \delta v_k''} \;\; (0 < \eta < 1)\,.$$

We next examine the second stage of the game. In period 1, anticipating the parent's reaction function (15), the child solves the following problem, given p and S:

$$\max_{D,\,k}\; u_k(D-(1-p)k) + v_k(Y_k(k)-(1+r)D + A(k,\,D,\,S))$$
$$s.t.\;\; D \le \bar{D}(Y_p).$$

Since we suppose the liquidity constraint is not binding in this section, the FOCs for this problem are given by

(19) $$u_k'(D-(1-p)k) - v_k'(Y_k(k)-(1+r)D + A(k,\,D,\,S)) \cdot \left[(1+r) - \frac{\partial A}{\partial D}\right] = 0\,,$$

(20) $$-u_k'(D-(1-p)k)\cdot(1-p) + v_k'(Y_k(k)-(1+r)D + A(k,\,D,\,S)) \cdot \left[Y_k'(k) + \frac{\partial A}{\partial k}\right] = 0\,.$$

Equations (19) and (20) imply

(21) $$Y_k'(k) - (1+r)(1-p) = 0\,.$$

From (19) and (21), we obtain the child's reaction functions:

(22) $$D = D(p,\,S) = \begin{cases} D^{+}(p,\,S), & \text{if } \partial A/\partial D = \partial A^{+}/\partial D, \\ D^{0}(p,\,S), & \text{if } \partial A/\partial D = 0, \end{cases}$$

(23) $$k = k(p,\,S) = \begin{cases} k^{+}(p,\,S), & \text{if } \partial A/\partial k = \partial A^{+}/\partial k, \\ k^{0}(p,\,S), & \text{if } \partial A/\partial k = 0. \end{cases}$$

The properties of $D(p,\,S)$ and $k(p,\,S)$ are summarized as follows:

(24) $$\frac{\partial D^{+}}{\partial p} = -k\rho - \frac{Y_k'(k)}{Y_k''(k)}\,,$$

(25) $$\frac{\partial D^+}{\partial S} = 1 - \rho > 0,$$

(26) $$\frac{\partial D^0}{\partial p} = -\frac{Y_k'(k)}{Y_k''(k)} > 0,$$

(27) $$\frac{\partial D^0}{\partial S} = 0,$$

(28) $$\frac{\partial k^+}{\partial p} = \frac{\partial k^0}{\partial p} = -\frac{1+r}{Y_k''(k)} > 0,$$

(29) $$\frac{\partial k^+}{\partial S} = \frac{\partial k^0}{\partial S} = 0,$$

where

$$\rho \equiv \frac{u_k''}{u_k'' + (1-\eta)^2(1+r)^2 v_k''} \ (0 < \rho < 1).$$

When $A > 0$, from (19) and $\partial A / \partial D = \partial A^+ / \partial D > 0$, we obtain

(30) $$u_k'(C_k^1) - (1+r)v_k'(C_k^2) < 0,$$

from which, we obtain the following proposition:

Proposition 1 (Lindbeck and Weibull, 1988).

If $A > 0$, then the child in families with non-binding liquidity constraint over-consumes in period 1. Hence, the Samaritan's dilemma arises in the competitive equilibrium.

Furthermore, comparing (13) with (21) yields the following proposition:

Proposition 2.

If $p = 0$, then the child chooses her parent's first-best level of education investment. If $p > 0$, the child chooses the level of education investment higher than her parent's first-best level.

4.2. First Stage: Parental savings and parental share of education

expenditures

We now examine the optimizing behavior of the parent at the first stage. The parent maximizes

$$
\begin{aligned}
U_p &= u_p[Y_p - S - pk(p,\, S)] \\
&\quad + v_p[(1+r)S - A(k(p,\, S),\, D(p,\, S),\, S)] \\
&\quad + \delta\{u_k[D(p,\, S) - (1-p)k(p,\, S)] \\
&\quad + v_k[Y_k(k(p,\, S)) - (1+r)D(p,\, S) + A(k(p,\, S),\, D(p,\, S),\, S)]\}
\end{aligned} \tag{31}
$$

with respect to S and p.

4.2.1. The case in which the non-negativity constraint on A is not binding

In this case, (31) is rewritten as

$$
\begin{aligned}
U_p &= u_p[Y_p - S - pk^+(p,\, S)] \\
&\quad + v_p[(1+r)S - A^+(k^+(p,\, S),\, D^+(p,\, S),\, S)] \\
&\quad + \delta\{u_k[D^+(p,\, S) - (1-p)k^+(p,\, S)] \\
&\quad + v_k[Y_k(k^+(p,\, S)) - (1+r)D^+(p,\, S) + A^+(k^+(p,\, S),\, D^+(p,\, S),\, S)]\}.
\end{aligned} \tag{32}
$$

Noting (29), we obtain the FOC for maximizing (32) with respect to S as

$$
\begin{aligned}
&-u_p' + v_p' \cdot [(1+r) - A_D^+ D_S^+ - A_S^+] \\
&+\delta\{u_k' \cdot D_S^+ - v_k' \cdot [(1+r)D_S^+ - A_D^+ D_S^+ - A_S^+]\} = 0.
\end{aligned} \tag{33}
$$

Using (14) with equality and (19) with $\partial A / \partial D = \partial A^+ / \partial D$, we can rewrite (33) as

$$
-u_p' + v_p' \cdot [(1+r) - A_D^+ D_S^+] = 0\,. \tag{34}
$$

From $A_D^+ D_S^+ > 0$, we derive the following proposition:

Proposition 3.

If $A > 0$, the parent over-consumes in period 1, relative to her first-best allocation.

Proposition 3 suggests that the child's strategic behavior causes distortions not only in her own consumption allocation but also in that of her parent's.

In order to derive the parental share of education expenditure in the equilibrium, we examine the form of (32) in the pU_p-plain. Differentiating

(32) with respect to p yields

$$\frac{\partial U_p}{\partial p} = -u_p' \cdot [k^+(p,\ S) + pk_p^+] - v_p' \cdot [A_k^+ k_p^+ + A_D^+ D_p^+]$$

(35)
$$+\delta\{u_k' \cdot [D_p^+ + k^+(p,\ S) - (1-p)k_p^+]$$
$$+v_k' \cdot [Y_k'(k)k_p^+ - (1+r)D_p^+ + A_k^+ k_p^+ + A_D^+ D_p^+]\}.$$

Substituting (19) with $\partial A / \partial D = \partial A^+ / \partial D$ and (20) with $\partial A / \partial k = \partial A^+ / \partial k$ into (35) yields

(36)
$$\frac{\partial U_p}{\partial p} = (-u_p' + \delta u_k')k - v_p' \cdot [A_k^+ k_p^+ + A_D^+ D_p^+] - pk_p^+ u_p'.$$

The first term in (36) represents the direct effect of an increase in p and the second term represents the effect of a decrease in A, which is the reaction to an increase in p. Both effects cancel each other out and the sum of the first and second terms always becomes zero.[3] The third term in (36) shows that, when p rises, the parent's utility decreases because the child increases the education investment, thus reducing the parent's consumption. Hence, from (36), we obtain

(37)
$$\left(\frac{\partial U_p}{\partial p}\right)_{p=0} = 0, \quad \left(\frac{\partial U_p}{\partial p}\right)_{p>0} = -pk_p^+ u_p' < 0.$$

4.2.2. The case in which the non-negativity constraint on A is binding

In this case, (31) is rewritten as

$$U_p = u_p[Y_p - S - pk^0(p,\ S)] + v_p[(1+r)S]$$

(38)
$$+\delta\{u_k[D^0(p,\ S) - (1-p)k^0(p,\ S)]$$
$$+v_k[Y_k(k^0(p,\ S)) - (1+r)D^0(p,\ S)]\}.$$

---

[3] Substituting (14) and (19) with $\partial A / \partial D = \partial A^+ / \partial D$ into (34) yields

$$-u_p' + \delta u_k' + v_p' \rho A_D^+ = 0.$$

Substituting the above equation into the first and second terms in (36) yields

$$(-u_p' + \delta u_k')k^+ - v_p' \cdot (A_k^+ k_p^+ + A_D^+ D_p^+)$$
$$= -v_p' \cdot [A_k^+ k_p^+ + A_D^+ (D_p^+ + \rho k^+)] = 0.$$

Using (27) and (29), we obtain the FOC for maximizing (38) with respect to S as

(39) $$-u'_p + (1+r)v'_p = 0.$$

Differentiating (38) with respect to p yields

(40) $$\frac{\partial U_p}{\partial p} = -u'_p \cdot (k^0 + pk^0_p) + \delta\{k^0 u'_k + [u'_k - v'_k \cdot (1+r)]D^0_p + [-u'_k \cdot (1-p) + v'_k \cdot Y'_k(k)]k^0_p\}.$$

Substituting (19) with $\partial A/\partial D = 0$ and (20) with $\partial A/\partial k = 0$ into (40) yields

(41) $$\frac{\partial U_p}{\partial p} = (-u'_p + \delta u'_k)k^0 - pk^0_p u'_p.$$

From (14) with strict inequality, (19) with $\partial A/\partial D = 0$ and (39), we have

(42) $$-u'_p + \delta u'_k < 0.$$

From (28), (41) and (42), we have

(43) $$\frac{\partial U_p}{\partial p} < 0,$$

for all p, when the non-negativity constraint on A is binding.

4.2.3. Competitive equilibrium

We define p_0 as p that satisfies (14) with equality when $A = 0$:

(44) $$-v'_p[(1+r)S] + \delta v'_k[Y_k(k(p_0, S)) - (1+r)D(p_0, S)] = 0.$$

Thus, $A = 0$ is the interior solution when $p = p_0$.[4]

Figure 1 shows that U_p jumps at $p = p_0$. While the Samaritan's dilemma arises for $0 \le p \le p_0$ because (14) holds with equality and thus $\partial A/\partial D = \partial A^+/\partial D > 0$ holds in (19), the dilemma is not present for $p_0 < p \le 1$ because (14) holds with strict inequality and thus $\partial A/\partial D = 0$ holds in (19). The resolution of the Samaritan's dilemma has a positive effect on the parental utility. On the other hand, k is excessive relative to the parental first best at $p = p_0$, and the increase in p from p_0 to $\lim_{\varepsilon \to 0}(p_0 + \varepsilon)$ raises k further. Although this has a negative effect on the parental utility, this effect is

[4] The proof of $p_0 < 1$ is shown in Appendix.

very small and can be neglected because ε is infinitely close to zero. Thus, the parental utility is higher when $p = \lim_{\varepsilon \to 0}(p_0 + \varepsilon)$ than when $p = p_0$. This is formally stated in the following lemma.

Lemma 1. $\lim_{\varepsilon \to 0}[U_p \big|_{p=p_0} - U_p \big|_{p=p_0+\varepsilon}] < 0$

Proof: See Appendix.

From (37), (43) and Lemma 1, we obtain the form of (31) as shown in Figure 1. The following proposition summarizes the results obtained in this section:

Proposition 4.
The parental share of education expenditure in the competitive equilibrium is either $p* = 0$ or $p* = p_0(+\lim_{\varepsilon \to 0} \varepsilon)$. If $p* = 0$, the child over-consumes in period 1 (giving rise to the Samaritan's dilemma) while the child chooses her parent's first-best level of education investment. If $p* = p_0(+\lim_{\varepsilon \to 0} \varepsilon)$, the child chooses the level of the education investment higher than her parent's first-best level while the child's consumption allocation is efficient.

This result is similar to that of Bruce and Waldman (1990) in that the parent is forced to choose between two types of inefficiency.[5]

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[5] Based on a model in which the parent chooses inter vivos transfers as well as bequests, and the child chooses not only savings but also actions that affect the level of family income, Bruce and Waldman (1990) show that, while the Samaritan's dilemma arises when bequests are operative, there is no Samaritan's dilemma but the rotten kid theorem fails when inter vivos transfers are operative but bequests are not.

5. Families with binding liquidity constraint

In this section, we examine the behavior of families whose borrowings take a corner solution.

5.1. Second and third stages: Ex-post transfers, borrowings and education investment

The third stage in this case is the same as that in the case of families with non-binding liquidity constraint described in the previous section, except that here the FOC with respect to A is assumed to be always satisfied with equality.[6]

In the second stage, the child in each family faces the following problem:

$$\max_{D,\, k}\ u_k(D-(1-p)k)+v_k(Y_k(k)-(1+r)D+A(k,\ D,\ S))$$
$$s.t.\ D \le \bar{D}(Y_p).$$

Since we suppose the liquidity constraint to be binding in this section, the FOCs for this problem are

$$u_k'(D-(1-p)k)-v_k'(Y_k(k)-(1+r)D+A(k,\ D,\ S))\cdot\left[(1+r)-\frac{\partial A}{\partial D}\right]>0, \tag{45}$$

$$-u_k'(D-(1-p)k)\cdot(1-p)+v_k'(Y_k(k)-(1+r)D+A(k,\ D,\ S))\cdot\left[Y_k'(k)+\frac{\partial A}{\partial k}\right]=0. \tag{46}$$

From (46), we obtain the child's reaction function:

$$k=k^{+}(p,\ S,\ \bar{D}(Y_p)), \tag{47}$$

where

$$\frac{\partial k^{+}}{\partial p}=\frac{u_k''k(1-p)-u_k'}{SOC(k)}>0, \tag{48}$$

$$\frac{\partial k^{+}}{\partial S}=\frac{-v_k''\cdot(1-\eta)^2(1+r)Y_k'}{SOC(k)}<0, \tag{49}$$

---

[6] While the Samaritan's dilemma disappears if the parents choose $A=0$ in the case of non-binding liquidity constraint, such a choice cannot lead to an efficient intertemporal consumption allocation in the case of binding liquidity constraint. We hence assume that A takes an interior solution in the equilibrium in this section.

(50) $$\frac{\partial k^{+}}{\partial \bar{D}}=\frac{u_k''\cdot(1-p)+v_k''\cdot(1+r)(1-\eta)^2 Y_k'}{SOC(k)}>0,$$

and $SOC(k)=u_k''\cdot(1-p)^2+v_k''\cdot(1-\eta)^2(Y_k')^2+v_k'\cdot(1-\eta)Y_k''<0$.

While (45) does not determine the sign of $u_k'(c_k^1)-v_k'(c_k^2)(1+r)$ in this case, (46) can be rewritten as

(51) $$\frac{u_k'(c_k^1)}{v_k'(c_k^2)}=\frac{(1-\eta)Y_k'(k)}{1-p},$$

and (51) derives the following proposition:

Proposition 5.

1. The child under-consumes in period 1 if $\dfrac{(1-\eta)Y_k'(k)}{1-p}>1+r$.

2. The child's consumption allocation is efficient if $\dfrac{(1-\eta)Y_k'(k)}{1-p}=1+r$.

3. The child over-consumes in period 1 if $\dfrac{(1-\eta)Y_k'(k)}{1-p}<1+r$.

In the last case in Proposition 5, the Samaritan's dilemma arises even though the liquidity constraint is binding. This is because the strategic incentive of the child to obtain more transfers from the parent is strong enough to overcome the effect of the binding liquidity constraint.

Equations (45) and (46) imply

(52) $$Y_k'(k)>(1-p)(1+r),$$

which derives the following proposition on the education investment:

Proposition 6.

If $p=0$, the child chooses the level of education investment lower than the parental first-best level.

It should be noted that whether the level of education investment is too high or too low is indeterminate if $p>0$.

5.2. First Stage

In the first stage, the problem for the parent in each family is to maximize

$$
\begin{aligned}
U_p &= u_p[Y_p - S - pk^+(p,\ S,\ \bar{D}(Y_p))] \\
&\quad + v_p[(1+r)S - A^+(k^+(p,\ S,\ \bar{D}(Y_p)),\ \bar{D}(Y_p),\ S)] \\
&\quad + \delta\{u_k[\bar{D}(Y_p) - (1-p)k^+(p,\ S,\ \bar{D}(Y_p))] \\
&\quad + v_k[Y_k(k^+(p,\ S,\ \bar{D}(Y_p))) - (1+r)\bar{D}(Y_p) + A^+(k^+(p,\ S,\ \bar{D}(Y_p)),\ \bar{D}(Y_p),\ S)]\}
\end{aligned} \tag{53}
$$

with respect to S and p.

5.2.1. Parental savings

The FOC with respect to S is as follows:

$$
\begin{aligned}
&-u_p' + v_p' \cdot (1+r) + (-v_p' + \delta v_k')A_S^+ \\
&+\{(-u_p' p - v_p' A_k^+) + \delta[-u_k' \cdot (1-p) + v_k' \cdot (Y_k' + A_k^+)]\}k_S^+ = 0.
\end{aligned} \tag{54}
$$

Substituting (14) with equality and (46) into (54) yields

$$
-u_p' + v_p' \cdot (1+r) - (u_p' p + v_p' A_k^+)k_S^+ = 0. \tag{55}
$$

5.2.2. Parental share of education expenditure

In order to derive the parental share of education expenditure in the equilibrium, we examine the form of (53) in the pU_p-plain. Differentiating (53) with respect to p yields

$$
\begin{aligned}
\frac{\partial U_p}{\partial p} &= -u_p' \cdot [k^+(p,\ S,\ \bar{D}(Y_p)) + pk_p^+] - v_p' A_k^+ k_p^+ \\
&\quad + \delta\{u_k' \cdot [k^+(p,\ S,\ \bar{D}(Y_p)) - (1-p)k_p^+] \\
&\quad + v_k' \cdot [Y_k'(k)k_p^+ + A_k^+ k_p^+]\}.
\end{aligned} \tag{56}
$$

Substituting (46) into (56) yields

$$
\frac{\partial U_p}{\partial p} = (-u_p' + \delta u_k')k - (u_p' p + v_p' A_k^+)k_p^+. \tag{57}
$$

Using (55), we can rewrite (57) as

(58)
$$\frac{\partial U_p}{\partial p} = (-u'_p + \delta u'_k)k - (-u'_p + v'_p \cdot (1+r))\left(\frac{k_p^+}{k_S^+}\right)$$
$$= \frac{v'_p}{(1+pk_S^+)(1-p)}\{[-(1-p)(1+r) + (1-\eta)Y'_k + (p-\eta)Y'_k k_S^+]k$$
$$+ (1-p)[\eta Y'_k - p(1+r)]k_p^+\}.$$

We now examine the sign of $\partial U_p / \partial p$ when $p = 0$. From (58), we have[7]

(59)
$$\left(\frac{\partial U_p}{\partial p}\right)_{p=0} = v'_p \cdot \{-(kk_S^+ - k_p^+ + k)\eta Y'_k + [Y'_k - (1+r)]k\} > 0.$$

We define k_0 as k when $p = 0$. Noting that $Y'_k(k_0) > 1+r$ (Proposition 6), we have the following two cases:

(60)
$$(1-\eta)Y'_k(k_0) < 1+r < Y'_k(k_0),$$

(61)
$$1+r < (1-\eta)Y'_k(k_0) < Y'_k(k_0).$$

Since differentiating (46) with $p = 0$ with respect to k and Y_p yields

(62)
$$\frac{\partial k_0}{\partial Y_p} = \frac{u''_k + v''_k \cdot (1+r)(1-\eta)^2 Y'_k}{u''_k + v''_k \cdot (1-\eta)^2 (Y'_k)^2 + v' \cdot (1-\eta) Y''_k} \bar{D}'(Y_p) > 0,$$

we have

(63)
$$\frac{\partial Y'_k(k_0)}{\partial Y_p} < 0, \quad \frac{\partial (1-\eta)Y'_k(k_0)}{\partial Y_p} < 0.$$

Therefore, (60) holds for families with a relatively higher Y_p and (61) holds for families with a relatively lower Y_p.

First, we consider families with (60) satisfied. The changes in $Y'_k(k(p))$ and $(1-\eta)Y'_k(k(p))/(1-p)$ when p rises are given by

(64)
$$\frac{\partial Y'_k(k(p))}{\partial p} = Y_k''(k)\frac{\partial k^+}{\partial p} < 0,$$

(65)
$$\frac{\partial}{\partial p}\left(\frac{(1-\eta)Y'_k(k(p))}{(1-p)}\right) = \frac{(1-\eta)Y''_k(k)k_p^+}{1-p} + \frac{(1-\eta)Y'_k(k)}{(1-p)^2}$$
$$= \frac{(1-\eta)}{(1-p)^2 SOC(k)}\left[u''_k \cdot (1-p)^2 Y'_k (1-\sigma) + v''_k \cdot (1-\eta)^2 (Y'_k)^3\right] > 0.$$

---

[7] The derivation of (59) is shown in the Appendix.

The sign of (65) depends on the assumption that $\sigma \equiv -kY_k''/Y_k' < 1$.[8] Under (64) and (65), as p increases from zero, both $Y_k'(k(p))$ and $(1-\eta)Y_k'(k(p))/(1-p)$ approach $1+r$. Noting (63), we can thus distinguish three categories of families in this case, according to Y_p:

(i) $(1-\eta)Y_k'(k(p))/(1-p) < 1+r$ holds for p that satisfies $Y_k'(k(p)) = 1+r$,

(ii) $(1-\eta)Y_k'(k(p))/(1-p) = 1+r$ holds for p that satisfies $Y_k'(k(p)) = 1+r$,

(iii) $(1-\eta)Y_k'(k(p))/(1-p) > 1+r$ holds for p that satisfies $Y_k'(k(p)) = 1+r$.

Defining $\tilde{Y}_p$ as Y_p of families categorized into (ii), (63) implies that (i) corresponds to families with $Y_p > \tilde{Y}_p$ and (iii) corresponds to families with $Y_p < \tilde{Y}_p$.

We examine the determination of p in each category in turn. The following lemma is useful for this.

Lemma 2.

1. If $Y_k'(k) = 1+r$ and $\dfrac{(1-\eta)Y_k'(k)}{1-p} \geq (\leq) 1+r$, then $\dfrac{\partial U_p}{\partial p} \leq (\geq) 0$,

2. If $\dfrac{(1-\eta)Y_k'(k)}{1-p} = 1+r$ and $Y_k'(k) \geq (\leq) 1+r$, then $\dfrac{\partial U_p}{\partial p} \geq (\leq) 0$.

Proof: See Appendix

(i) $Y_p > \tilde{Y}_p$

Figure 2 illustrates the parental utility function (53) in the pU_p-plain. When $p = 0$, we have $\partial U_p/\partial p > 0$ from (59). Raising p from zero, we attain p_1, which represents p satisfying $Y_k'(k) = 1+r$. When $p = p_1$, $(1-\eta)Y_k'(k)/(1-p) < 1+r$ holds, and Lemma 2.1 implies that $\partial U_p/\partial p > 0$. Raising p further from p_1, we reach to p_2, which represents p satisfying $(1-\eta)Y_k'(k)/(1-p) = 1+r$. When $p = p_2$, $Y_k'(k) < 1+r$ holds, and Lemma 2.2

---

[8] When $Y_k(k)$ takes the Cobb-Douglas functional form, we have $\sigma < 1$. That is, given $Y_k = Bk^{\alpha}$ $(B > 0,\ 0 < \alpha < 1)$, we have $\sigma = -Y_k''k/Y_k' = 1-\alpha < 1$.

implies that $\partial U_p / \partial p < 0$. Therefore, under an assumption that $\partial^2 U_p / \partial p^2 < 0$, the equilibrium solution p^* must be located between p_1 and p_2, and thus $(1-\eta)Y_k'(k)/(1-p) < 1+r$ and $Y_k'(k) < 1+r$ simultaneously hold for p^*. From Proposition 5, this implies the emergence of Samaritan's dilemma (the child over-consumes in period 1), with excessive amounts being invested in education in the equilibrium.

The intuition behind this result is as follows. Under the efficient level of education investment that satisfies $Y_k'(k) = 1+r$, since the Samaritan's dilemma arises, the parent increases p in order to induce to her child to pursue higher education. This is because the education investment reallocates resources forwards, and thus reduces distortions caused by the Samaritan's dilemma.

(ii) $Y_p = \tilde{Y}_p$

When $p = 0$, we have $\partial U_p / \partial p > 0$ from (59). Raising p from zero, we attain p^*, where $(1-\eta)Y_k'(k(p))/(1-p) = 1+r$ and $Y_k'(k(p)) = 1+r$ are simultaneously satisfied. Therefore, both the intertemporal consumption allocation and the education investment are efficient. In addition, since $p = \eta$ holds, the parental first-best is achieved in the equilibrium.[9]

(iii) $Y_p < \tilde{Y}_p$

When $p = 0$, we have $\partial U_p / \partial p > 0$ from (59). Raising p from zero, we attain a level that satisfies $(1-\eta)Y_k'(k)/(1-p) = 1+r$. At this level of p, $Y_k'(k) > 1+r$ holds and Lemma 2.2 implies that $\partial U_p / \partial p > 0$. Raising p further, we reach a level that satisfies $(1-\eta)Y_k'(k)/(1-p) > 1+r$ and $Y_k'(k) = 1+r$. At this level of p, Lemma 2.1 implies that $\partial U_p / \partial p < 0$. Under

[9] From (55), we obtain $-u_p' + v_p' \cdot (1+r) = [v_p' / 1 + pk_S^+][-\eta Y_k' + p(1+r)]k_S^+$. Substituting $p = \eta$, $Y_k' = 1+r$ and $(1-\eta)Y_k'/(1-p) = 1+r$ into the above equation and (51) yields the first-best conditions.

an assumption that $\partial^2 U_p / \partial p^2 < 0$, therefore, $(1-\eta)Y_k'(k)/(1-p) > 1+r$ and $Y_k'(k) > 1+r$ simultaneously hold at the equilibrium solution p^*. From Proposition 5, this implies that the child under-consumes in period 1, and that the education investment is insufficient in the equilibrium.

In contrast to category (i), under the efficient level of education investment, the child of families in this category under-consumes in the first period because she is more highly liquidity constrained than the child in families with $Y_p > \tilde{Y}_p$. The parent, therefore, decreases p in order to induce the child to invest less in education and consume more in the first period.

As for the families with (61) being satisfied, their Y_p is lower than that of families in category (iii), and the property of equilibrium in (iii) can be applied to them.

The following proposition summarizes the above analysis:

Proposition 7.

1. The Samaritan's dilemma and the over-investment in education simultaneously arise for families with $Y_p > \tilde{Y}_p$.
2. The parental first best is achieved for families with $Y_p = \tilde{Y}_p$.
3. The insufficiency of the filial consumption in period 1 and the under-investment in education simultaneously arise for families with $Y_p < \tilde{Y}_p$.

In contrast to the case of non-binding liquidity constraint (Proposition 4), Proposition 7 suggests that, in the case of binding liquidity constraint, the property of equilibrium is different from that in Bruce and Waldman (1990). While either the intertemporal allocation of consumption or the level of filial action is efficient from the family perspective in Bruce and Waldman (1990), neither of them is efficient in this case (except for families with $Y_p = \tilde{Y}_p$).

6. The effects of income on education and welfare

In this section, based on the results obtained in sections 4 and 5, we clarify the differences in education and welfare between families with different incomes.

First of all, under a quite natural assumption, we show that, Y_p of the families with non-binding liquidity constraint is higher than Y_p of the families with binding liquidity constraint. Define $\hat{Y}_p$ as Y_p of families whose FOC with respect to D, (19), is satisfied with equality for $D = \bar{D}(Y_p)$. In other words, the child's most-preferred level of borrowings in a family with $\hat{Y}_p$, which is denoted by $D^{**}(\hat{Y}_p)$, is just equal to the upper limit of how much she can borrow, namely $\bar{D}(\hat{Y}_p)$. Assuming that $d\bar{D}(Y_p)/dY_p > dD^{**}(Y_p)/dY_p$,[10] we find that the liquidity constraint is binding if $Y_p < \hat{Y}_p$, but is not if $Y_p \geq \hat{Y}_p$.

From Propositions 4 and 7, we obtain the following proposition.

Proposition 8.

Whether the investment in a child's education is too much or too little relative to the parental first best depends on the parental income level:

1. k^* is equal to or greater than k^F, if $Y_p \geq \hat{Y}_p$,
2. k^* is greater than k^F, if $\tilde{Y}_p < Y_p < \hat{Y}_p$,
3. k^* is equal to k^F, if $Y_p = \tilde{Y}_p$,
4. k^* is smaller than k^F, if $Y_p < \tilde{Y}_p$.

Furthermore, our model has implications for the incentive problems in the family, especially for the rotten kid theorem (Becker, 1974). To show this, we define the second-best solution for the parent by contrasting the second best to the first best presented in Section 3. That is, we refer to the solution

[10] From (22), (25) and (34), we have

$$\frac{dD^{**}(Y_p)}{dY_p} = \frac{\partial D^{**}}{\partial p^*}\frac{\partial p^*}{\partial Y_p} + \frac{\partial D^{**}}{\partial S^*}\frac{\partial S^*}{\partial Y_p}$$

$$= (1-\rho)\frac{u_p''}{u_p'' + (1+r)^2(1-\eta)[1-\eta(1-\rho)]v_p''}.$$

This is smaller than 1, and likely to be very small. Hence, to assume that $d\bar{D}(Y_p)/dY_p > dD^{**}(Y_p)/dY_p$ may be acceptable.

where the liquidity constraint is binding as the second best.[11] The following proposition is derived as a consequence of Proposition 7.

Proposition 9

There exist families in which the parental welfare is higher in the competitive equilibrium than in the parental second-best solution if the liquidity constraint is binding.

Proof: Proposition 7 implies that, at least for families whose Y_p is $\tilde{Y}_p$ or in the neighborhood of $\tilde{Y}_p$, the utility level of the parent is higher in the competitive equilibrium than in the parental second best.

The intuition behind Proposition 9 is as follows. Lindbeck and Weibull (1988, pp. 1180-81) argue that compulsory savings systems could be welfare-improving when the Samaritan’s dilemma is present. Liquidity constrains affect the intertemporal consumption allocation of an individual in the same way as compulsory savings systems do. In the Samaritan’s dilemma, the child faces the marginal cost of borrowing that is lower than the market interest rate due to the response of ex-post transfers from her parent, and this motivates the child to consume inefficiently large amounts in period 1. However, if the liquidity constraint is binding, it prevents the child from freely allocating consumption between periods 1 and 2, and may contribute to achieve a more efficient consumption allocation. Under the parental second best, the filial consumption in period 1 is insufficient due to the liquidity constraint. Hence, for a class of families in which the distortions caused by the Samaritan’s dilemma cancel out the distortions caused by the liquidity constraint, parental welfare turns out to be higher in the competitive equilibrium than in the parental second best.

The rotten kid theorem asserts that, if altruistic parental transfers are operative, then, even though a child is not altruistic toward her parent, the

[11] The conditions for the parental second best are shown in Appendix.

child chooses the level of her action that is desirable for the parent (i.e., the level the parent would choose if the parent were able to directly control her child's action). Therefore, the parent obtains the same utility level, whether or not she can choose her child's action level. On the other hand, several papers such as those of Hirshleifer (1977) and Bergstrom (1989) point out the limits of generality within which the rotten kid theorem applies, and show that, in more general cases, the parent is placed on a lower utility level when she cannot choose her child's action level than when she can. In contrast, Proposition 9 suggests that, as a result of the child's strategic behavior, the parent may be better off in the competitive equilibrium than in the second-best solution, where the parent chooses her child's education level.

7. Conclusion

Considering pure altruism as the relevant transfer motive, we show that the investment in education can be too much or too little, depending on the income level of the family. In obtaining such a result, the child's strategic behavior in consumption allocation plays a major role. In our model, two types of transfers from the parent to the child are considered: one is the financial contribution of the parent to the child's education cost during the child's youth, and the other is the ex-post transfers, which occur after the child's income has been realized. The latter transfers provide an incentive for the child to consume too much in her youth, thus engendering the Samaritan's dilemma. Whether or not the parent faces the Samaritan's dilemma depends on the parent's income, and, in families facing the dilemma, the parent behaves so as to induce her child to pursue a higher education. This is because the education investment reallocates resources forwards, and counteracts the Samaritan's dilemma.

The results obtained in this paper also have implications for the rotten kid theorem. In some families with binding liquidity constraint, the parental welfare in the equilibrium is higher than that in the parent's second best.

Appendix

Proof of the existence of p_0 such that $p_0 < 1$

First, we show the negative relationship between A and p, which is a necessary condition for the existence of p_0. If $A > 0$, from (15), (22) and (23), we have $A = A^+(k^+(p, S), D^+(p, S), S)$. Differentiating this equation with respect to p and substituting (16)-(18), (24), (25), (28) and (29) into the resulting equation yields

$$\begin{aligned} \frac{dA^+}{dp} &= \frac{\partial A^+}{\partial k}\frac{\partial k^+}{\partial p} + \frac{\partial A^+}{\partial D}\frac{\partial D^+}{\partial p} + \left(\frac{\partial A^+}{\partial k}\frac{\partial k^+}{\partial S} + \frac{\partial A^+}{\partial D}\frac{\partial D^+}{\partial S} + \frac{\partial A^+}{\partial S}\right)\frac{\partial S}{\partial p} \\ &= \eta Y_k'(k)\frac{(1+r)}{Y_k''(k)} + (1+r)\eta\left(-k\rho - \frac{Y_k'(k)}{Y_k''(k)}\right) + [(1+r)\eta(1-\rho) + (1-\eta)(1+r)]\frac{\partial S}{\partial p} \\ &= -(1+r)\eta\rho k + (1+r)(1-\eta\rho)\frac{\partial S}{\partial p}, \end{aligned} \tag{A1}$$

where

$$\begin{aligned} \frac{\partial S}{\partial p} &= \frac{-u_p''(k + pk_p) - v_p''(1+r)^2\eta\rho k(1-\eta(1-\rho))}{u_p'' + v_p''(1+r)^2\eta\rho(1-\eta(1-\rho))} \\ &= -k - \frac{u_p'' pk_p}{u_p'' + v_p''(1+r)^2\eta\rho(1-\eta(1-\rho))} < 0. \end{aligned}$$

From (A1), we have $dA^+/dp < 0$.

Next, we prove $p_0 < 1$. Equation (21) implies that $k \to \infty$ as $p \to 1$. However, since k cannot exceed Y_p, $k = Y_p$ holds when p is larger than a certain value. In such a case, (1) implies that

$$(1-p)Y_p = C_p^1 + S.$$

We thus have $S \to 0$ as $p \to 1$. This implies that

$$\lim_{p\to 1}\{-v_p'[(1+r)S] + \delta v_k'[Y_k(Y_p) - (1+r)D + A]\} < 0, \tag{A2}$$

because $\lim_{C_p^2 \to 0} v_p'(C_p^2) = \infty$ is assumed. On the other hand, from the definition, p_0 satisfies

$$-v_p'[(1+r)S] + \delta v_k'[Y_k(k(p,S)) - (1+r)D(p,S)] = 0. \tag{A3}$$

Differentiating the LHS of (A3) with respect to p yields

$$\text{(A4)} \quad \begin{aligned} \frac{\partial(-v_p' + \delta v_k')_{A=0}}{\partial p} &= -v_p'' \cdot (1+r)\frac{\partial S}{\partial p} + \delta v_k''[Y_k' \frac{\partial k^0}{\partial p} - (1+r)\frac{\partial D^0}{\partial p}] \\ &= -v_p'' \cdot (1+r)\frac{\partial S}{\partial p} < 0. \end{aligned}$$

From (A2)-(A4), we obtain $p_0 < 1$.

Proof of *Lemma 1*

For $p = p_0$, we define the following function with dummy variable θ:

$$\text{(A5)} \quad u_k'(D - (1-p_0)k) - v_k'(Y_k(k) - (1+r)D)\left[(1+r) - \theta A_D^+\right] = 0,$$

$$\text{(A6)} \quad -(1-p_0)u_k'(D - (1-p_0)k) + v_k'(Y_k(k) - (1+r)D)\left[Y_k'(k) + \theta A_k^+\right] = 0,$$

$$\text{(A7)} \quad u_p'(Y_p - S - p_0 k) - v_p'((1+r)S)\left[(1+r) - \theta A_D^+ D_S^+\right] = 0.$$

Equations (A5) and (A6) imply

$$\text{(A8)} \quad Y_k'(k) - (1+r)(1-p_0) = 0.$$

From (A5)-(A7) (or (A5), (A7) and (A8)), we obtain $(D(\theta), k(\theta), S(\theta))$. From (A8), k is independent of θ. In the equilibrium, D and S satisfy (A5) and (A7) with $\theta = 0$ when $\partial A / \partial D = 0$, whereas they satisfy (A5) and (A7) with $\theta = 1$ when $\partial A / \partial D = \partial A^+ / \partial D$.

Differentiating (A5), (A7) and (A8) yields

$$\text{(A9)} \quad \frac{dk}{d\theta}(\equiv k'(\theta)) = 0,$$

$$\text{(A10)} \quad \frac{dD}{d\theta}(\equiv D'(\theta)) = \frac{-(1+r)\eta v_k'}{u_k'' + (1+r)^2(1-\theta\eta)v_k''} > 0,$$

$$\text{(A11)} \quad \frac{dS}{d\theta}(\equiv S'(\theta)) = \frac{(1+r)(1-\rho)\eta v_p'}{u_p'' + (1+r)^2[1-\theta\eta(1-\rho)]v_p''} < 0.$$

Noting that $A = 0$ when $p = p_0$, the parent's utility function is given by

$$\text{(A12)} \quad \begin{aligned} \left(U_p\right)_{p=p_0} &= u_p[Y_p - S(\theta) - p_0 k(\theta)] + v_p[(1+r)S(\theta)] \\ &+ \delta\{u_k[D(\theta) - (1-p_0)k(\theta)] + v_k[Y_k(k(\theta) - (1+r)D(\theta)]\}. \end{aligned}$$

Differentiating (A12) with respect to θ yields

$$\left(\frac{\partial U_p}{\partial \theta}\right)_{p=p_0} = p_0 k'(\theta) + [-u'_p + (1+r)v'_p]S'(\theta)$$

(A13) $$+\delta\{\left[u'_k - v'_k \cdot (1+r)\right]D'(\theta) + \left[-u'_k \cdot (1-p_0) + v'_k \cdot \left(Y'_k(k)\right)\right]k'(\theta)\}.$$

From (A5)-(A7), (A9)-(A11) and (14), we have

(A14) $$\left(\frac{\partial U_p}{\partial \theta}\right)_{p=p_0} = [D_S^+ S'(\theta)v'_p - D'(\theta)v'_k]\theta A_D^+ < 0.$$

When θ moves from $\theta = 1$ to $\theta = 0$, the parent's utility increases.

Derivation of sign of (59)

Substituting (46), (48) and (49) into $kk_S^+ - k_p^+ + k$ yields

(A15) $$kk_S^+ - k_p^+ + k = \frac{1}{[SOC(k)]_{p=0}}\{v'_k \cdot (1-\eta)Y'_k(1-\sigma) + kv''_k \cdot (1-\eta)^2 Y'_k[Y'_k - (1+r)]\},$$

where $[SOC(k)]_{p=0} = u''_k + v''_k \cdot (1-\eta)^2 (Y'_k)^2 + v'_k \cdot (1-\eta)Y''_k < 0$ and $\sigma = -Y''_k k / Y'_k$.

Substituting (A15) into (59) yields

$$\left(\frac{\partial U_p}{\partial p}\right)_{p=0} = v'_p \cdot \{-(kk_{S_p}^+ - k_{\hat{p}}^+ + k)\eta Y'_k + [Y'_k - (1+r)]k\}$$

$$= \frac{v'_p}{[SOC(k)]_{p=0}}\{-[v'_k \cdot (1-\eta)Y'_k(1-\sigma) + kv''_k \cdot (1-\eta)^2 Y'_k(Y'_k - (1+r))]\eta Y'_k$$

$$+[SOC(k)]_{p=0}[Y'_k - (1+r)]k\}$$

$$= \frac{v'_p}{[SOC(k)]_{p=0}}\{-[v'_k \cdot (1-\eta)Y'_k(1-\sigma)] + [kv''_k \cdot (1-\eta)^3 (Y'_k)^2 (Y'_k - (1+r))]\eta Y'_k$$

$$+[u''_k + v'_k \cdot (1-\eta)Y''_k][Y'_k - (1+r)]k\}.$$

Since $\sigma < 1$ is assumed and $Y'_k > 1+r$ holds for $p = 0$ (Proposition 6), $[SOC(k)]_{p=0} < 0$ implies

(A16) $$\left(\frac{\partial U_p}{\partial p}\right)_{p=0} > 0.$$

Proof of *Lemma 2*

1. Substituting $Y'_k = 1+r$ into (58) yields

(A17)
$$\left(\frac{\partial U_p}{\partial p}\right)_{Y_k'=1+r} = \frac{v_p' \cdot (1+r)(p-\eta)}{(1+pk_S^+)(1-p)}[(1+k_S^+)k-(1-p)k_p^+] = \frac{v_p' \cdot (1+r)(p-\eta)}{(1+pk_S^+)(1-p)} \cdot \frac{v_k' \cdot (1-\eta)Y_k'(1-\sigma)}{SOC(k)}.$$

From (A17), noting that $\sigma < 1$ is assumed and $1+pk_S^+ > 0$ holds when $Y_k' = 1+r$, the sign of $(\partial U_p / \partial p)_{Y_k'=1+r}$ depends on the sign of $p-\eta$. Since $(1-\eta)Y_k'(k)/(1-p) \geq (\leq) 1+r$ is equivalent to $p-\eta \geq (\leq) 0$ when $Y_k'(k) = 1+r$, $SOC(k) < 0$ implies that, if $(1-\eta)Y_k'(k)/(1-p) \geq (\leq) 1+r$,

(A18)
$$\left(\frac{\partial U_p}{\partial p}\right)_{Y_k'=1+r} \leq (\geq) 0.$$

2. Substituting $(1-\eta)Y_k'/(1-p) = 1+r$ into (58) yields

(A19)
$$\left(\frac{\partial U_p}{\partial p}\right)_{\frac{(1-\eta)Y_k'}{1-p}=1+r} = \frac{v_p' \cdot (1+r)(p-\eta)}{(1+pk_S^+)(1-\eta)}(kk_S^+ - k_p^+).$$

From (A19), assuming $1+pk_S^+ > 0$,[12] the sign of $(\partial U_p / \partial p)_{(1-\eta)Y_k'(k)/(1-p)=1+r}$ depends on the sign of $p-\eta$ because $kk_S^+ - k_p^+ < 0$. Since $Y_k'(k) \geq (\leq) 1+r$ is equivalent to $p-\eta \leq (\geq) 0$ when $(1-\eta)Y_k'(k)/(1-p) = 1+r$, we have that, if $Y_k'(k) \geq (\leq) 1+r$,

(A20)
$$\left(\frac{\partial U_p}{\partial p}\right)_{\frac{(1-\eta)Y_k'}{1-p}=1+r} \geq (\leq) 0.$$

Second-best conditions for the parent

Assuming that the liquidity constraint $D \leq \bar{D}$ is binding, we consider the

[12] If $Y_k' \geq 1+r$, then we have $1+pk_S^+ > 0$, but, if $Y_k' < 1+r$, then we do not necessarily have $1+pk_S^+ > 0$. In this case, however, if $\eta \leq 2-(1+r)/Y_k'$ (namely, if the difference between Y_k' and $1+r$ is small enough), then we have $1+pk_S^+ > 0$.

parent's problem in which she chooses $\left\{C_p^1,\ C_p^2,\ C_k^1,\ C_k^2,\ k\right\}$ so as to maximize her utility subject to $0 \le p \le 1$ as well as the budget constraints **(1)-(4)**. We can write the Lagrangian as:

$$L(S,\ A,\ k,\ p,\ \bar{D},\ \lambda) = u_p(Y_p - S - pk) + v_p[(1+r)S - A] \\ \delta\{u_k[\bar{D} - (1-p)k] + v_k[Y_k(k) - (1+r)\bar{D} + A]\} + \lambda(1-p). \tag{A21}$$

The corresponding FOCs are

$$\frac{\partial L}{\partial S} = -u_p'(C_p^1) + (1+r)v_p'(C_p^2) = 0\,, \tag{A22}$$

$$\frac{\partial L}{\partial A} = -v_p'(C_p^2) + \delta v_k'(C_k^2) = 0\,, \tag{A23}$$

$$\frac{\partial L}{\partial k} = -pu_p'(C_p^1) - (1-p)\delta u_k'(C_k^1) + \delta v_k'(C_k^2)Y_k'(k) = 0\,, \tag{A24}$$

$$\frac{\partial L}{\partial p} = k[-u_p'(C_p^1) + \delta u_k'(C_k^1)] - \lambda \le 0, \tag{A25}$$

$$p\frac{\partial L}{\partial p} = p\{k[-u_p'(C_p^1) + \delta u_k'(C_k^1)] - \lambda\} = 0, \tag{A26}$$

$$\frac{\partial L}{\partial \lambda} = 1 - p \ge 0, \qquad \lambda\frac{\partial L}{\partial \lambda} = 0\,. \tag{A27}$$

Equations (A22) and (A23) imply

$$-u_p'(C_p^1) + (1+r)\delta v_k'(C_k^2) = 0\,. \tag{A28}$$

Since we assume that the liquidity constraint is binding, we have

$$u_k'(C_k^1) - (1+r)v_k'(C_k^2) > 0\,. \tag{A29}$$

Equations (A28) and (A29) imply

$$-u_p'(C_p^1) + \delta u_k'(C_k^1) > 0\,. \tag{A30}$$

From (A26), (A27) and (A30), we have

$$p = 1. \tag{A31}$$

Substituting (A28) and (A31) into (A24) yields

$$Y_k'(k) = (1+r)\,. \tag{A32}$$

Equations (A22), (A23), (A31) and (A32) derive the second-best solution.

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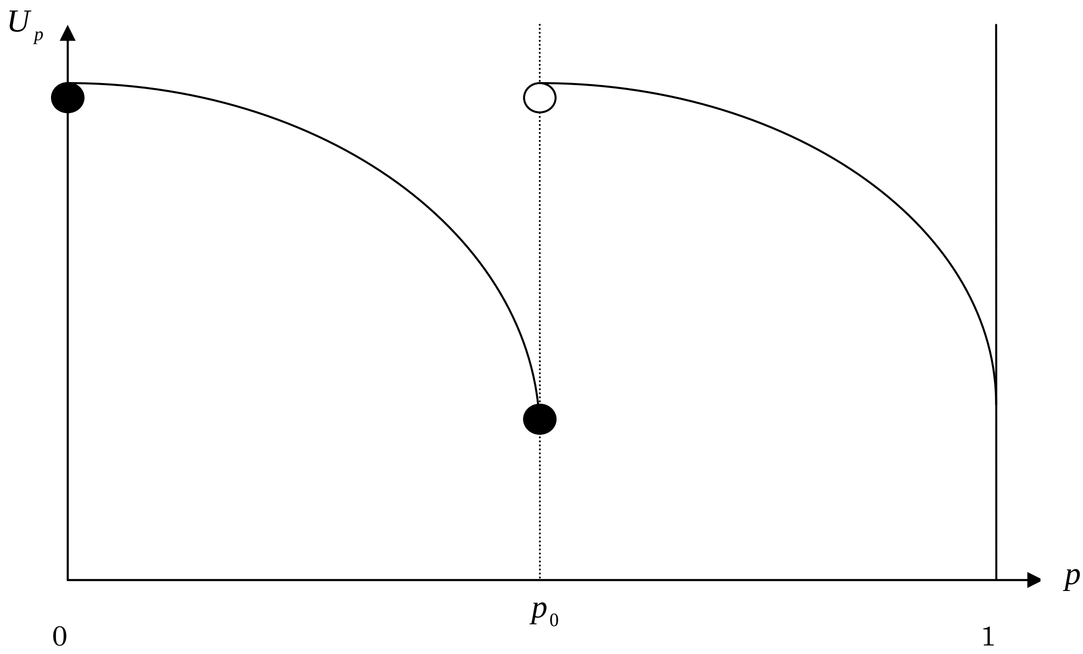

Figure 1

The parental utility function for families with non-binding liquidity constraint

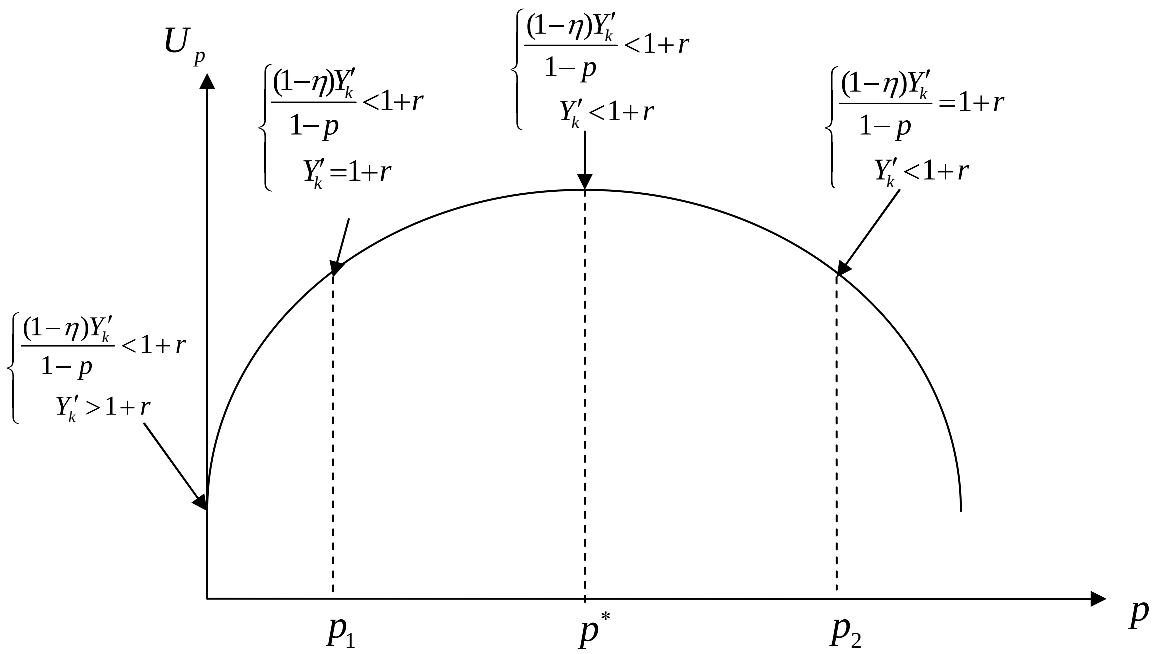

Figure 2

The parental utility function for families with binding liquidity constraint ($Y_p > \tilde{Y}_p$)